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Improved measurements and analysis of the current profile in Tokamak fusion plasmas

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Outline

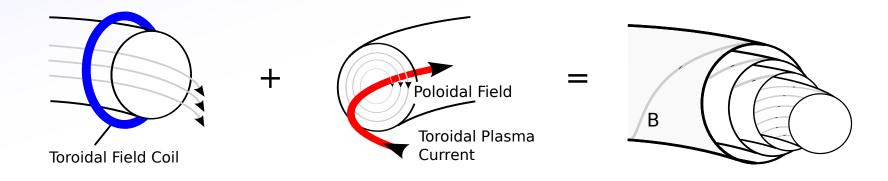
- Introduction
 - Flux surfaces and current profiles
 - Magnetic equilibrium
 - Bayesian analysis
- Bayesian equilibrium
 - Current-tomography
 - Current tomography + Grad-Shafranov
 - L-Mode reconstructions
 - H-Mode results
- Internal measurements
 - Motional Stark effect.
 - Coherence Imaging
 - Imaging MSE
 - Direct j_{ϕ} imaging.
- Integrated Data Analysis
 - Current diffusion
 - Imaging MSE comparison
 - Sawtooth models





Flux Surfaces

- The Tokamak: External toroidal field coils and a large current in the plasma result in a helical magnetic field.

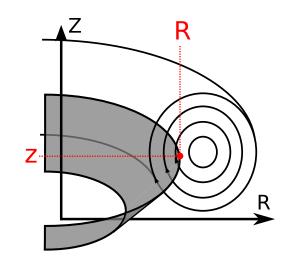


- Field lines form surfaces of constant magnetic flux

Poloidal magnetic flux:

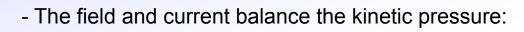
$$\psi(R,Z) = \int_{0,0}^{R,Z} B_{\theta}(\mathbf{r}) d\mathbf{r}$$

- Many plasma quantities are functions of ψ , e.g. n_e , T_e .
- Flux surfaces are the basis of our knowledge and used for:
 - Comparing/combining measurements ('mapping')
 - Basis of 1D transport calculations



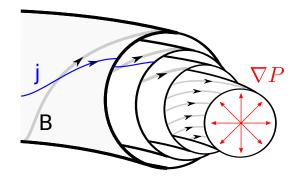


Current and stability



 $j \times B = \nabla P$

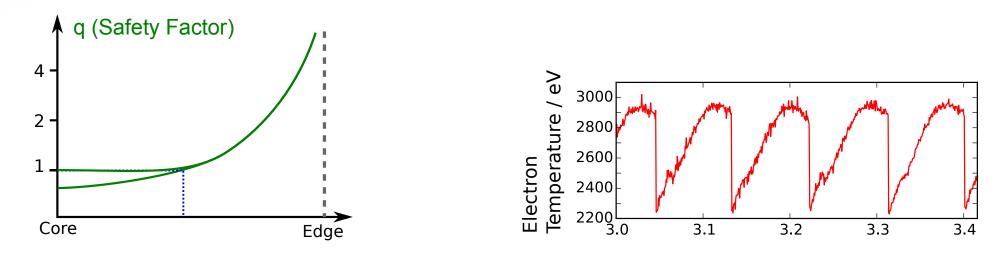
- The current distribution and resulting field are important for the plasma stability:



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e.g.: When the central q value falls below 1.0, the plasma core periodically suddenly expels particles and energy - known as a 'sawtooth' crash. The crash is a magnetic reconnection event, which occurs far more rapidly than explained by simple theoretical models.

...(we'll return to this later)



Magnetic Equilibrium

- How do we know **j** and **B**?

Assume: Axisymmetry + Isotropic pressure + No flow Define the 'poloidal current flux' *f*:

$$f(R,Z) = \int^{R,Z} j_{\theta}(\mathbf{r}) d\mathbf{r} \qquad f = RB_{\phi}$$

Kinetic pressure $p(\psi)$ and $f(\psi)$ are constant on flux surfaces. Decompose the force balance into toroidal and poloidal:

$$j \times B = \nabla p$$

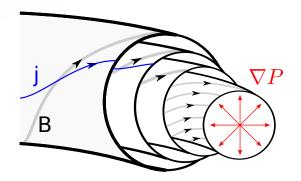
 $j_{\phi} = Rp' + \frac{\mu_0}{R}ff'$

Also known as the Grad-Shafranov equation:

$$-\frac{1}{R}\frac{\partial^2\psi}{\partial Z^2} - \frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right) = \mu_0 R \frac{\partial p}{\partial \psi} + \frac{\mu_0^2}{R} f \frac{\partial f}{\partial \psi}$$

For very simple $p(\psi)$ and $f(\psi)$ functions, one can solve the Grad-Shafranov equation for given boundary ψ .

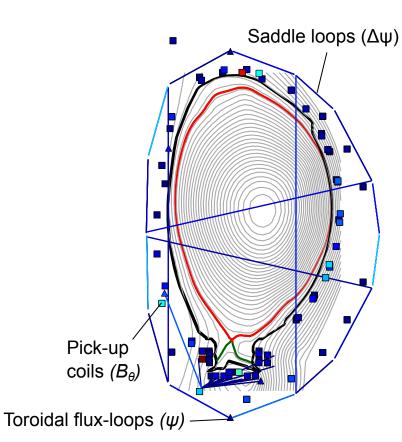
Boundary $\boldsymbol{\psi}$ calculated from magnetic pick-ups around plasma perimeter



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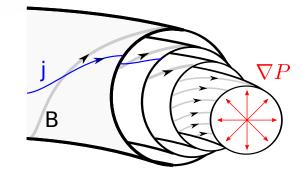
Magnetic Equilibrium

$$-\frac{1}{R}\frac{\partial^2\psi}{\partial Z^2} - \frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right) = \mu_0 R\frac{\partial p}{\partial \psi} + \frac{\mu_0^2}{R}f\frac{\partial f}{\partial \psi}$$

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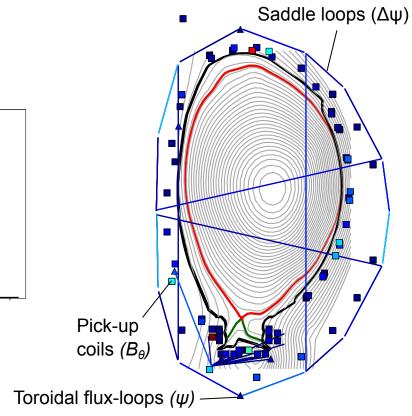
- Usually only converges for simple p, f functions.
- Difficult to deal with pedestal pressure/current.

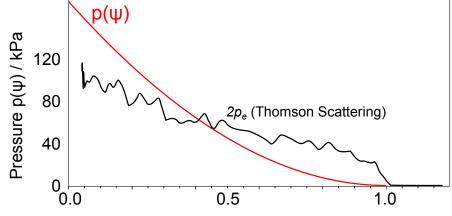


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but....

- Is the converged solution the only solution?

- Are the simplified *p*, *f* profiles over-constrained / under-constrained?

 $_{5\,/\,4\bar{5}}Are$ the data consistent with the assumptions?





Bayesian Inference

- Rigorous framework for dealing with the question:

What can we know about the plasma, given the data we measured?

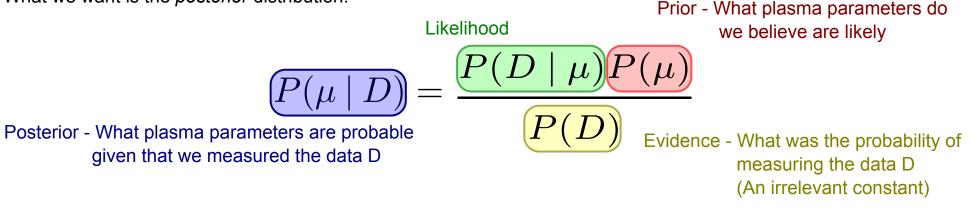
We need:

- $\mu\,$ a set of parameters describing the state of the plasma that we want to know.
- D a set of measured data.
- $P(D \mid \mu)$ The likelihood distribution: A model of what data might be measured given a certain set of plasma parameters.

Typically, a 'forward model' that gives the most likely data $\langle D \rangle = f(\mu)$ and a simple Gaussian distribution of uncertainty from measurement noise:

$$P(D \mid \mu) \propto exp \left[-(D - f(\mu))^2 / 2\sigma^2 \right]$$

What we want is the *posterior* distribution:



 $P(D \mid \mu) = \prod_{i} P(D_i \mid \mu)$

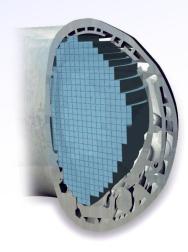
Any combination of diagnostics: 6/45

(More explanation and examples available for Q&A)



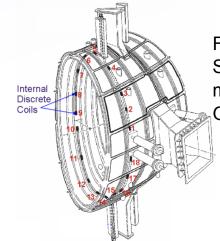
Current Tomography

- How do we apply this to current distribution?



R [m]

Physics Model: grid of axisymmetric current beams.



Forward Model / Likelihood: Simple prediction of magnetic diagnostics with Gaussian likelihood function.

