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Improved measurements and analysis of the current profile in Tokamak fusion plasmas

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Outline

- Introduction
  - Flux surfaces and current profiles
  - Magnetic equilibrium
  - Bayesian analysis

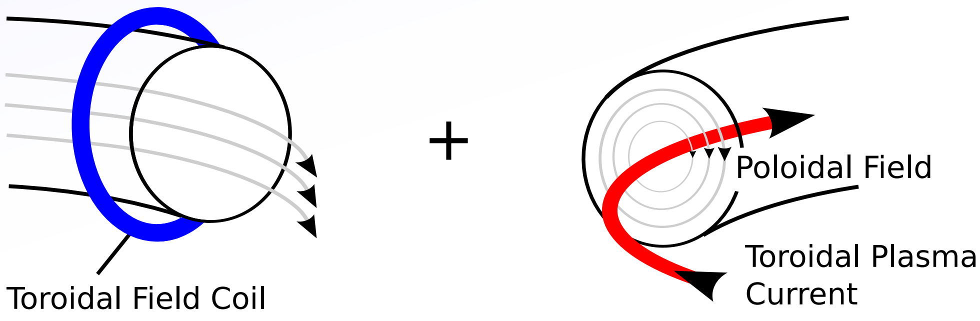
- Bayesian equilibrium
  - Current-tomography
  - Current tomography + Grad-Shafranov
  - L-Mode reconstructions
  - H-Mode results

- Internal measurements
  - Motional Stark effect.
  - Coherence Imaging
  - Imaging MSE
  - Direct j_{φ} imaging.

- Integrated Data Analysis
  - Current diffusion
  - Imaging MSE comparison
  - Sawtooth models

Flux Surfaces

- The Tokamak: External toroidal field coils and a large current in the plasma result in a helical magnetic field.



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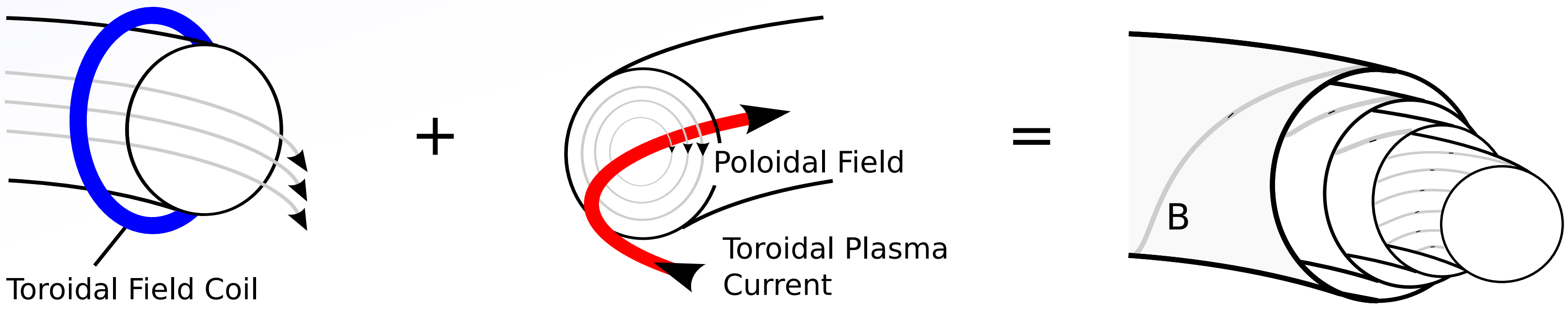

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  - Comparing/combining measurements ('mapping')
  - Basis of 1D transport calculations

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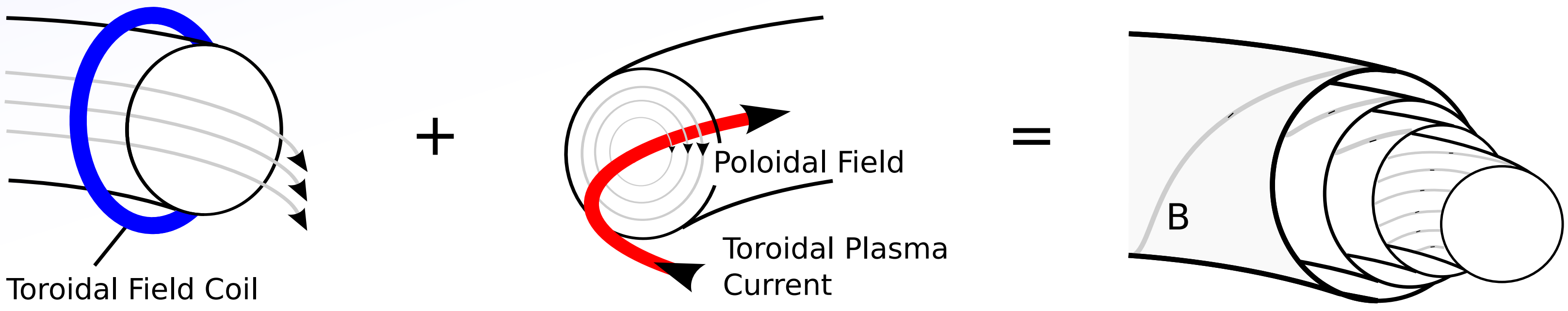

- Field lines form surfaces of constant magnetic flux

Poloidal magnetic flux:

$$\psi(R,Z) = \int_{0,0}^{R,Z} B_\theta(\mathbf{r})d\mathbf{r}$$

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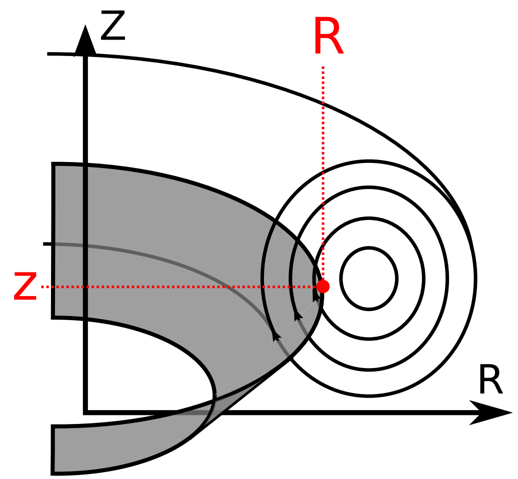
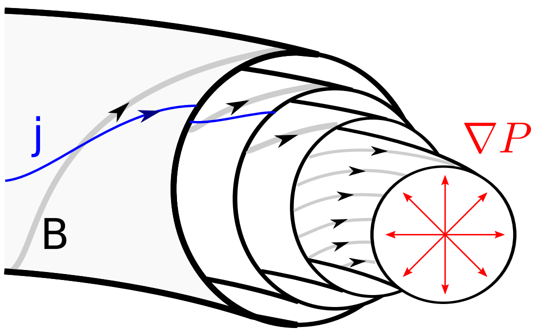


Current and stability

- The field and current balance the kinetic pressure:

$$j \times B = \nabla P$$



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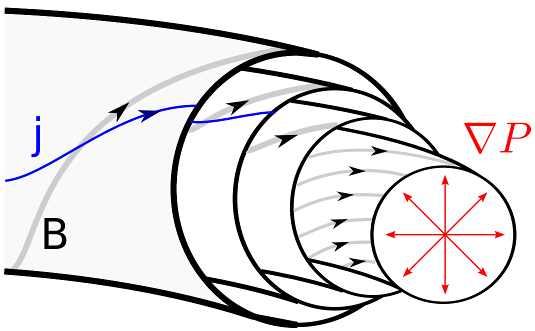

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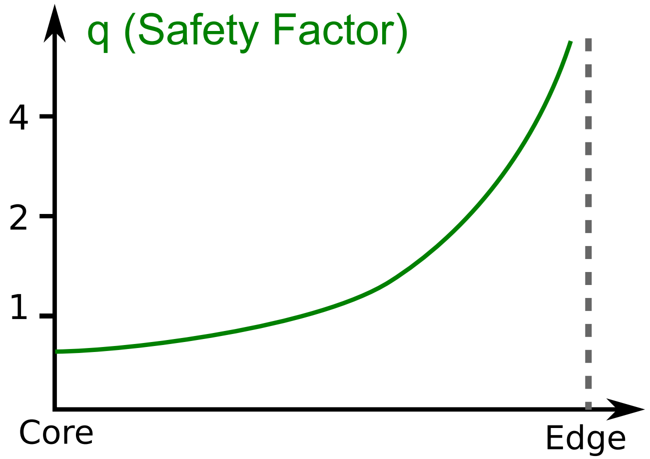

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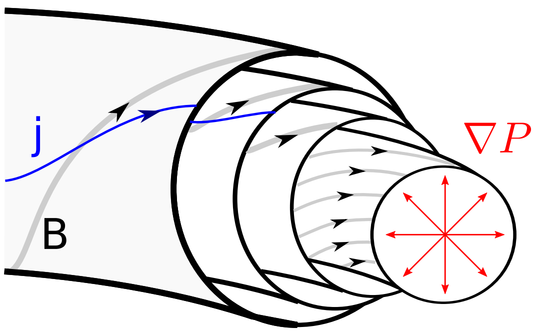

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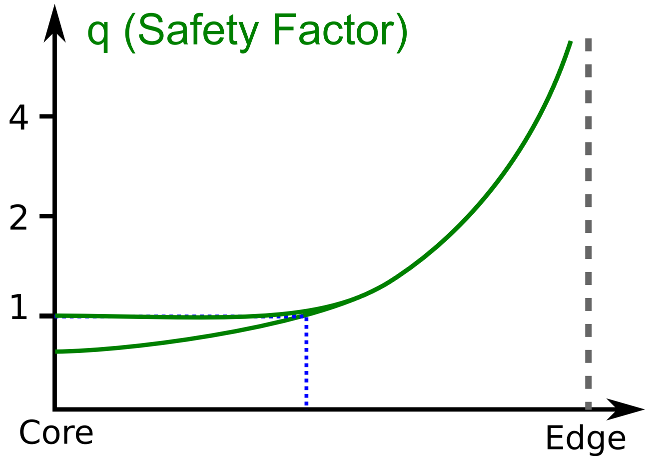
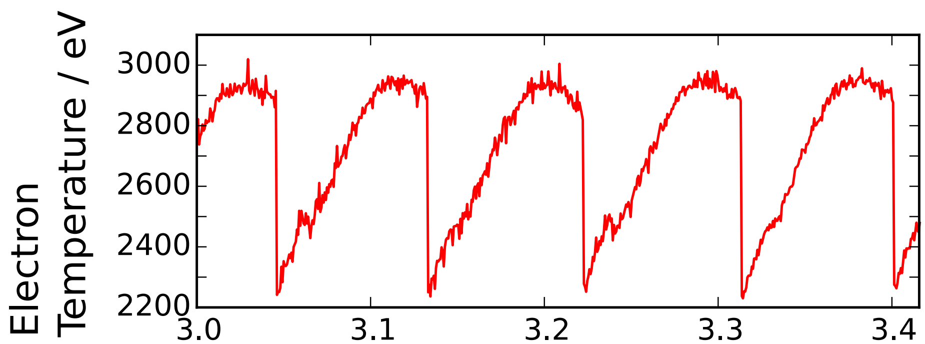




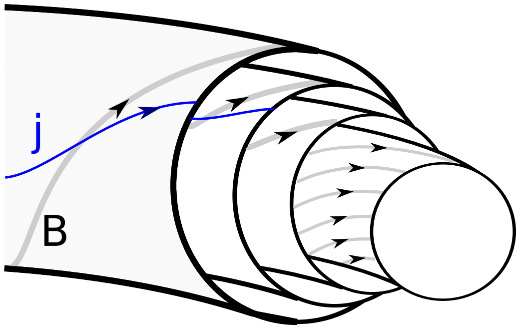
e.g.: When the central q value falls below 1.0, the plasma core periodically suddenly expels particles and energy - known as a 'sawtooth' crash. The crash is a magnetic reconnection event, which occurs far more rapidly than explained by simple theoretical models.