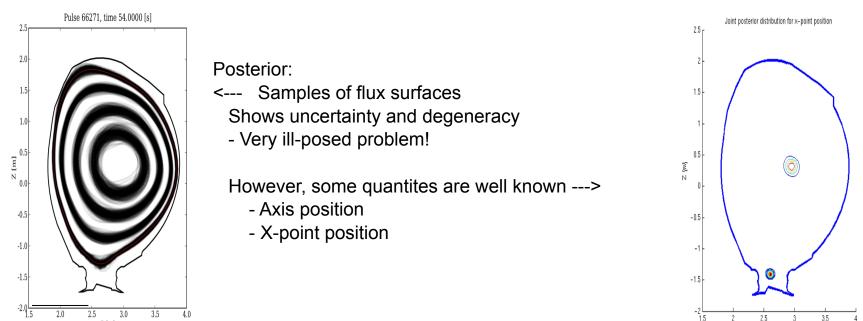
R [m]

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Prior: Simple regularisation of grid - neighboring current must be similar (first attempt)







Current Tomography + Equilibrium

- Can we re-introduce the force balance? --> Bayesian Equilibrium
- Force balance: We observe that the magnetic and pressure forces are approximately equal:

$$j \times B \approx \nabla p$$

 $P(\text{stable} \mid j_{\phi}, f, p) = exp\left(-\sum_{i,k}^{\prime} \left[(j \times B) - \nabla p\right]^2 / 2\sigma^2\right)$ How good should our equilibrium be?

- Now we can ask the question:

$$P(j_{\phi}, f, p \mid D_m, stable)$$

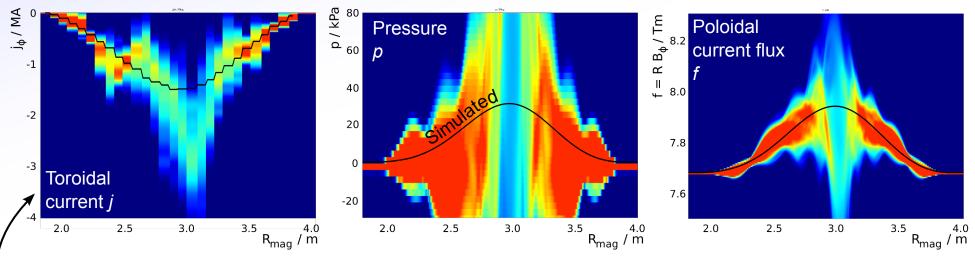
What space of plasma currents and pressures are consistent with the measurements and are close to equilibrium?



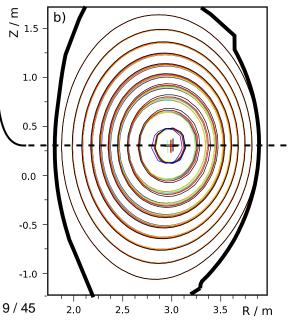


Bayesian Equilibrium

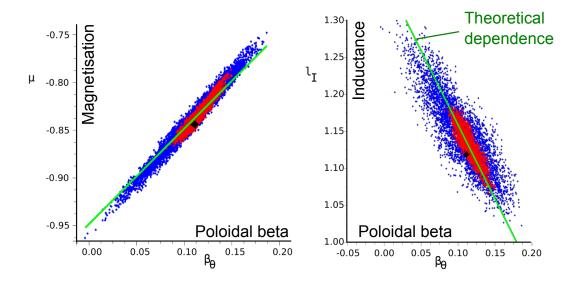
L-Mode plasmas: Low resolution current beam grid, fully explored posterior distribution:



Uncertainty is large in core due to degeneracy: Equilibrium doesn't tell us much Flux surfaces:



Samples of integral quantities reveal relations between 'Shafranov Integrals' that are well determined in simple analytical equilibrium solutions:





 J_{φ}

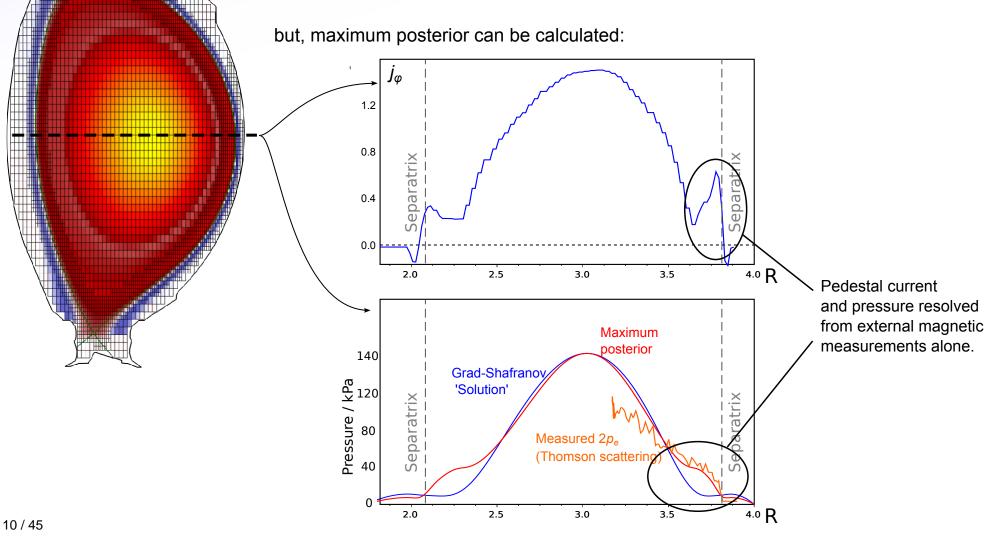


Bayesian Equilibrium

H-Mode plasmas: Very sharp changes in j and p require high-resolution current beam grid:

Too many parameters to explore the posterior (Monte-Carlo algorithm). Needs:

- More computation power
- Better algorithms

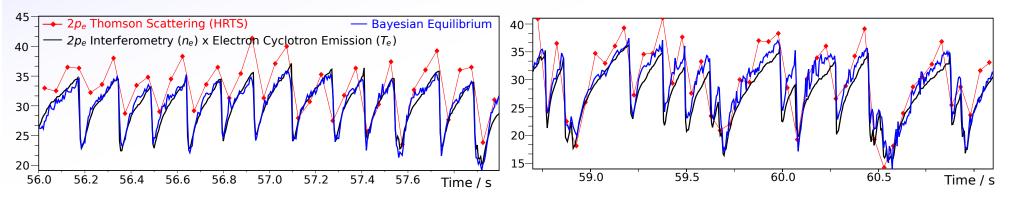




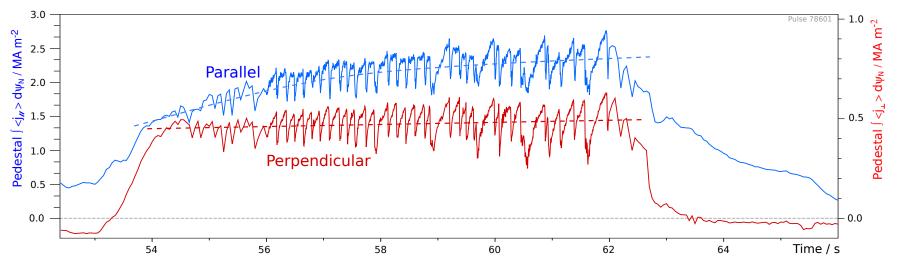


Pedestal Pressure

Flexible p, f profiles show that pedestal pressure can be very accurately measured with magnetic coils. - Matches kinetic measurements almost perfectly.



Pedestal parallel and perpendicular currents can be separated:



- Very good information on edge current, even from magnetics alone!



Max-Planck Institut für Plasmaphysik Greifswald / Garching Fusion Frontiers and Interfaces, 2019 Improved measurements and analysis of the current profile in tokamak Fusion plasmas



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E / 1.5 -N

1.0

0.5

0.0

-0.5

-1.0

2.0

2.5

3.0

- L-Mode reconstructions
- H-Mode results

- Rigorous determination of uncertainty
- Too computationally intensive for H-mode
- Need internal measurements!

R/m

3.5

- Internal measurements
 - Motional Stark effect.
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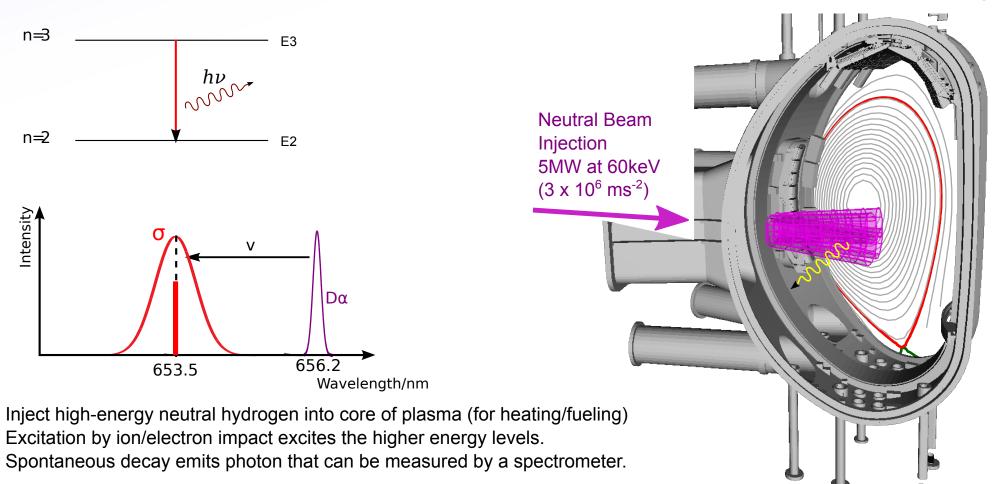
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Internal Measurements