...(we'll return to this later)

Magnetic Equilibrium

- How do we know ***j*** and ***B***?

Assume: Axisymmetry + Isotropic pressure + No flow
Define the 'poloidal current flux' *f*:



$$f(R,Z) = \int^{R,Z} j_\theta(\mathbf{r}) d\mathbf{r} \qquad f = RB_\phi$$

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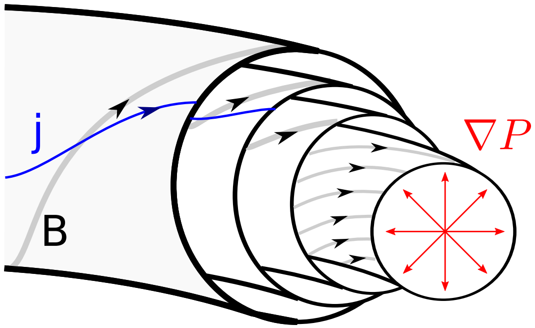

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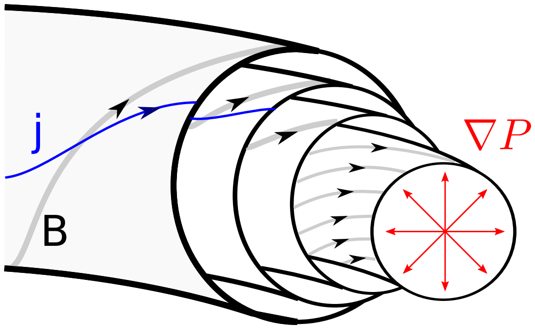

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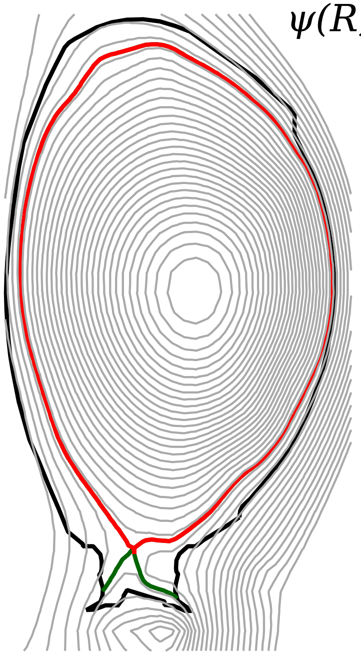

Also known as the Grad-Shafranov equation:

$$-\frac{1}{R}\frac{\partial^2\psi}{\partial Z^2} - \frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right) = \mu_0 R\frac{\partial p}{\partial\psi} + \frac{\mu_0^2}{R} f \frac{\partial f}{\partial\psi}$$

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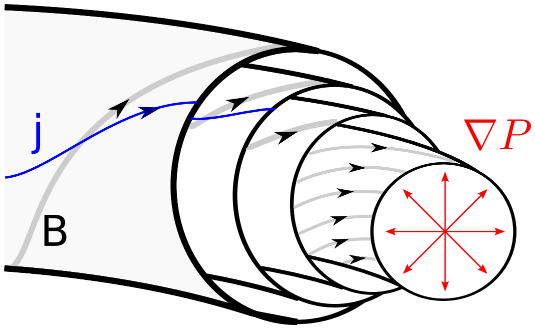

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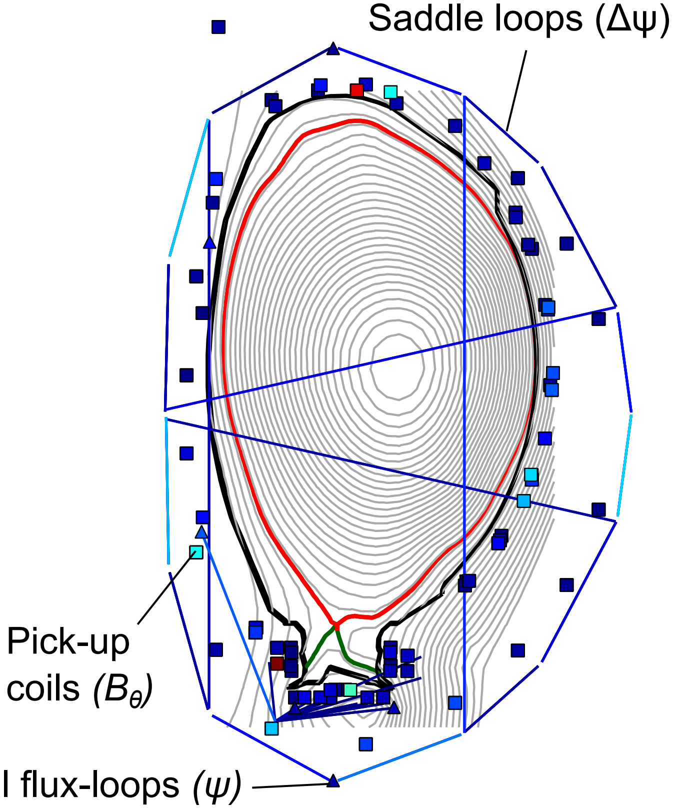

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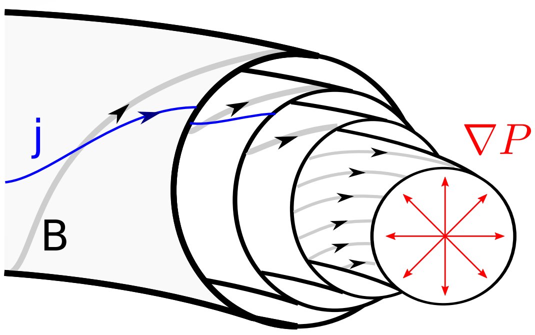

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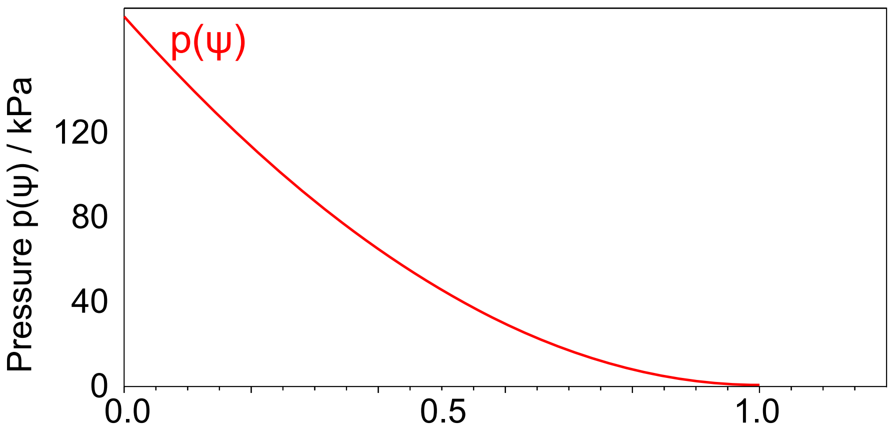
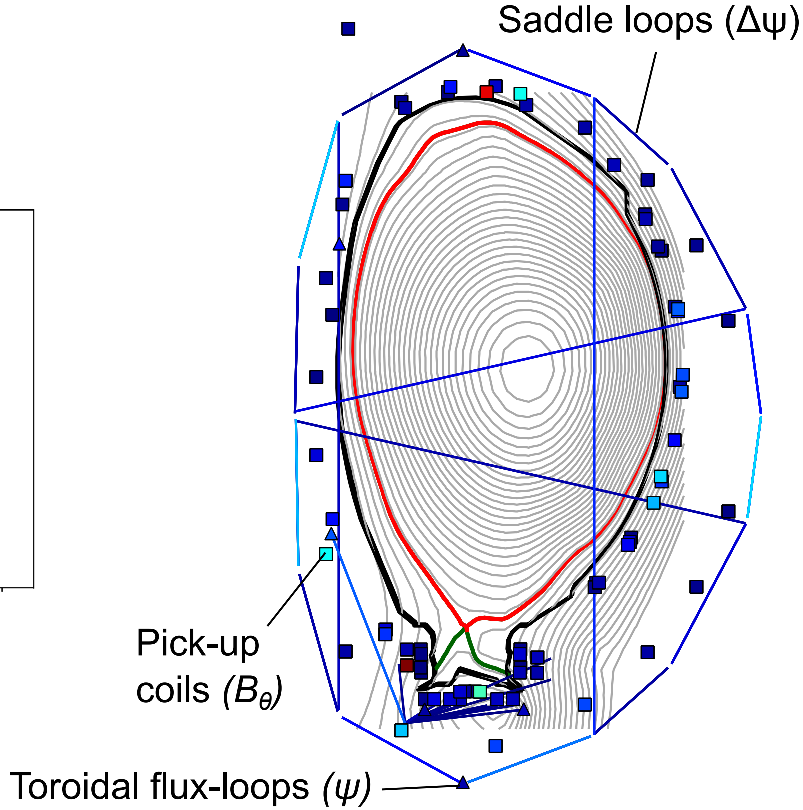
- Difficult to deal with pedestal pressure/current.





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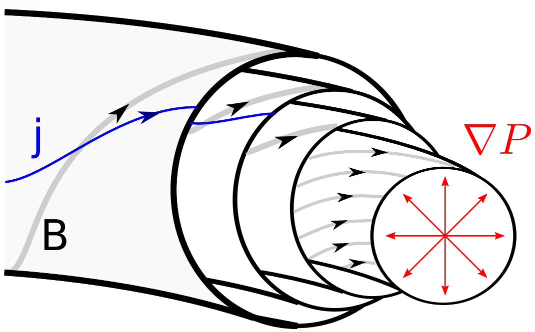

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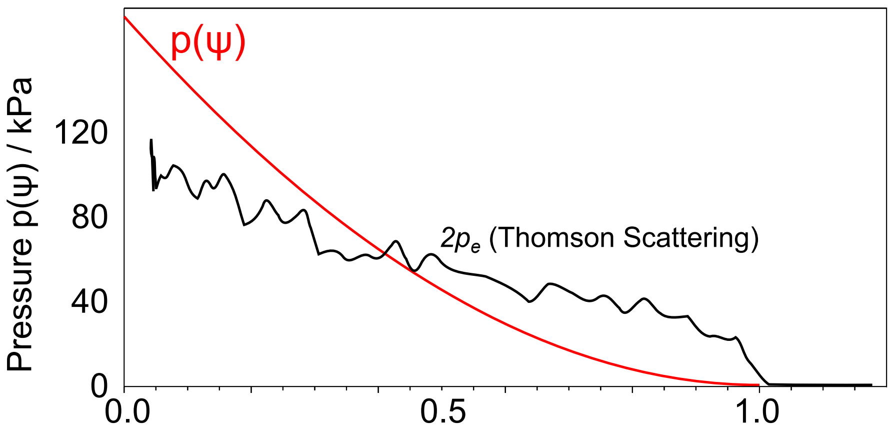

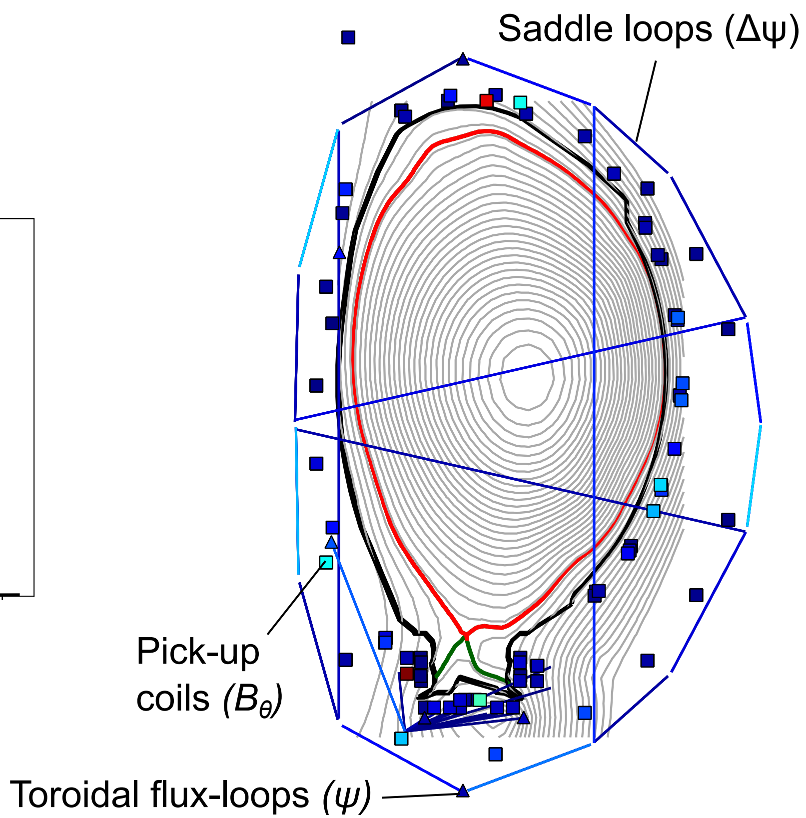

but....
- Is the converged solution the only solution?
- Are the simplified *p, f* profiles over-constrained / under-constrained?
- Are the data consistent with the assumptions?