How can we measure deep inside the plasma? Spectroscopy - observe the light emitted by atoms in the plasma:

e.g. Hydrogen Balmer-α line:

Magnetic Surfaces Plasma Edge



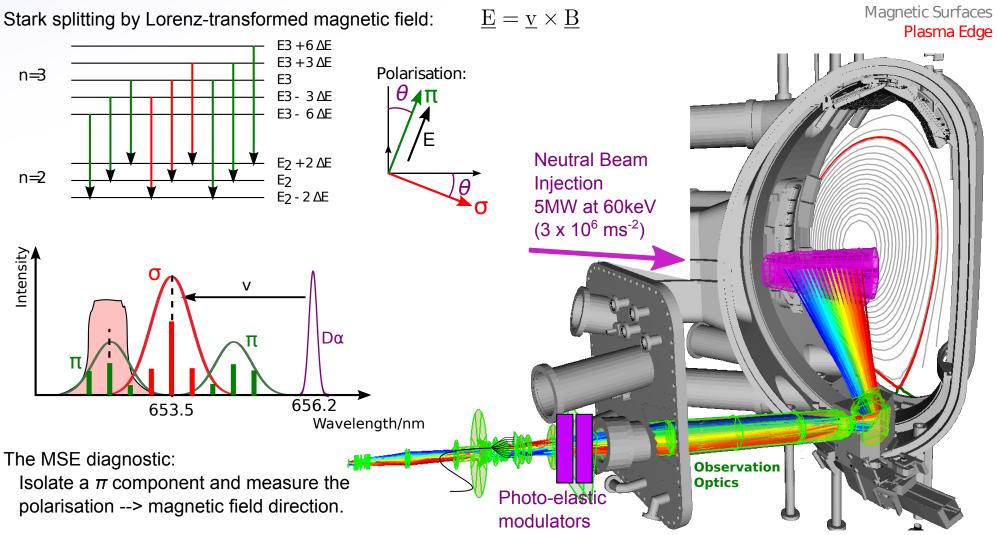




Motional Stark Effect Polarimetry

The atomic energy levels are modified by the local magnetic/electric fields:

- Zeeman splitting (magnetic field)
- Stark splitting (electric field):



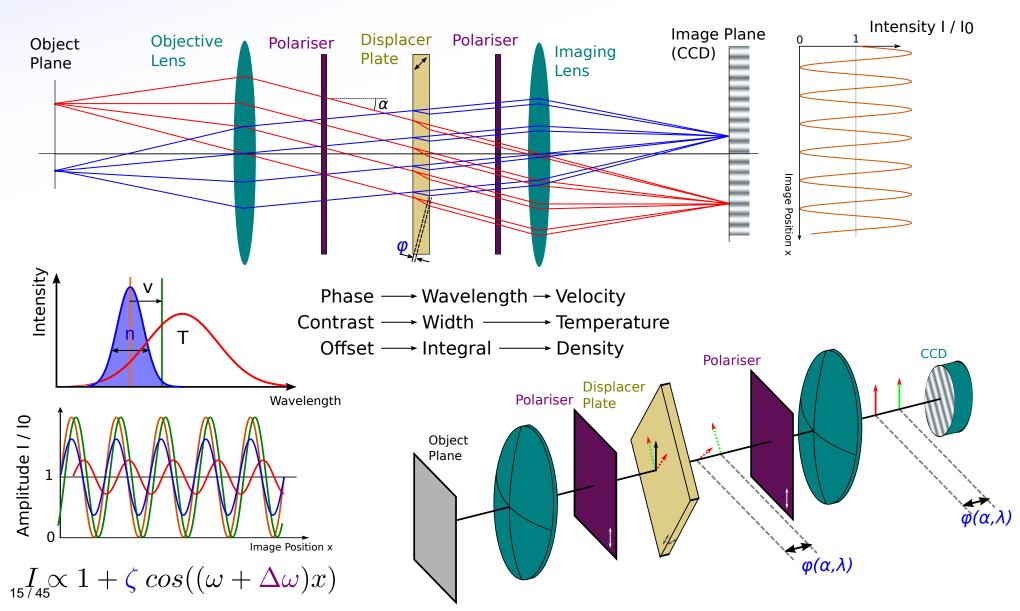
ASDEX Upgrade Vacuum Vessel





Coherence Imaging

Coherence Imaging: Spectroscopic technique, modulated in space and imaged with a CMOS camera. Also used for imaging spectroscopic moments







Multiplet Polarisation Coherence Imaging

Removing the first polariser gives a dependence on the initial polarisation:

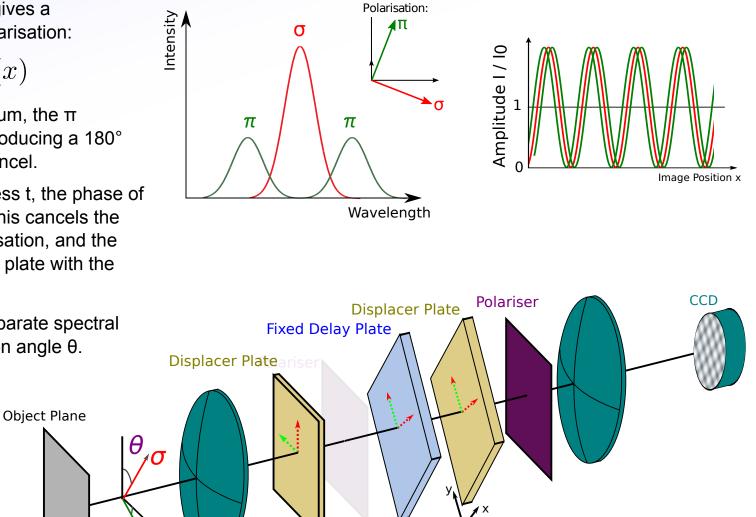
 $I \propto 1 + \zeta \cos 2\theta \cos(x)$

For the Stark/Zeeman spectrum, the π component is at 90° to σ , introducing a 180° phase shift, so they would cancel.

At some specific plate thickness t, the phase of the π wings is 180° from σ . This cancels the 180° from the opposite polarisation, and the patterns add. We add a delay plate with the optimal τ_0 .

However, we now need to separate spectral contrast ζ from the polariastion angle θ .

add another displacer at 45°. Combined effect adds 2 extra terms:



 $_{\rm 16/45} I \propto 1 + \zeta \cos 2\theta \cos(x) + \zeta \sin 2\theta \cos(x-y) - \zeta \sin 2\theta \cos(x+y)$

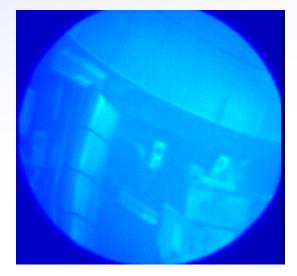
π



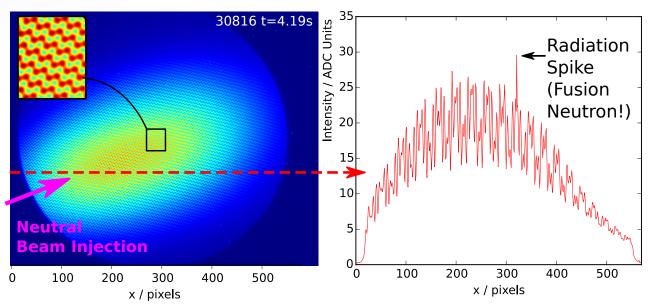


Imaging Motional Stark Effect results

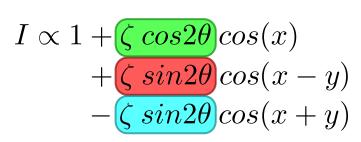
Raw image (without neutral beam)

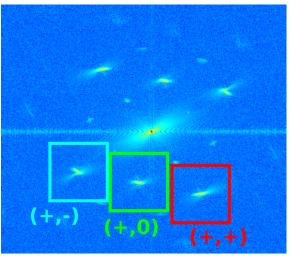


Raw image (with neutral beam)