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- Rigorous framework for dealing with the question:

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Any combination of diagnostics: $P(D \mid \mu) = \prod_i P(D_i \mid \mu)$

(More explanation and examples available for Q&A)

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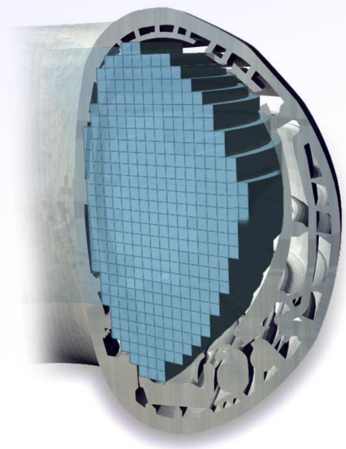

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grid of axisymmetric
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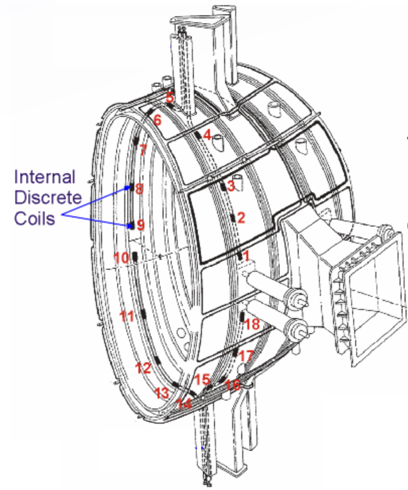

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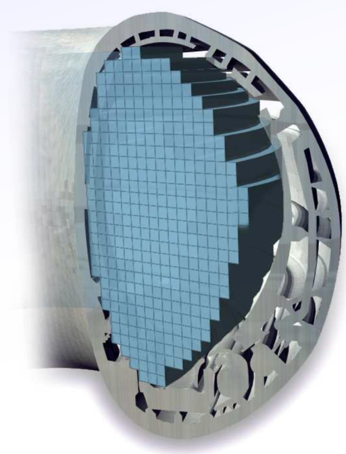

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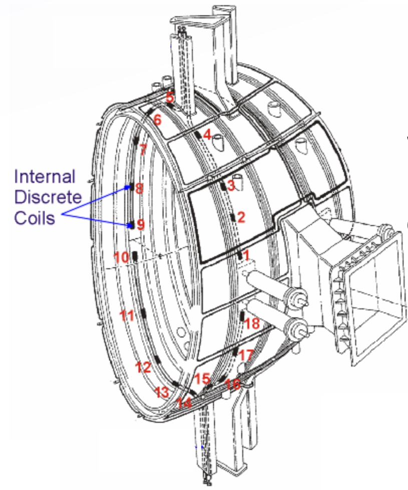

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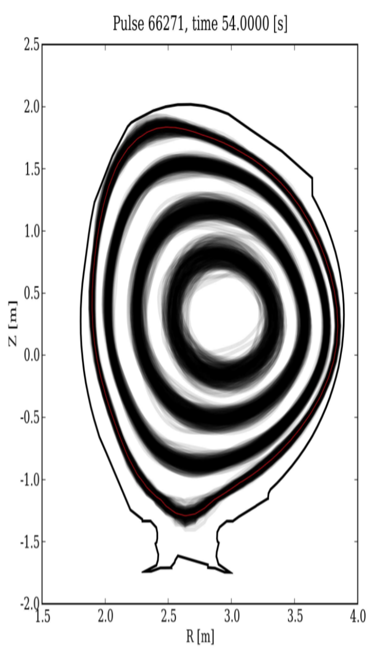

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Shows uncertainty and degeneracy
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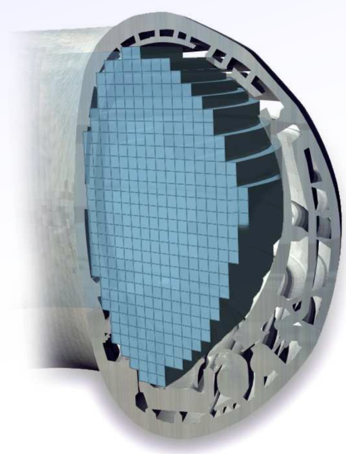

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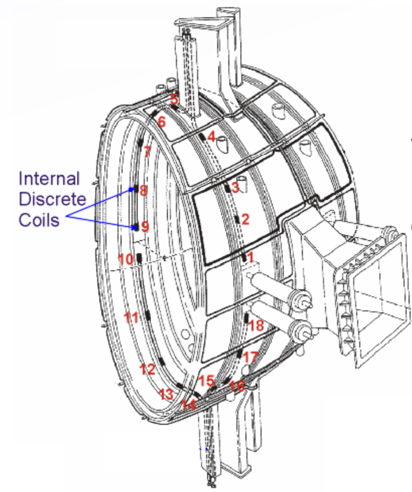

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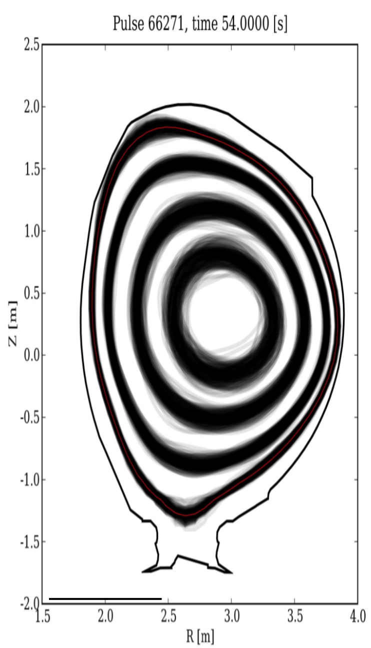

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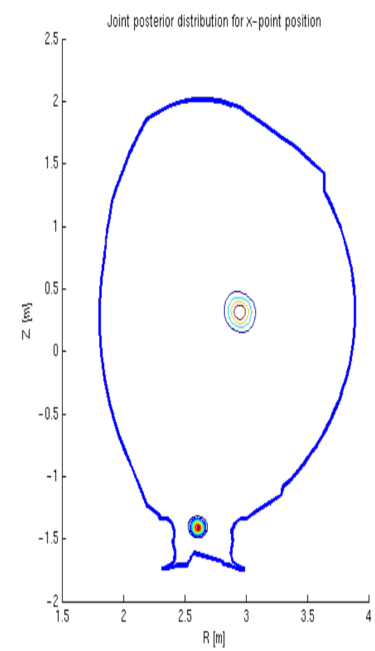
However, some quantities are well known --->
- Axis position
- X-point position



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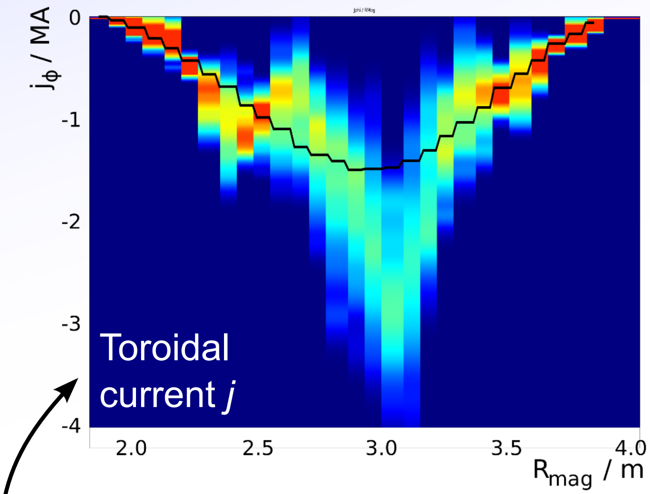
- Now we can ask the question:

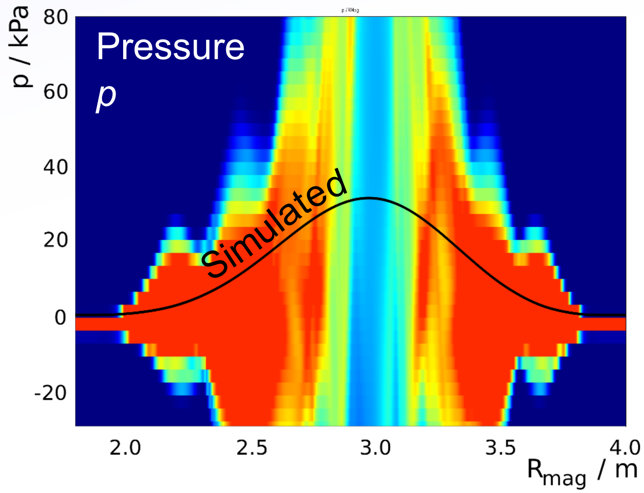
$$P(j_\phi, f, p \,|\, D_m, stable)$$

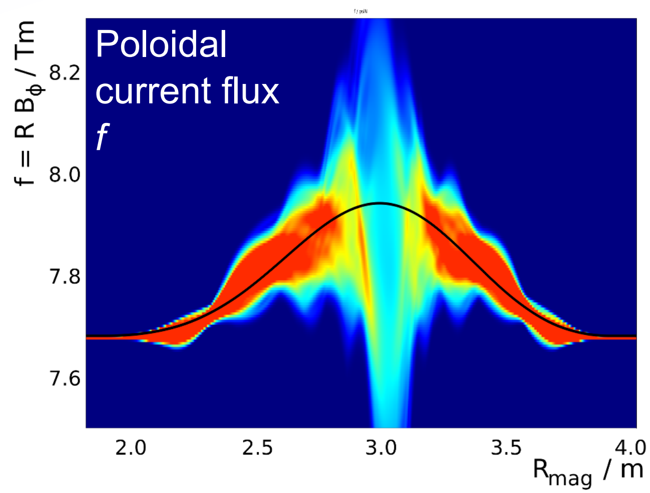
What space of plasma currents and pressures are consistent with the measurements and are close to equilibrium?