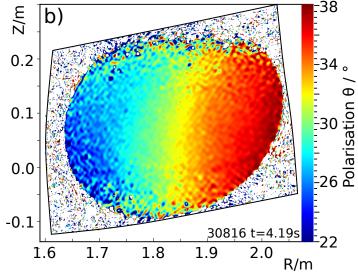


Fourier transform





Demodulated polarisation angle

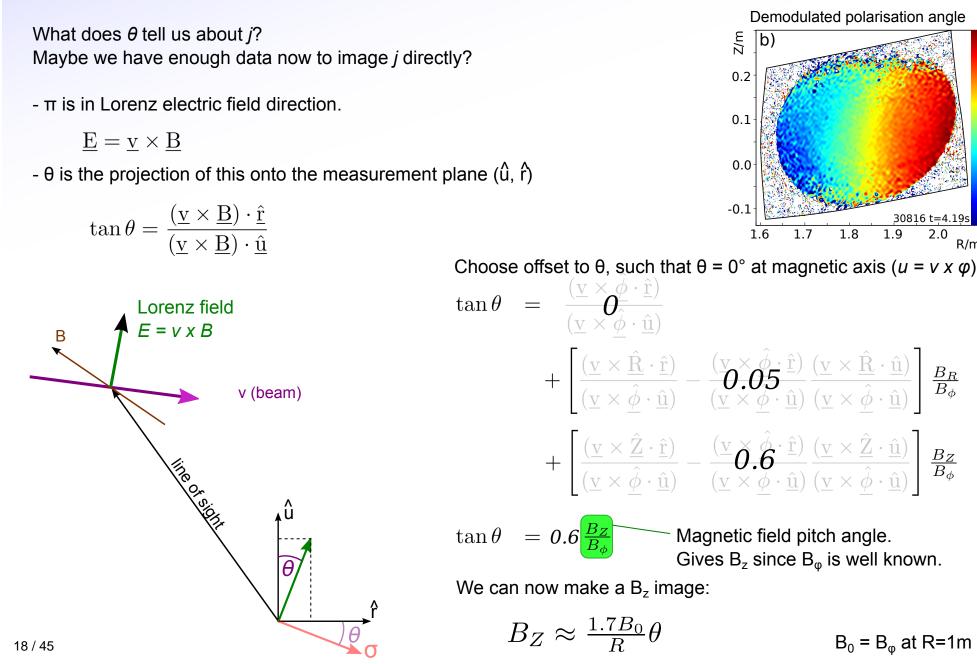






50 Polarisation

Imaging Motional Stark Effect results

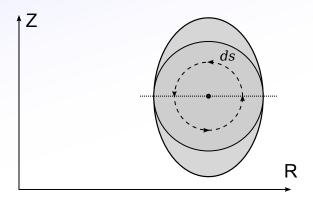




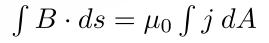


Imaging Motional Stark Effect results

What does B_z tell us about *j* in the core? Large aspect ratio approximation (assume core is a cylinder)



More generally elongation is important: [CC.Petty Nucl. Fus. 2002]



$$B = \frac{1}{2}\mu_0 jr$$

$$\frac{dB_Z}{dR} = \frac{1}{2}\mu_0 j$$

$$\mu_0 j \approx -\left(1 + \frac{1}{\kappa^2}\right) \frac{dB_Z}{dR}$$

 $\frac{dB_Z}{dR} = \frac{1.7B_0}{R} \left(\frac{d\theta}{dR} - \frac{\theta}{R}\right)$

$$\begin{array}{c} \underbrace{\mathsf{E}}_{\mathsf{N}} & \mathbf{b} \\ 0.2 \\ 0.1 \\ 0.0 \\ -0.1 \\ 1.6 \\ 1.7 \\ 1.8 \\ 1.9 \\ 2.0 \\ \mathsf{R/m} \end{array}$$

Demodulated polarisation angle

To first order, local *j* relates to local derivative of measurement

This is only an approximation! ... but we now understand that $d\theta/dR$ holds the information about *j*. What can we see in $d\theta/dR$ at the axis?

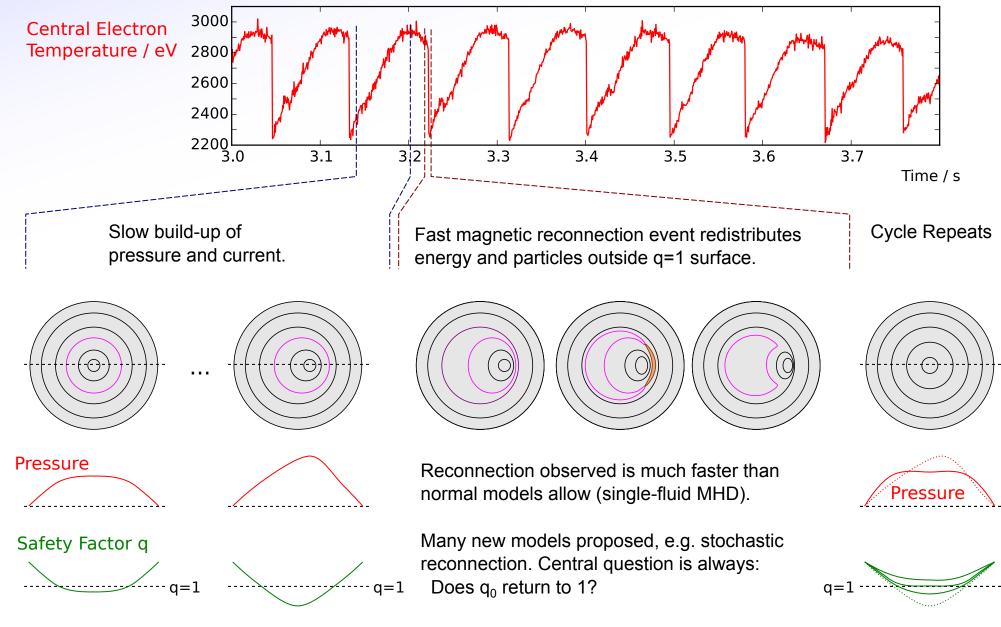
Central safety factor also requires location of centre:

$$q_0 \approx \frac{2B_\phi}{\mu_0 j_0 R} \approx \frac{2B_\phi^{1m}}{\mu_0 j_0 R_0^2}$$





Sawteeth - Magnetic Reconnection







PEMs-based

MSE

2.7

2.8

time / s

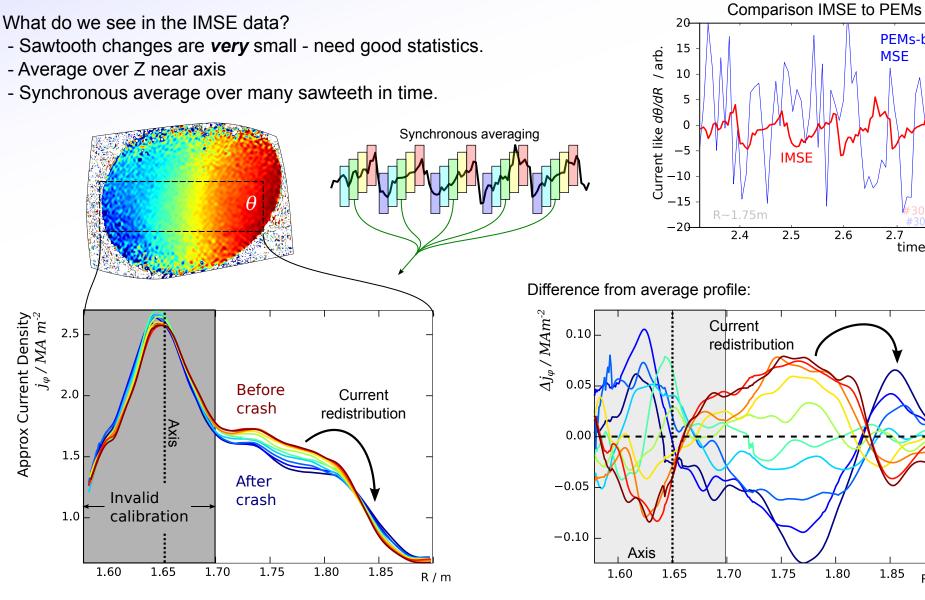
2.6

1.80

1.85

R/m

Sawteeth - Magnetic Reconnection



Current redistribution: $\Delta i \sim 0.050 \text{ MA m}^{-2}$ $_{21/4}$ easurements every ~3cm (resolution):

 $\Delta(d\theta/dR) \sim 0.7^{\circ}m^{-1}$ --> $\Delta\theta \pm 0.02^{\circ}$ required for $\Delta R=3cm$





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- Rigorous determination of uncertainty
- Too computationally intensive for H-mode
- Need internal measurements!

- -Excellent internal measurements.
- Good dynamics from very approximate derivation of Δj_{arphi}
- Calibration very difficult to required accuracy.
- Need to include in equilibrium





Integrated Data Analysis - Equilibrium

New approach to equilibrium at ASDEX Upgrade:

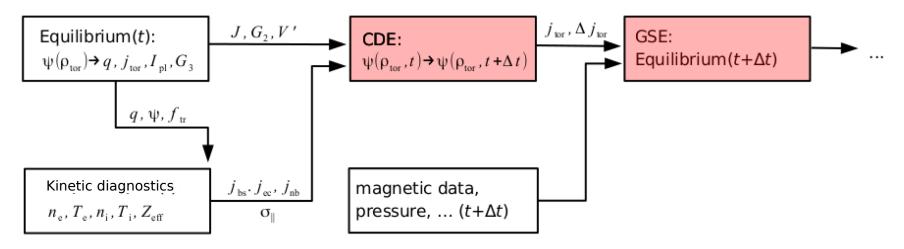
- Grad-Shafranov solver, but with rigorous treatment of errors
- Try to mitigate effect of nonphysical regularisation with as much realistic information as possible:
- Pressure constraints: n_e , T_e , T_i , Z_{eff} , fast-ions (from modeling)
- Geometric information (Inboard/outboard agreement of diagnostics)
- Current diffusion:
- Modeled current 'sources': ECCD, bootstrap, NBI.

Current Diffusion Equation (CDE):

$$\sigma_{\parallel} \frac{\partial \psi}{\partial t} = \frac{R_0 J^2}{\mu_0 \rho} \frac{\partial}{\partial \rho} \left(\frac{G_2}{J} \frac{\partial \psi}{\partial \rho} \right) - \frac{V'}{2\pi\rho} (j_{\rm bs} + j_{\rm cd})$$
Bootstrap current
Current drive (ECCD, NBI etc)

Provides a weak constraint on j_{φ} from expected evolution from previous time-points.

- i.e. physically realistic (and informative) prior information.







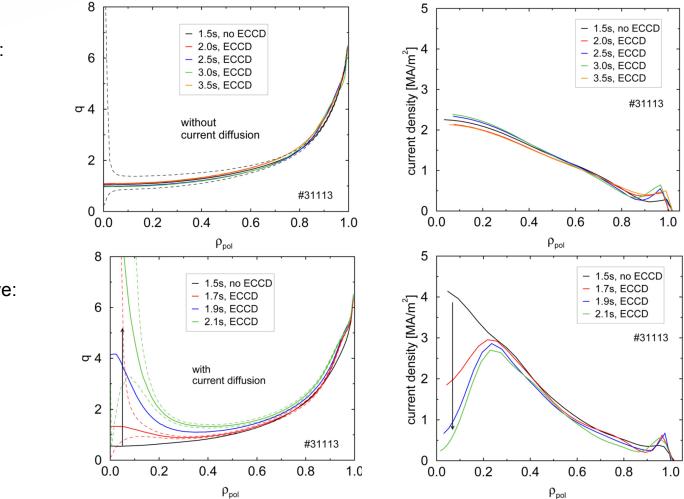
Integrated Data Analysis Equilibrium

Example: Counter-current Electron Cyclotron Current Drive (ECCD)

ECCD drives localised on-axis current

- Not seen by magnetics (small due to low area of centre)
- No effect on pressure profile = not seen by kinetic inputs

Regularised GS solution:



With CDE + Current Drive:

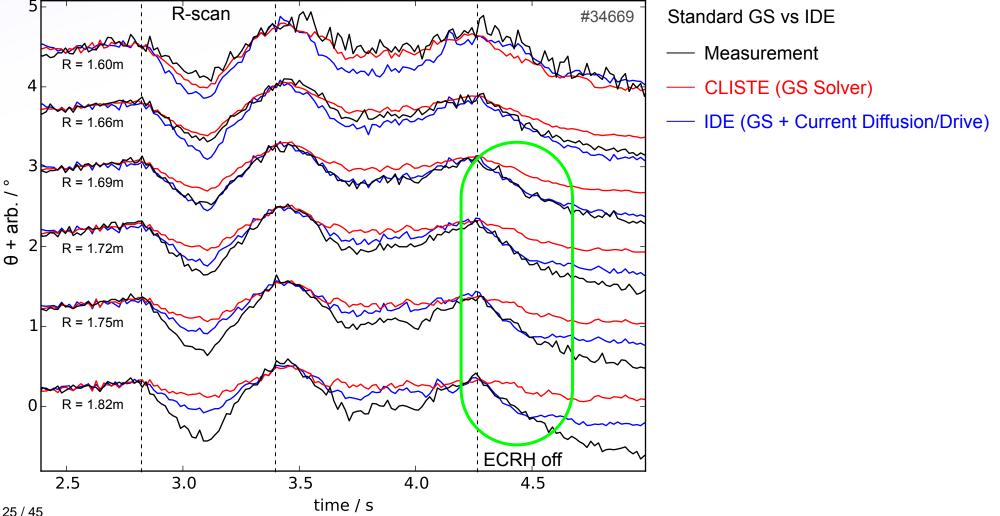




Integrated Equilibrium vs IMSE

- By comparing by IMSE, can see where IDE predicts more physics than the 'standard' GS solver:
- 1) During R-scan
- 2) ECRH switch-off

However, there is still physics only seen by diagnostic!

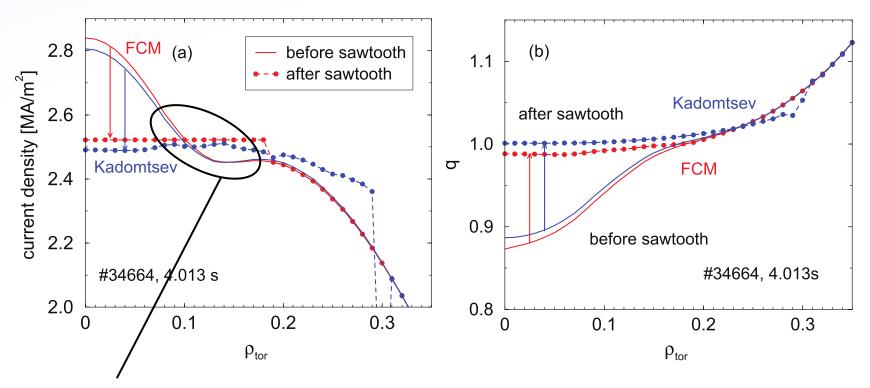






Integrated Equilibrium - Sawteeth

- During sawteeth (reconnection), current diffusion not applicable.
- Include different sawtooth models in equilibrium code and compare IMSE predictions to measurements.
- Kadomtsev: Complete reconnection. $q_0 \rightarrow 1$. Current outside q=1 surface.
- Flat-current model (FCM): Current conserved outside q=1, flat current density inside.



Current redistribution similar to seen in Δj_{φ} images.

Difference between models requires absolute $j_{\varphi} \sim 0.02 \text{ MA } m^{-2} \rightarrow d\theta/dR \sim 0.01^{\circ} (3 \text{ cm}^{-3})$



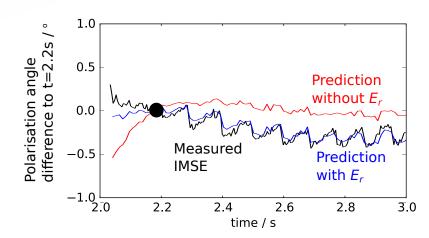
Integrated Equilibrium vs IMSE - Sawteeth

Required precision is so high, many other factors become important:

Plasma radial electric field:

 $E = v \times B + E_r$

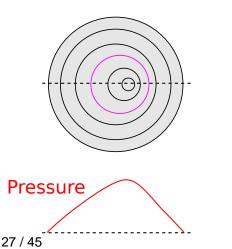
At some locations, ΔE_r during sawtooth dominates measurement:

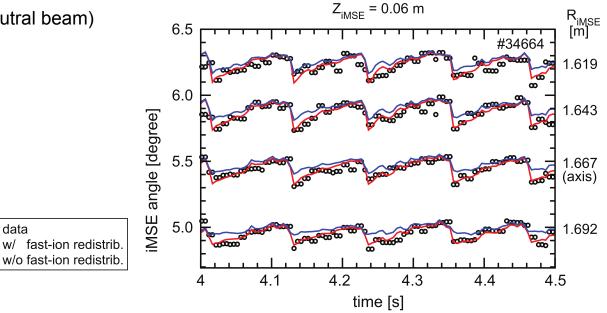


Shafranov shift:

Movement of plasma axis with pressure. (including redistribution of fast-ions from neutral beam)

> data 0







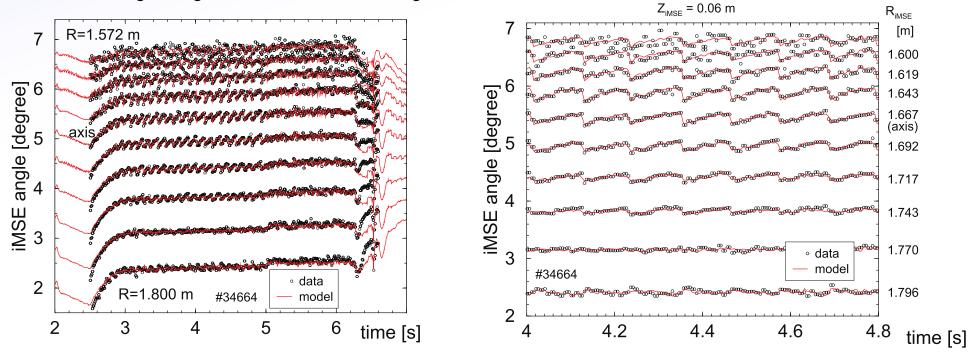


Integrated Equilibrium vs IMSE - Sawteeth

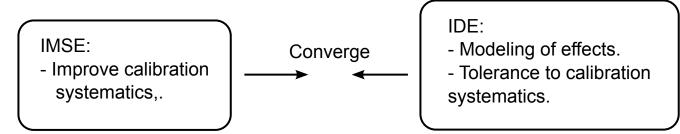
Required precision is so high, many other factors become important:

but...

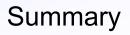
we now have good agreement between full integrated model and IMSE measurements for sawtooth evolution in θ .



- This is where we are - 'the state of the art ... science' What next?







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- Good dynamics from approximate derivation of $\Delta j \phi$

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- Calibration very difficult to required accuracy.
- Need to include in equilibrium
- Excellent tool for practical analysis with available data.
- Current diffusion provides realistic model of missing information when data incomplete.
- Sawtooth models in good agreement with IMSE evolution.
- Still need to converge IDE+IMSE to arrive at an absolute q.