Bayesian Equilibrium

L-Mode plasmas: Low resolution current beam grid, fully explored posterior distribution:

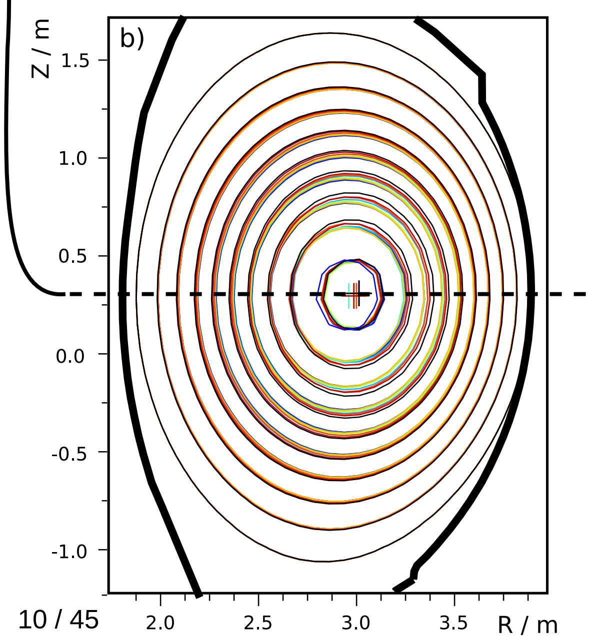






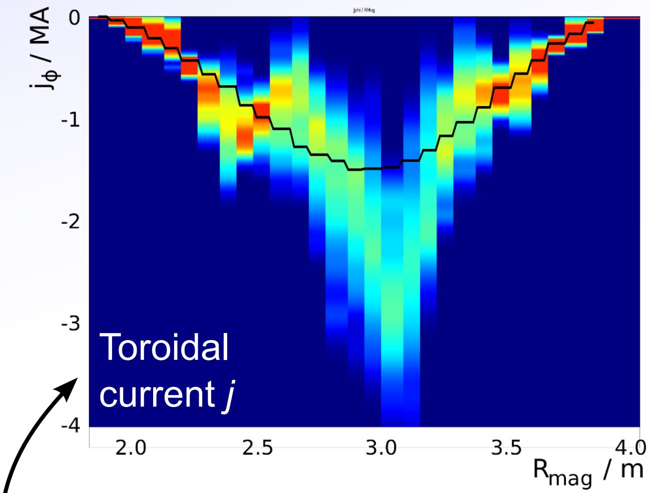
Uncertainty is large in core due to degeneracy: Equilibrium doesn't tell us much

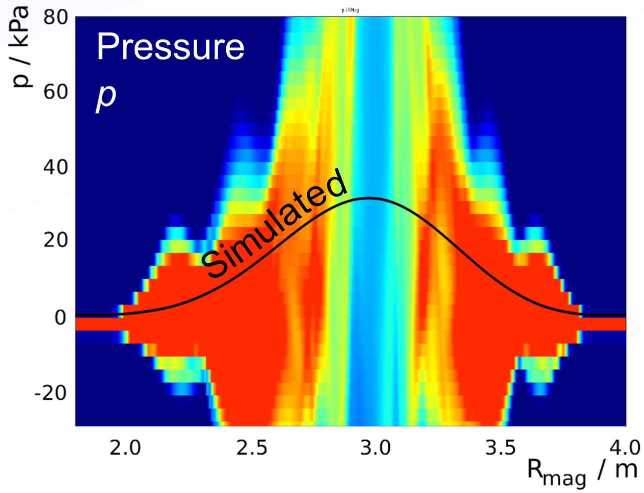
Flux surfaces:

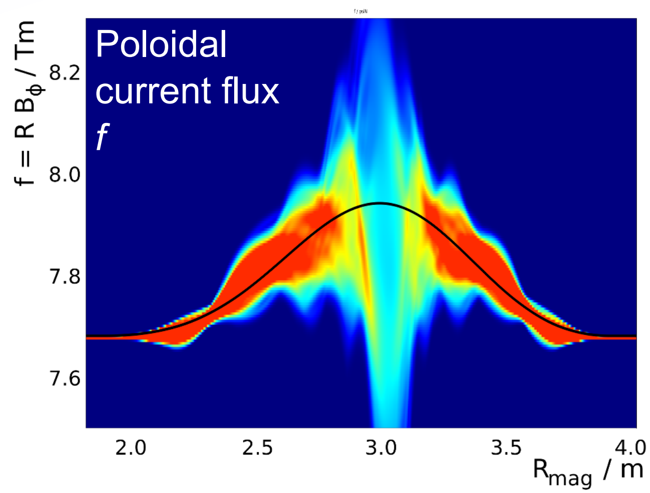


Bayesian Equilibrium

L-Mode plasmas: Low resolution current beam grid, fully explored posterior distribution:







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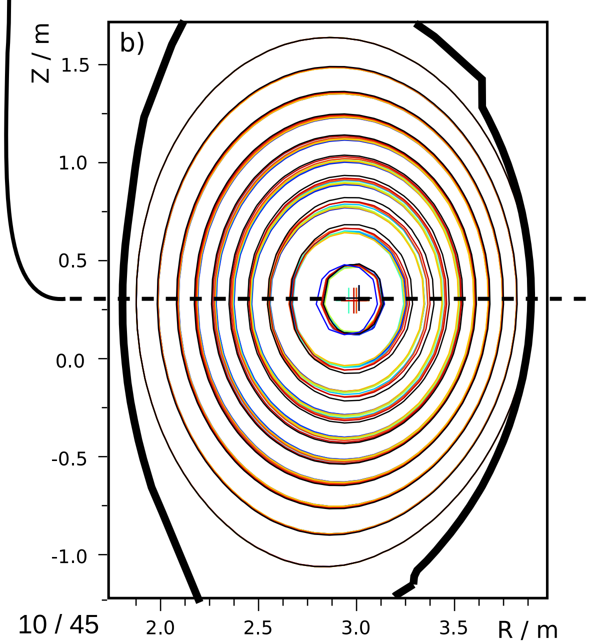
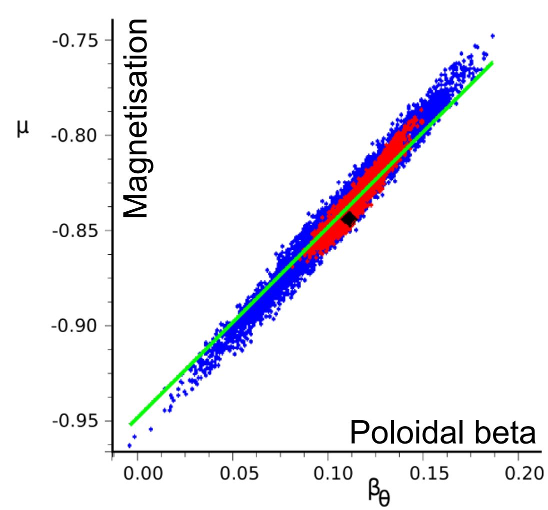
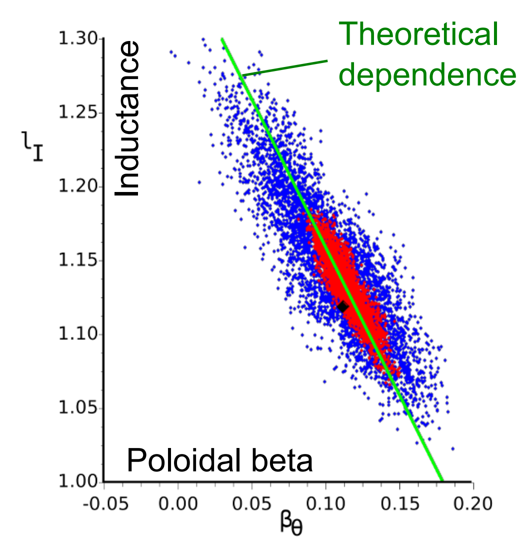


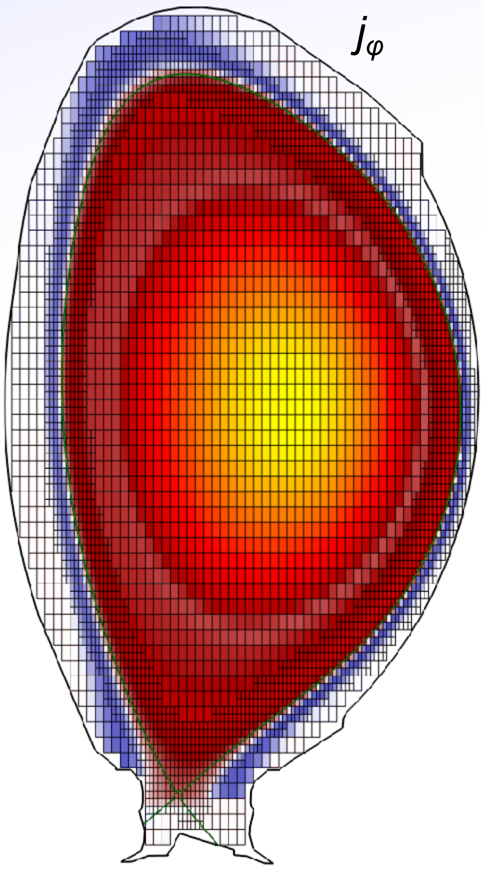
Samples of integral quantities reveal relations between 'Shafranov Integrals' that are well determined in simple analytical equilibrium solutions:





Bayesian Equilibrium

H-Mode plasmas: Very sharp changes in j and p require high-resolution current beam grid:



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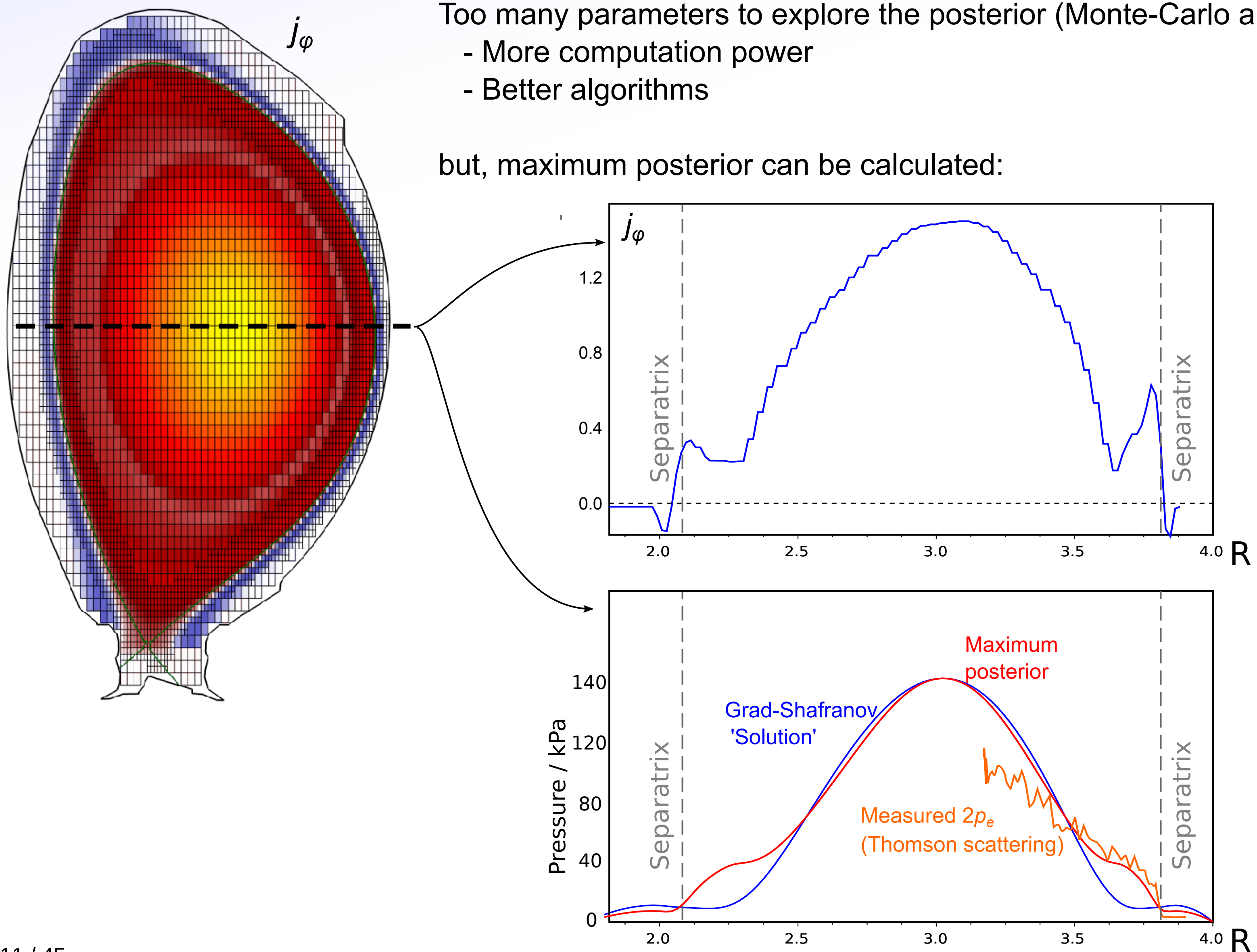

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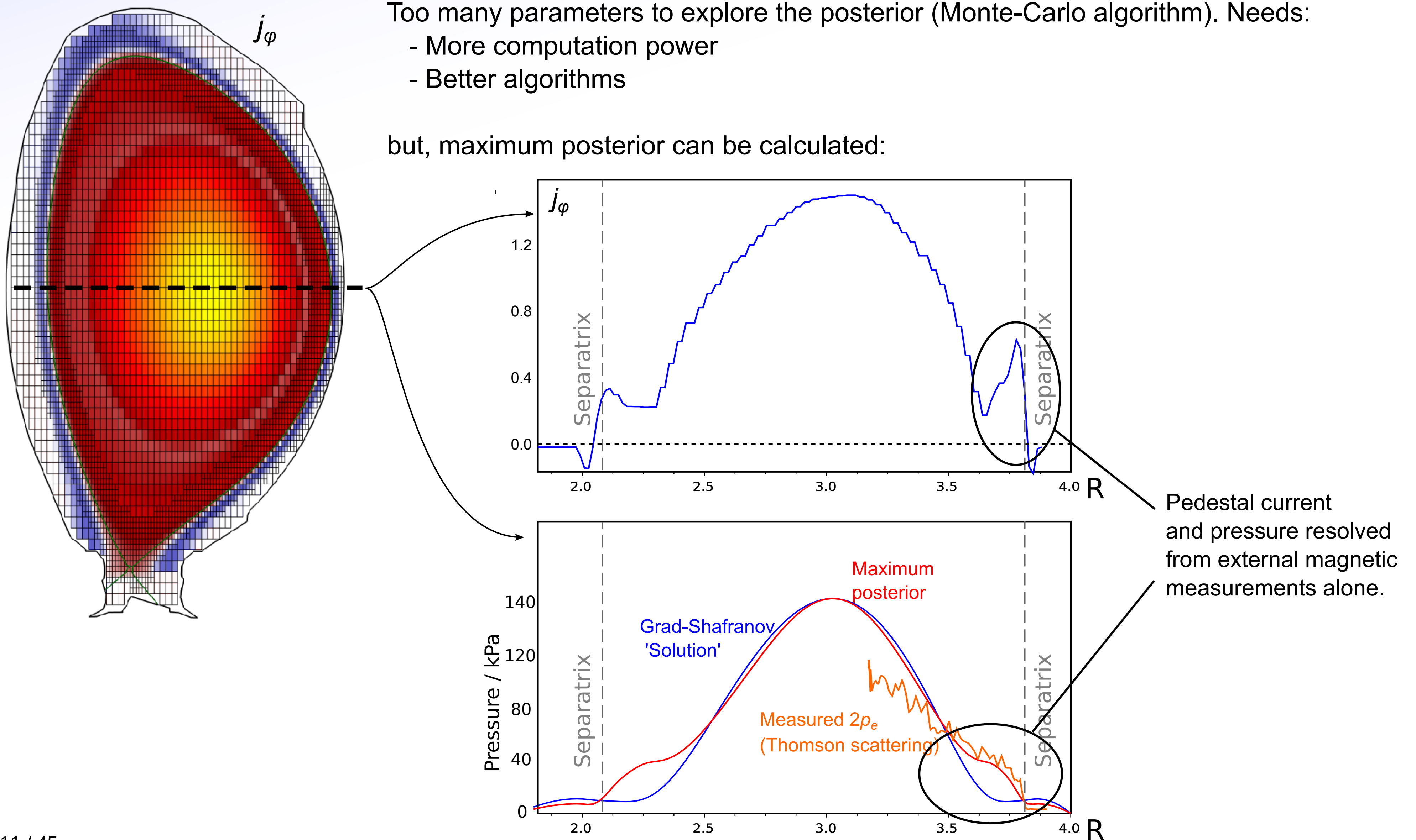
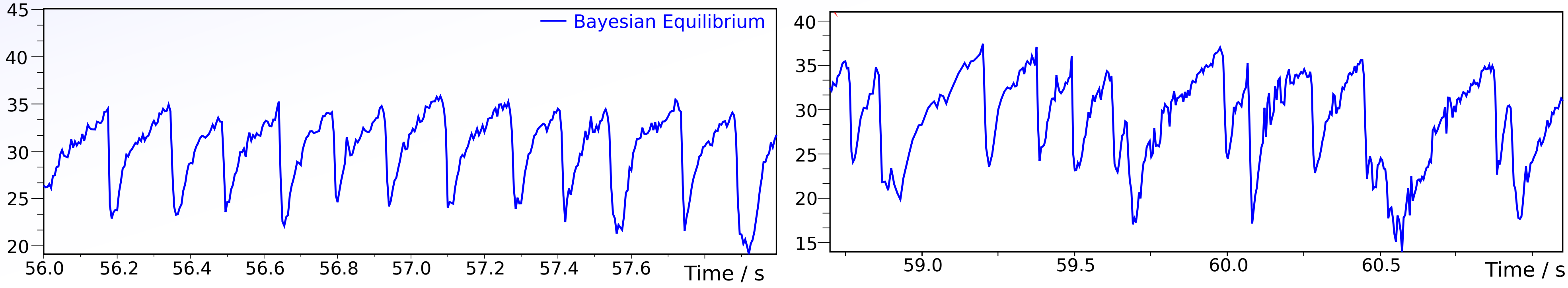


Pedestal Pressure

Flexible p, f profiles show that pedestal pressure can be very accurately measured with magnetic coils.



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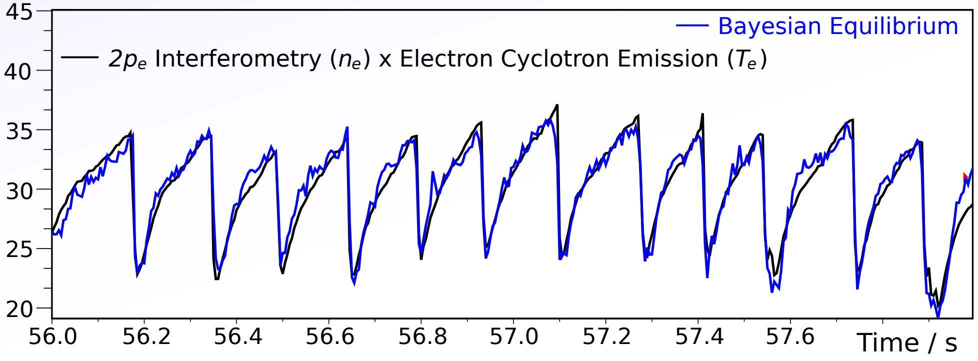

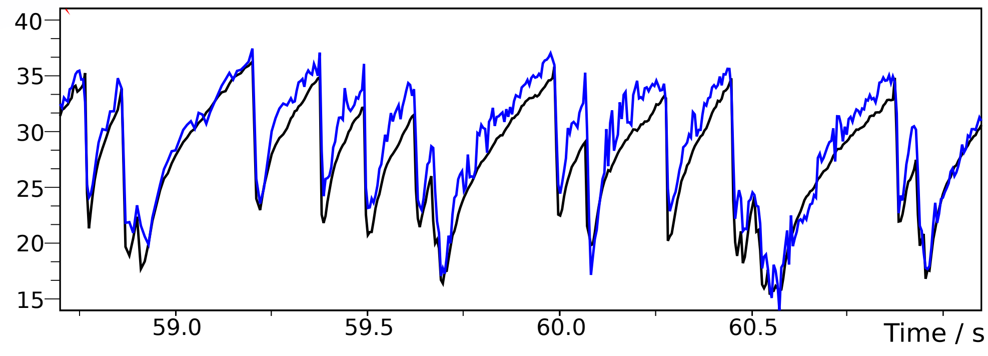

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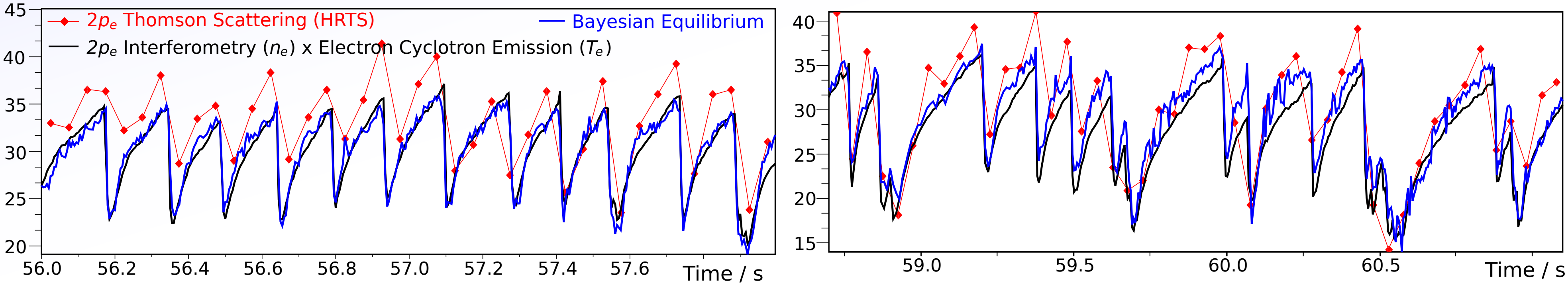

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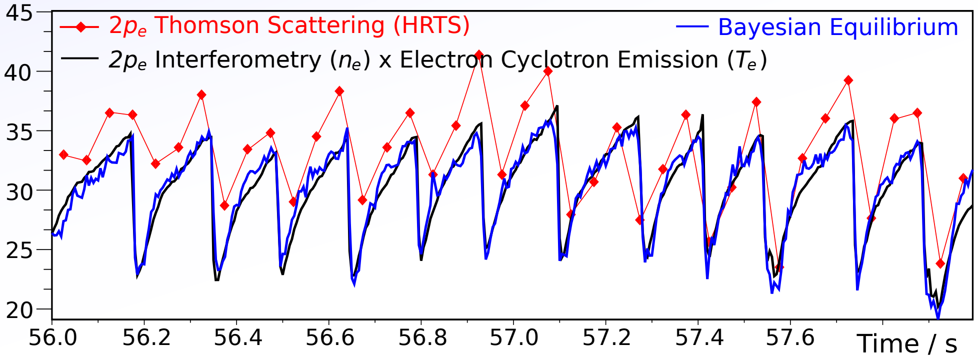
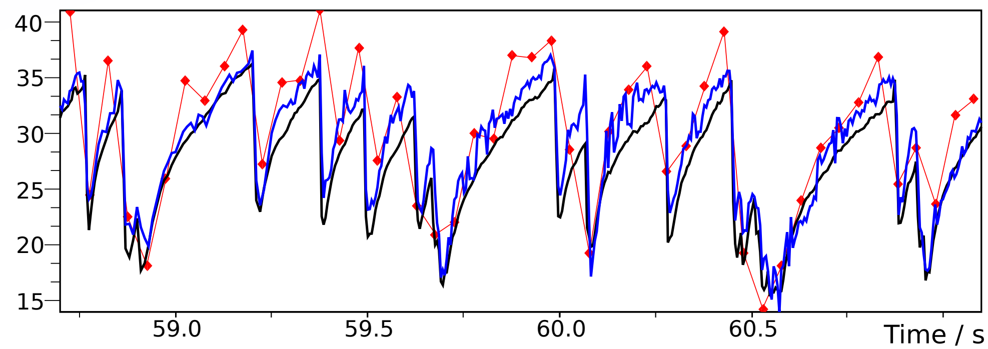
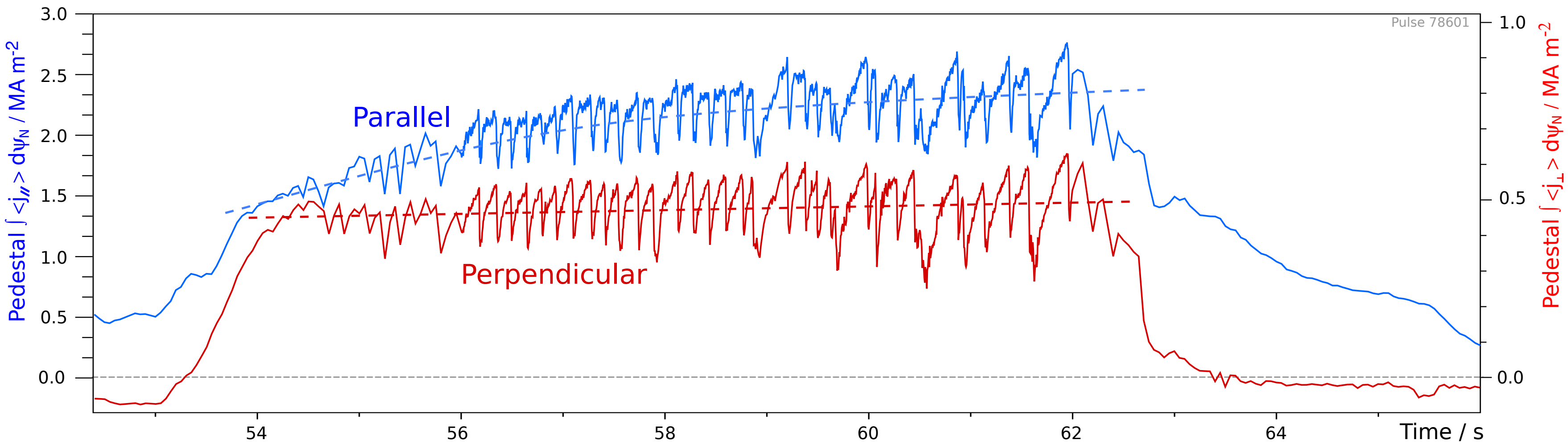