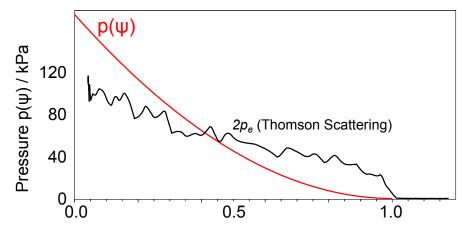


Magnetic Equilibrium

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Typical equilibrium code (Picard iterations

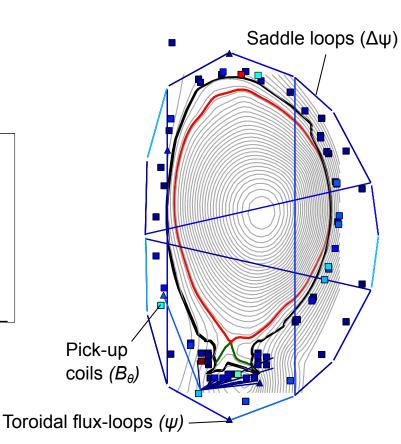
- Start with guess of $\psi_0(R,Z)$ and $p(\psi)$, $f(\psi)$
- Iterate:
 - Calculate new ψ_{i+1} from $p(\psi_i)$ and $f(\psi_i)$ through GS equation.
 - Optimise $p(\psi)$, $f(\psi)$ to best match ψ_{i+1} to magnetic measurements.
 - Repeat with new ψ_{i+1}
- This (and similar) schemes assumes validity of GS equation and attempts to find a consistent plasma $\psi_i = ?= \psi_{i+1}$.
- Usually only converges for simple *p*, *f* functions.
- Difficult to deal with pedestal pressure/current.



but....

- Is the converged solution the only solution?

- Are the simplified *p*, *f* profiles over-constrained / under-constrained? $_{30/45}$ Are the data consistent with the assumptions?







Bayesian Inference

A simple example with electron density:

Physics model:

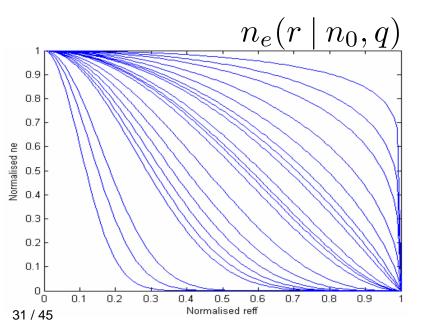
$$n_e(r) = n_0(1 - r^2)^q$$

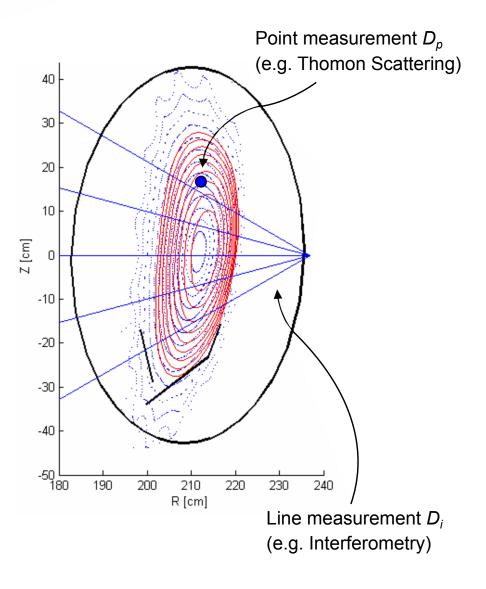
Parameters:

$$\mu = (n_0, q)$$

Forward model:

 $f(\mu) = \int n_e dl$

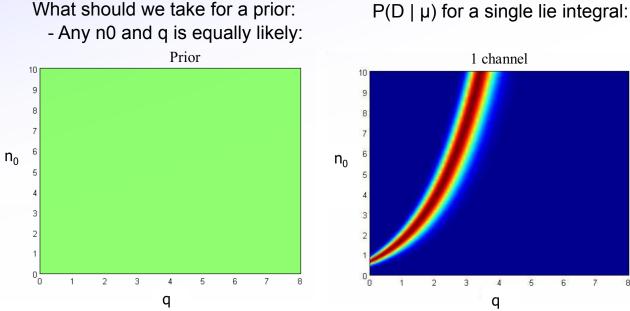




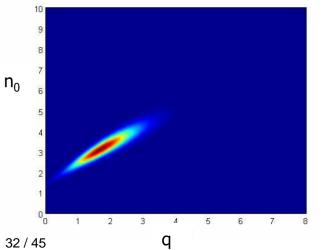


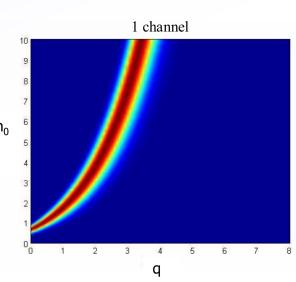
Bayesian Inference

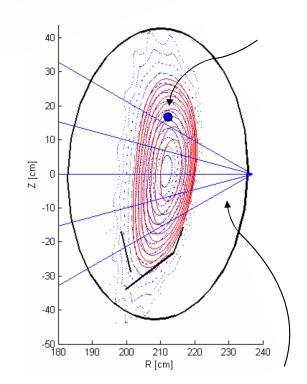
(C) EUROfusion ASDEX Upgrade



 $P(D \mid \mu)$ for 5 line integrals:

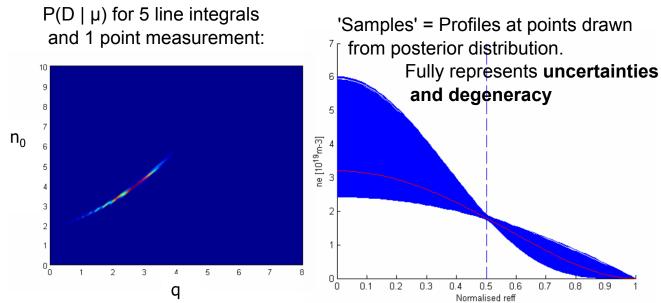






0.8

0.9



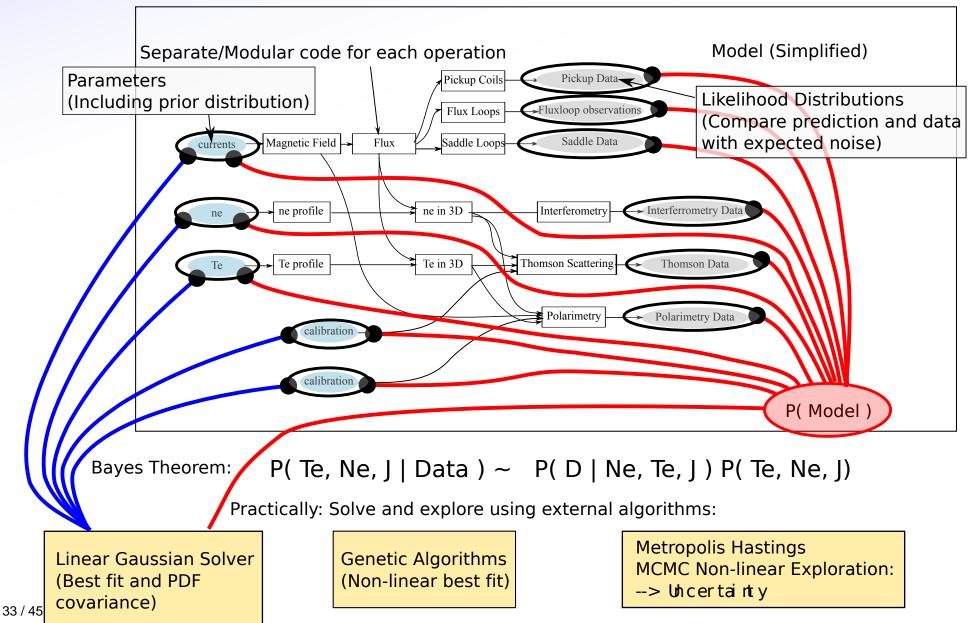
32 / 45





Forward modelling and Bayesian Inference

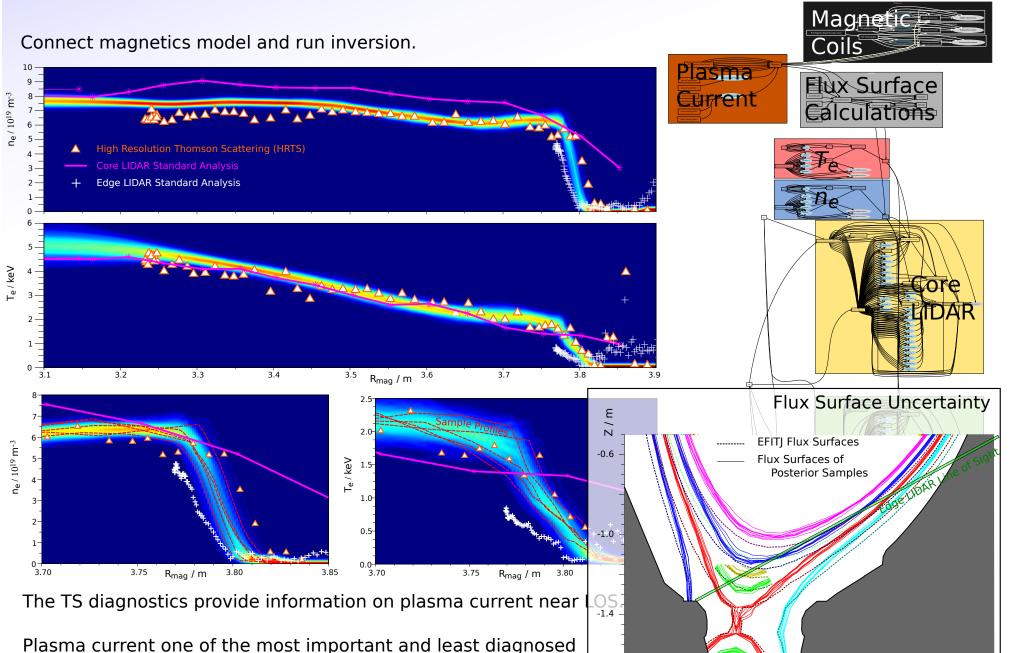
Minerva framework for Bayesian combined modelling:







^{3.6} R/m



2.0

2.4

2.8

3.2

₃patameters in Tokamaks.