Pedestal parallel and perpendicular currents can be separated:



- Very good information on edge current, even from magnetics alone!

Outline

- Introduction
  - Flux surfaces and current profiles
  - Magnetic equilibrium
  - Bayesian analysis

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  - Current-tomography
  - Current tomography + Grad-Shafranov
  - L-Mode reconstructions
  - H-Mode results

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  - Motional Stark effect.
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  - Imaging MSE
  - Direct j_φ imaging.

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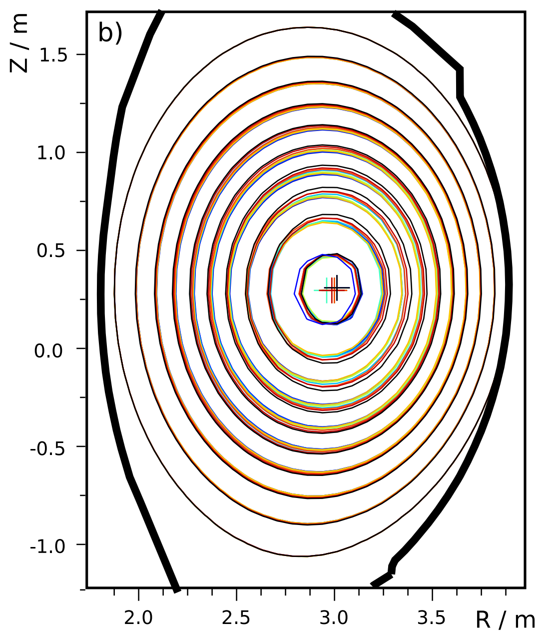

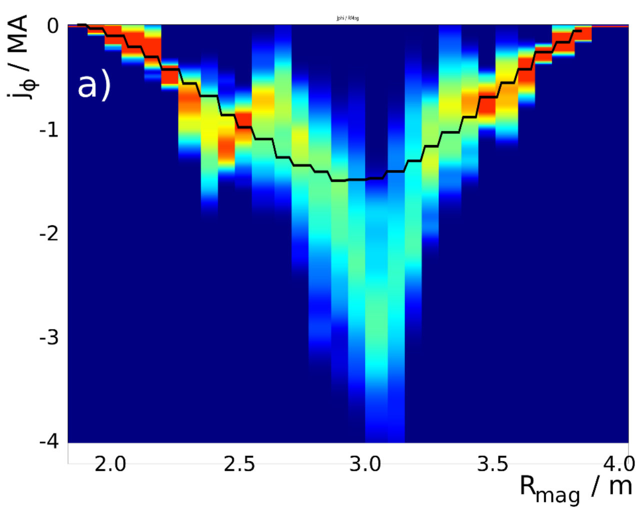

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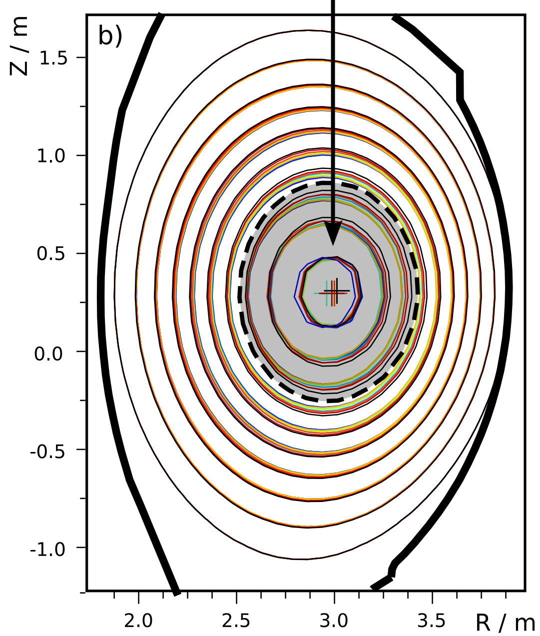

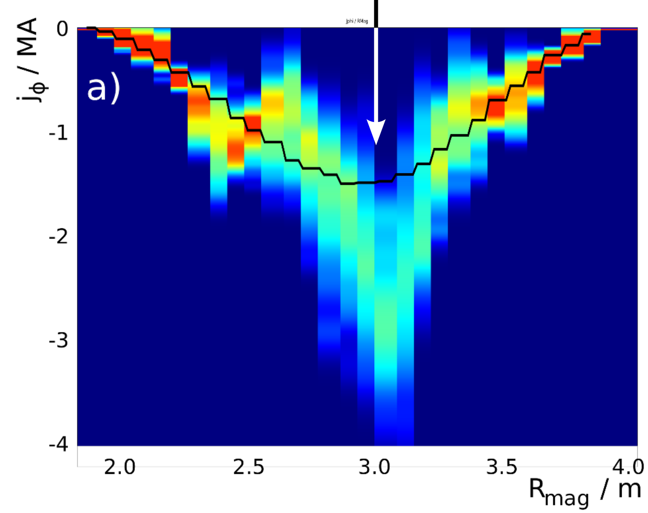

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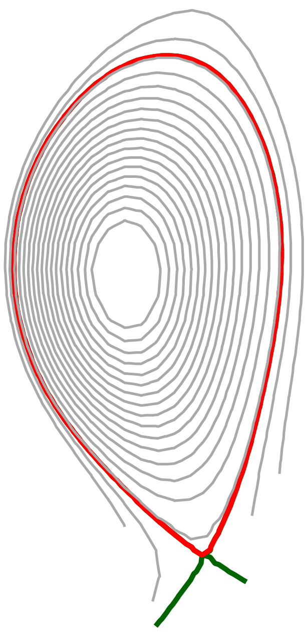
How can we measure deep inside the plasma?



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Spectroscopy - observe the light emitted by atoms in the plasma:

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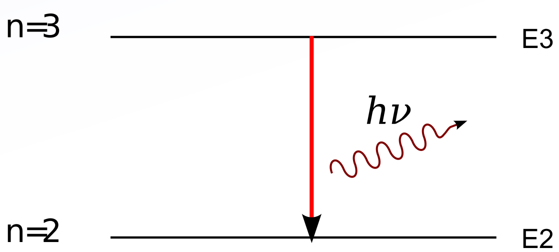

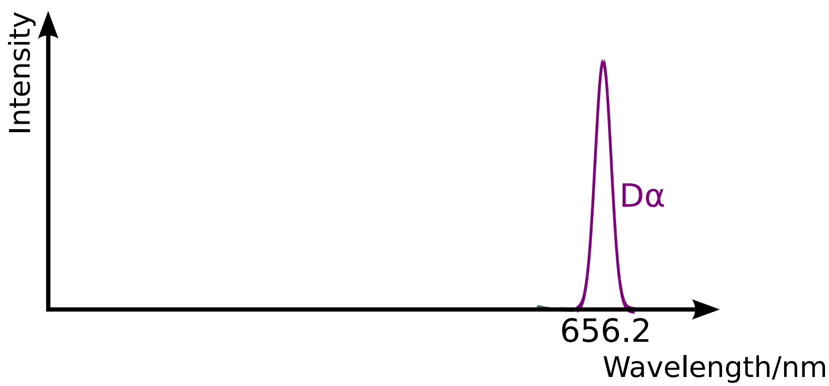

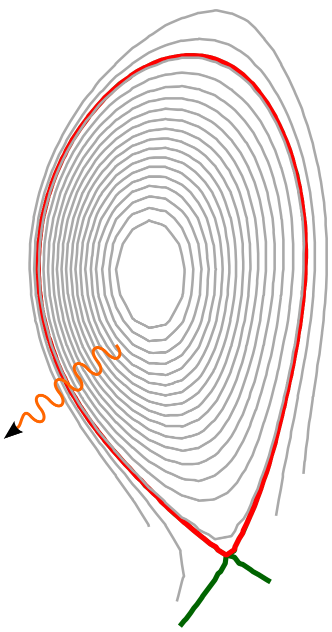

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Excitation by ion/electron impact excites the higher energy levels.
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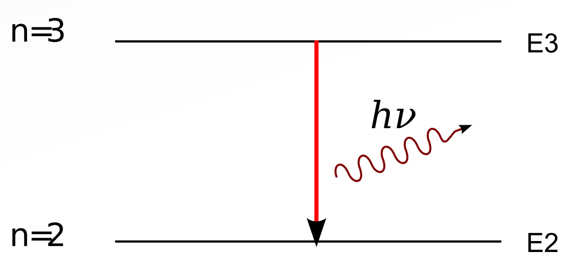