EUROfusion

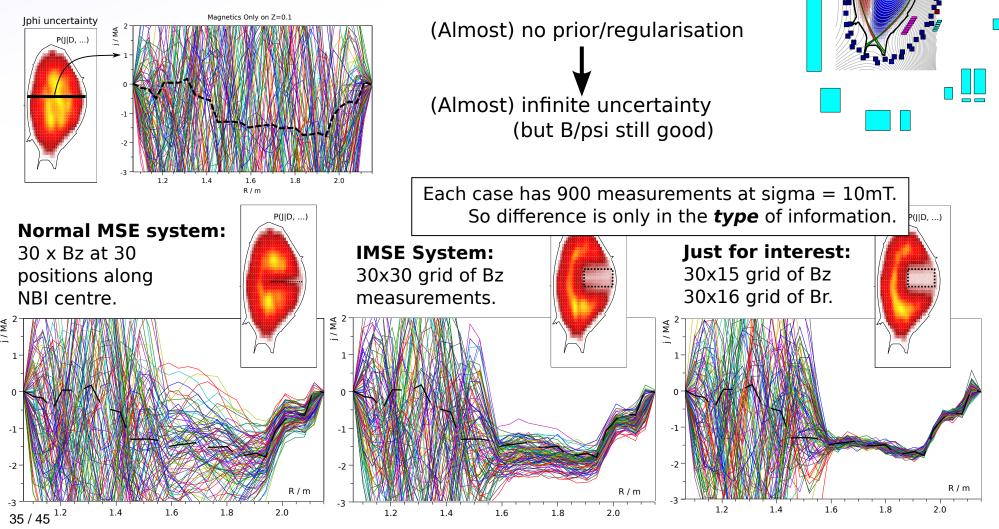
ASDEX

Upgrade

IMSE + Current Tomography

Put description of AUG coils and some pickups into Minerva so we can now do Current Tomorgraphy and Bayesian Equilibrium for AUG.

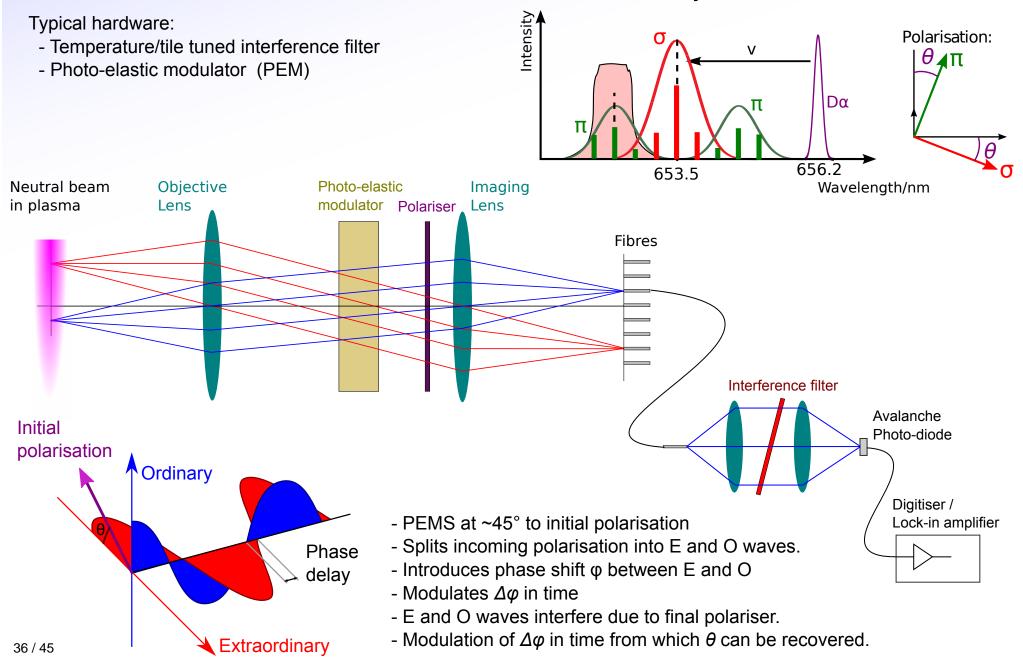
For magnetics only, we have the usual tomography situation:







Motional Stark Effect Polarimetry



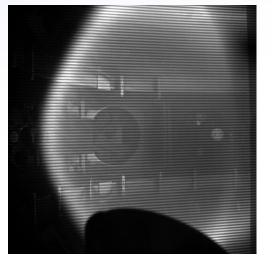


EUROfusion ASDEX

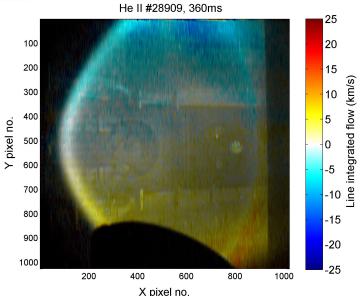
Coherence Imaging

Some results of neutral Helium flow in the (relatively) cold edge of MAST:

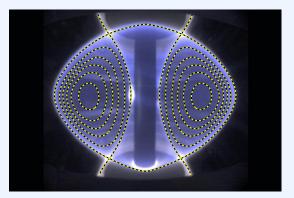
Raw Image:



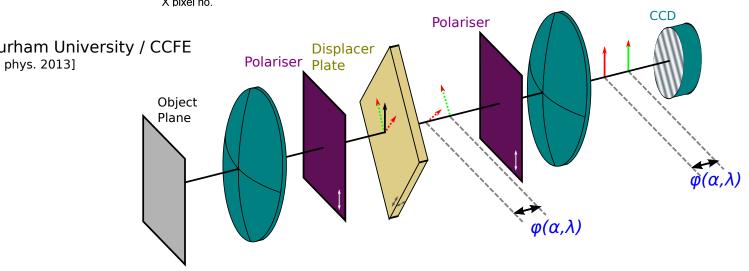
Helium Flow Velocity:



MAST Mega Amp Spherical Tokamak, CCFE, Culham, UK



MAST is a 'spherical' Tokamak. The torus has a very small major radius compared to it's minor radius, but is still a Tokamak.



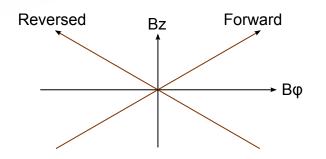
*With thanks to Scott Silburn, Durham University / CCFE [S. Silburn et. al. 40th EPS Conf. on plasma phys. 2013]



IMSE - Calibration

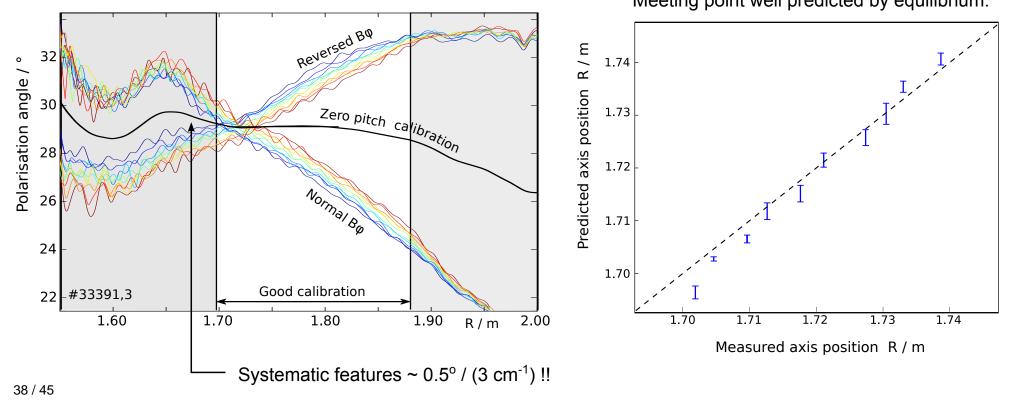
Absolute q_0 requires absolute $d\theta/dR$ How can we calibrate θ , (or $d\theta/dR$)?

- Run the same plasma with reversed field --> Reversed pitch angle
- Also scan axis position to confirm meeting point (magnetic axis) agrees with 0 pitch angle.



Plasma moved by 5cm to scan axis position.

Meeting point well predicted by equilibrium:

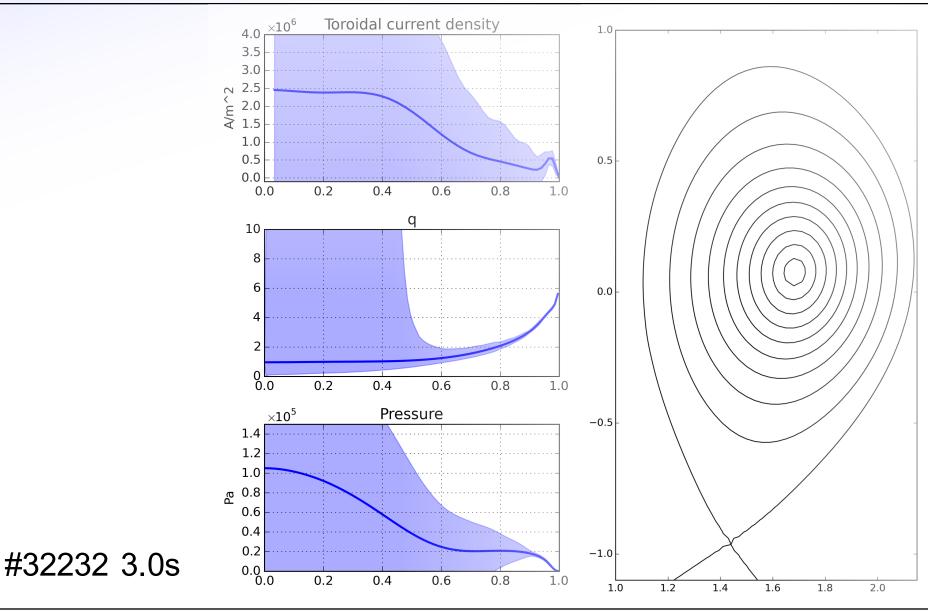


EUROfusion ASDEX Upgrade





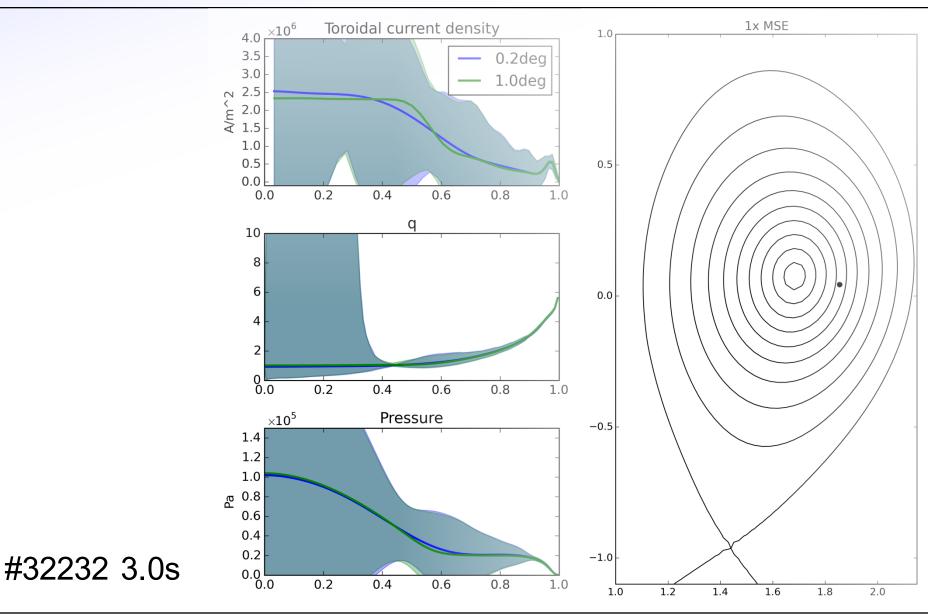
Uncertainties in j/q without internal measurements







Uncertainties in j/q 1 MSE LOS

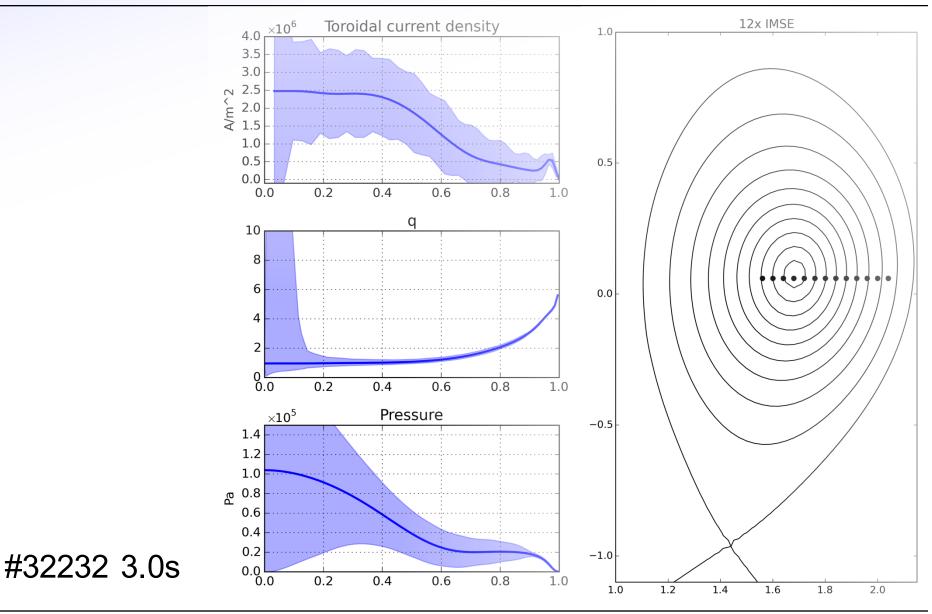


40/45A. Bock





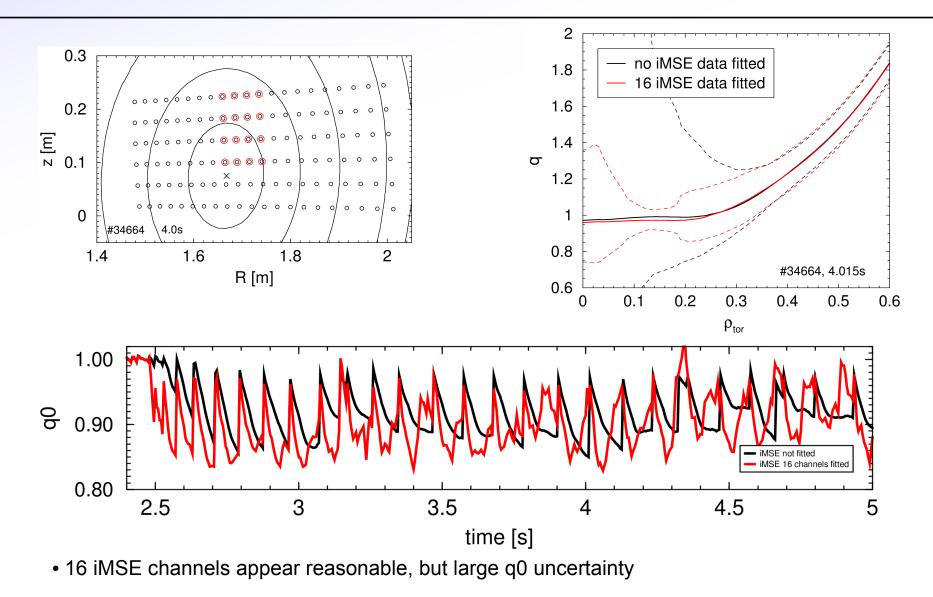
Uncertainties in j/q 12 IMSE LOS







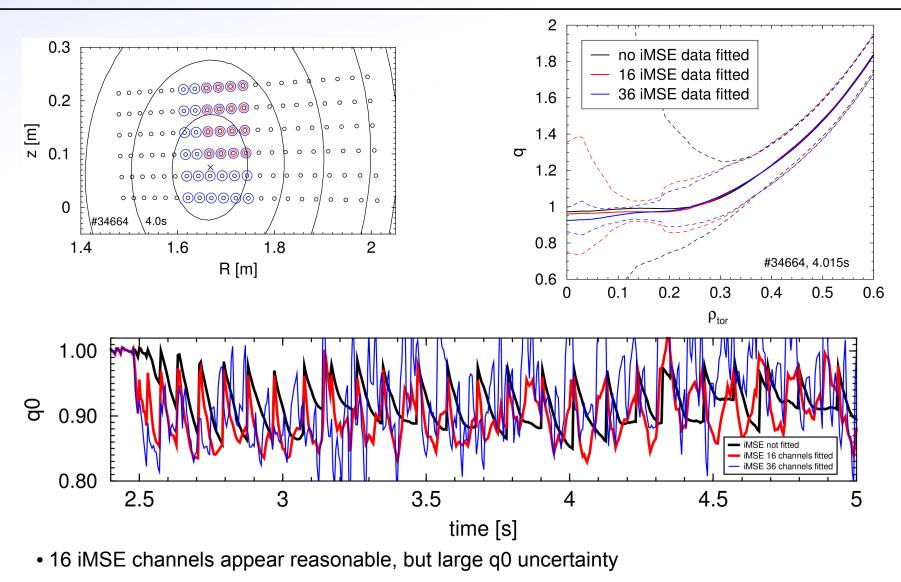
Status: iMSE fit (2)







Status: iMSE fit (3)



- 36 iMSE channels not conclusive
- 43/45 Note: q0 estimation is the most challenging problem in current profile reconstruction!



Max-Planck Institut für Plasmaphysik Greifswald / Garching Fusion Frontiers and Interfaces, 2019 Improved measurements and analysis of the current profile in tokamak Fusion plasmas



Title

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