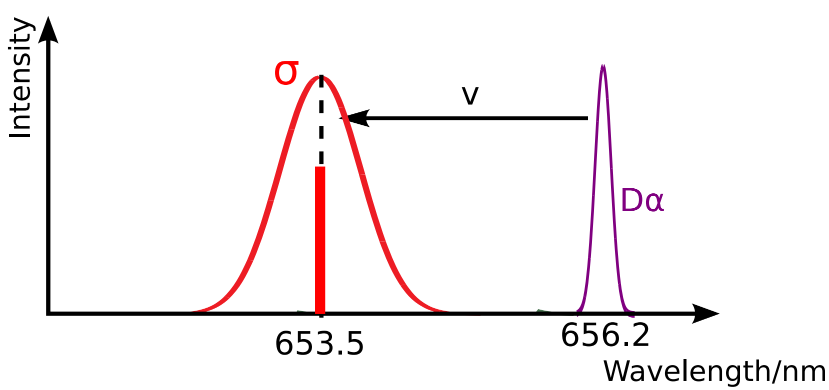

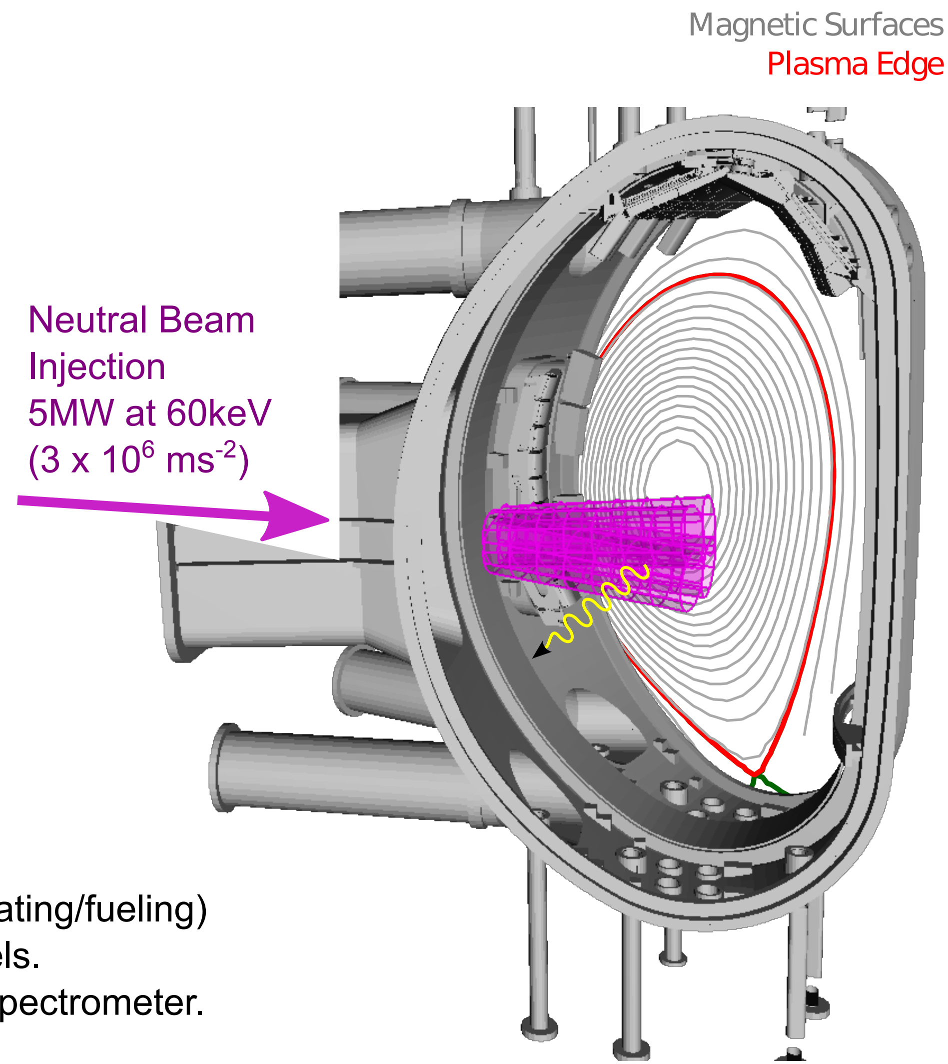

ASDEX Upgrade Vacuum Vessel

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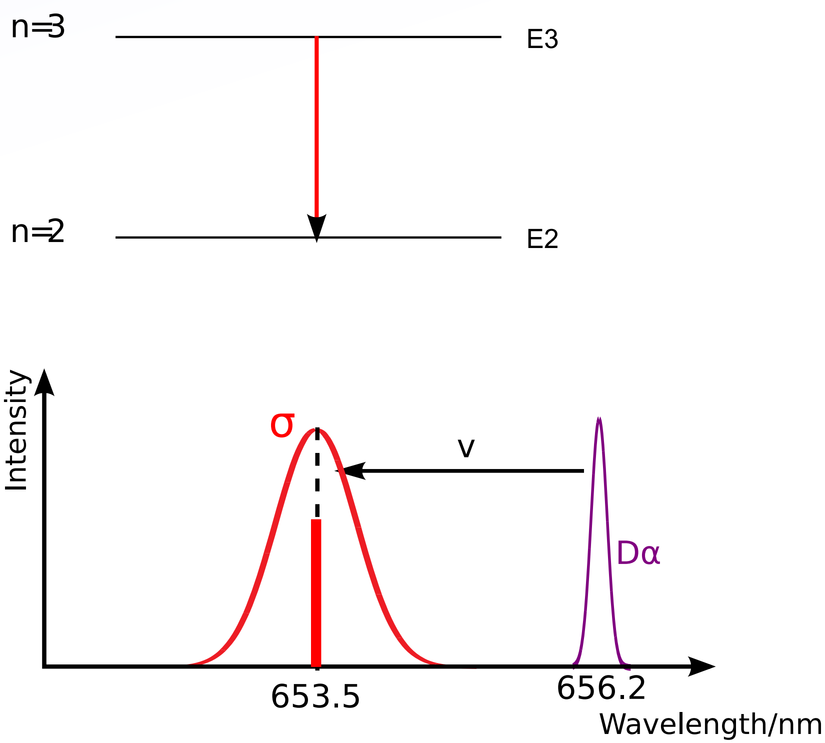
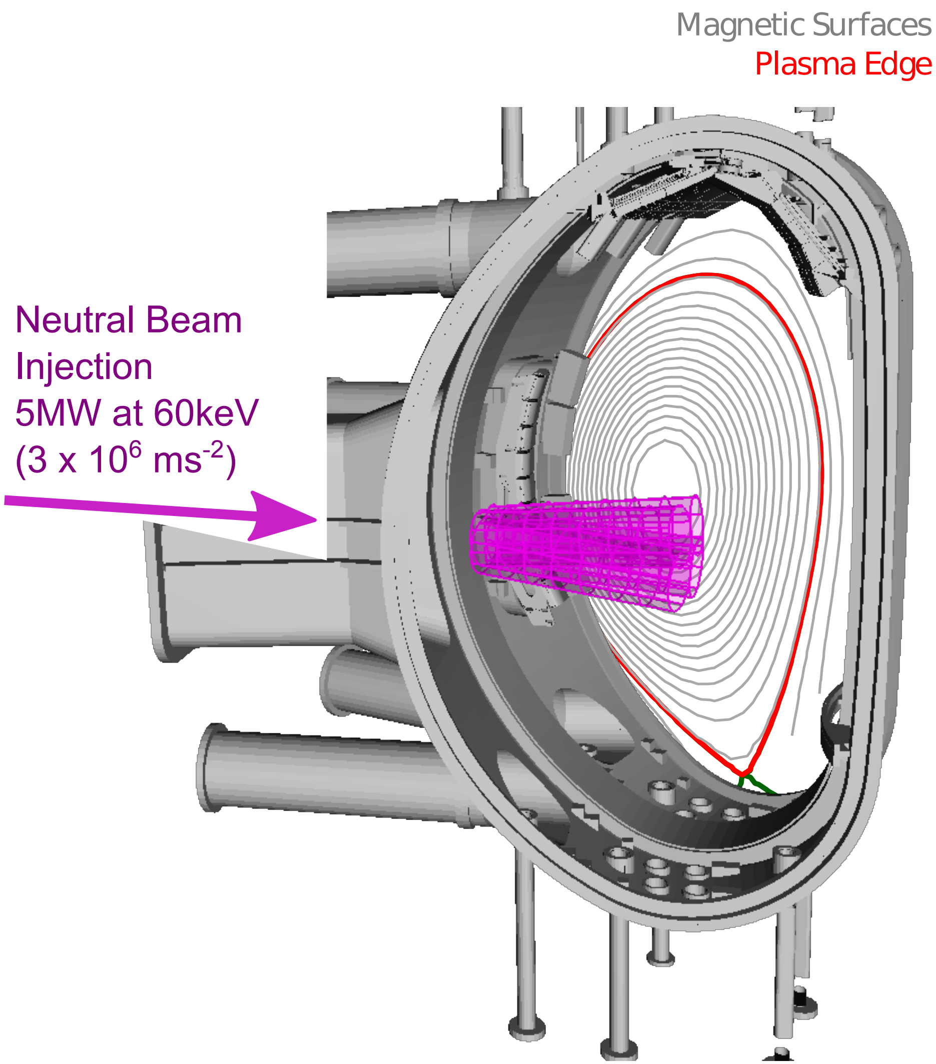
Motional Stark Effect Polarimetry





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The atomic energy levels are modified by the local magnetic/electric fields:

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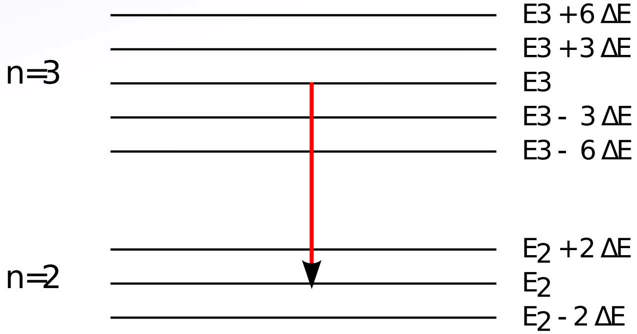
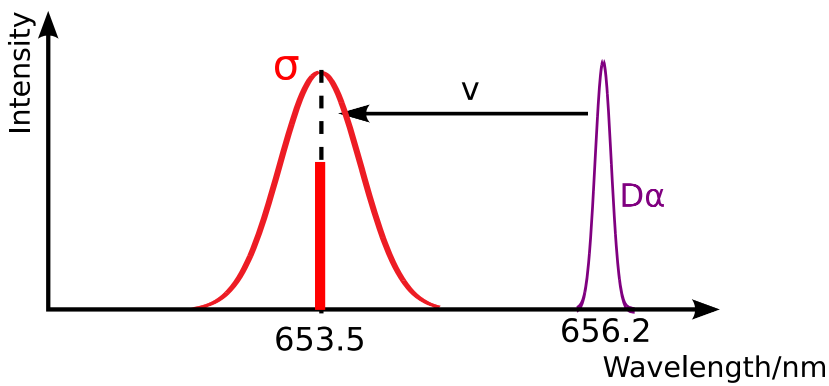
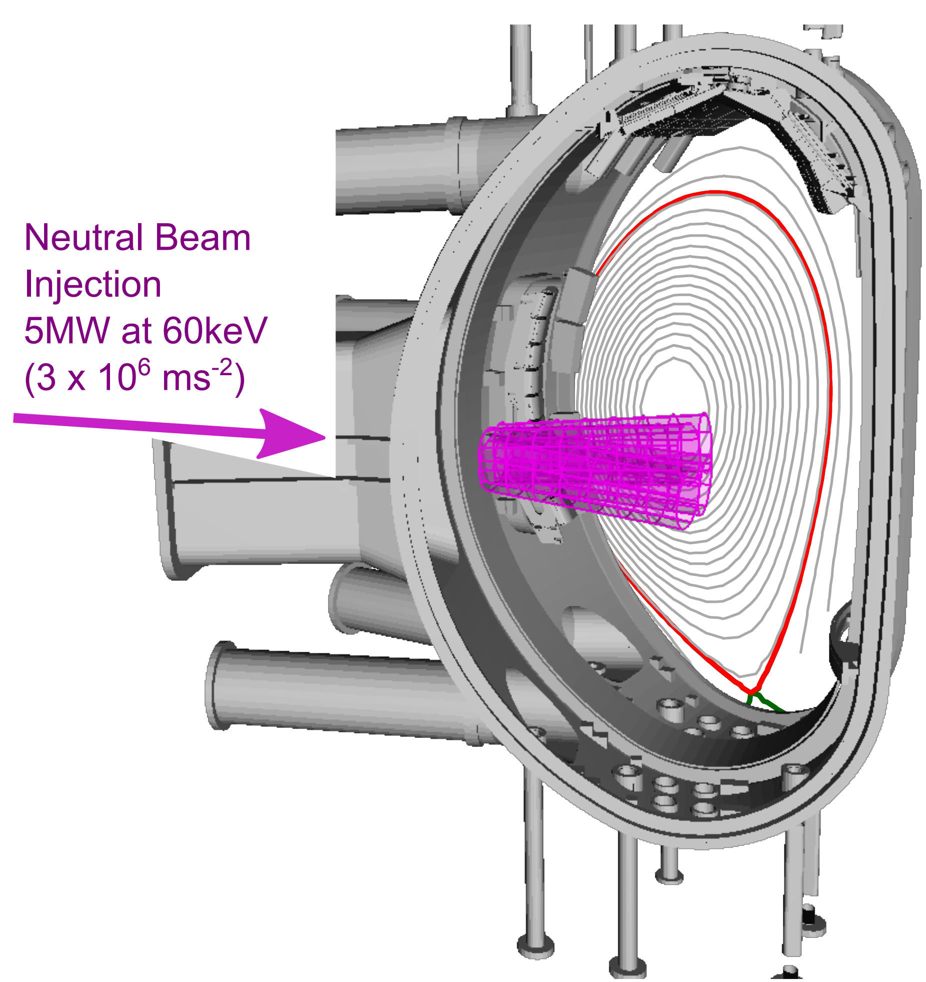
Stark splitting by Lorenz-transformed magnetic field: $\underline{E} = \underline{v} \times \underline{B}$







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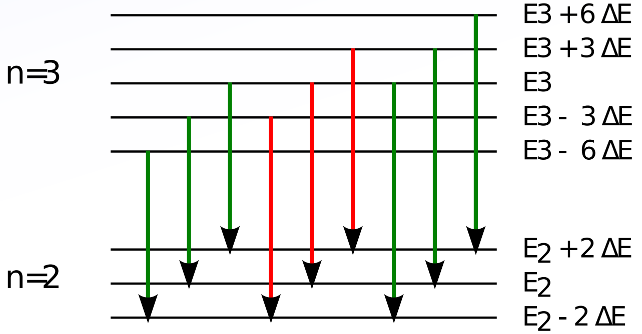

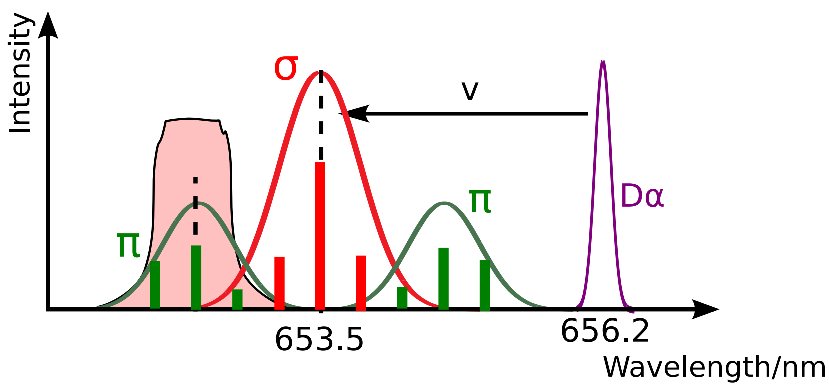

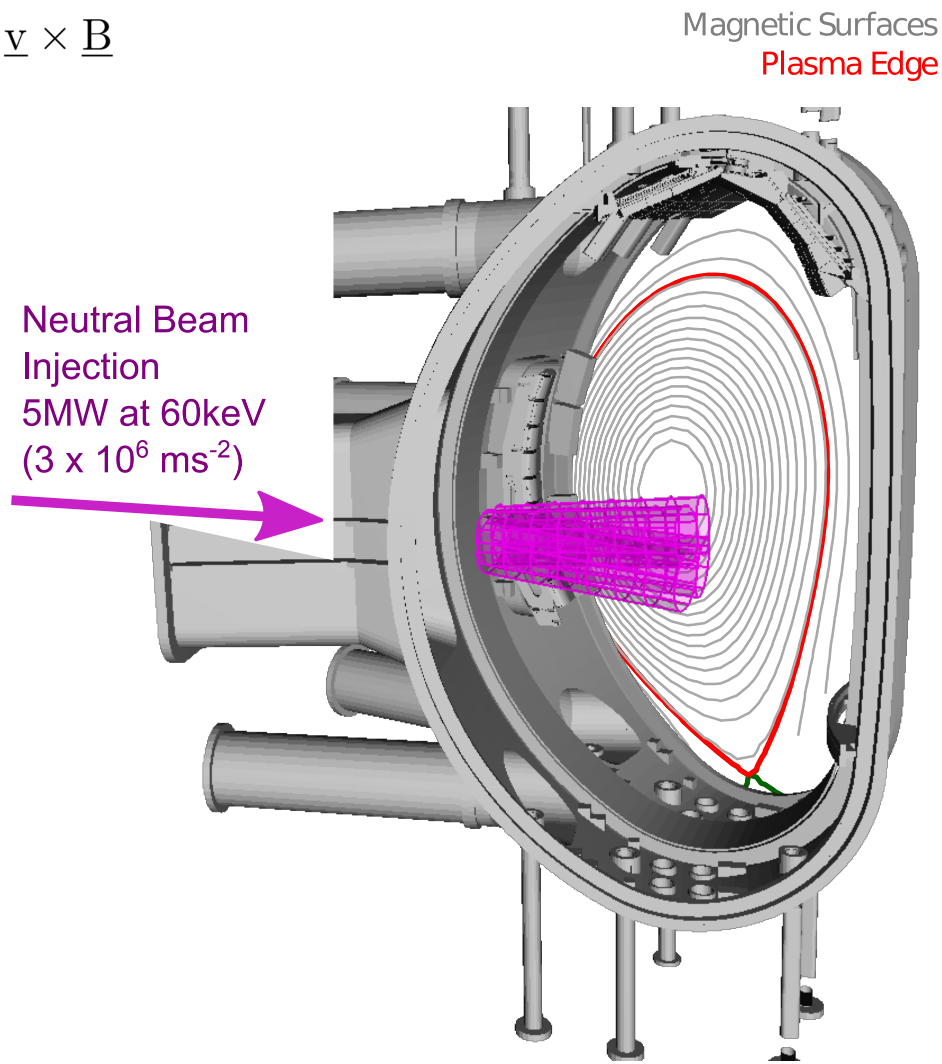

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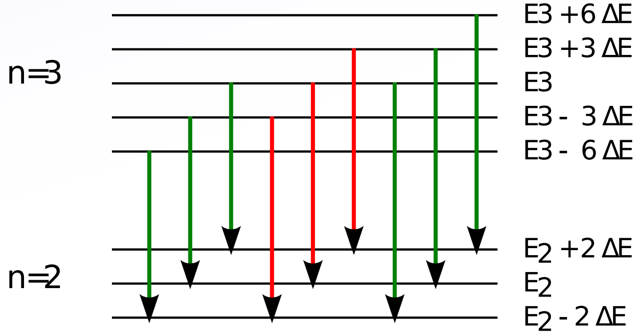

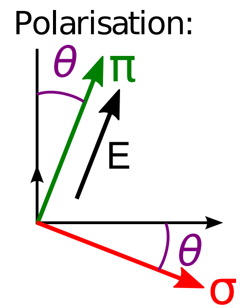

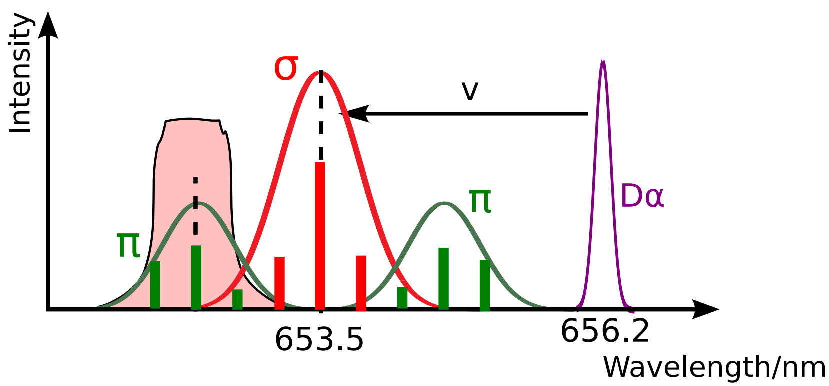

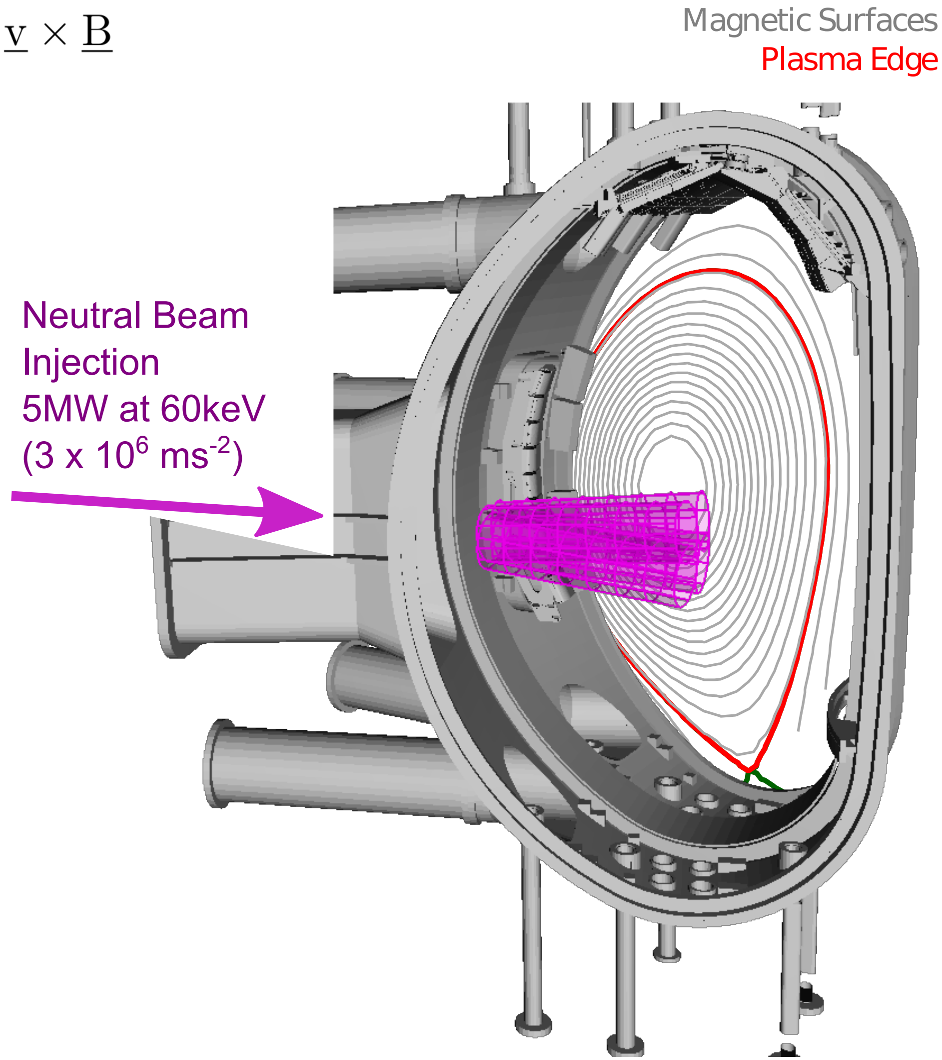

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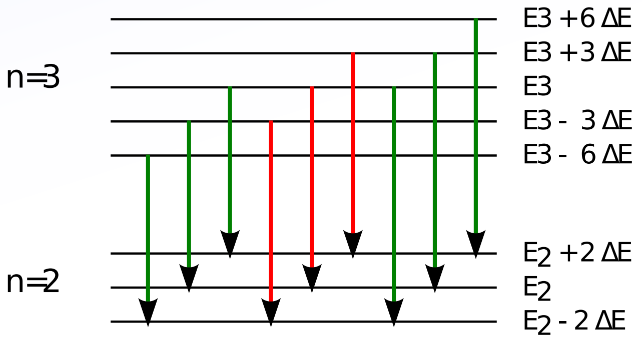
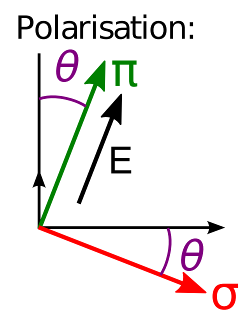
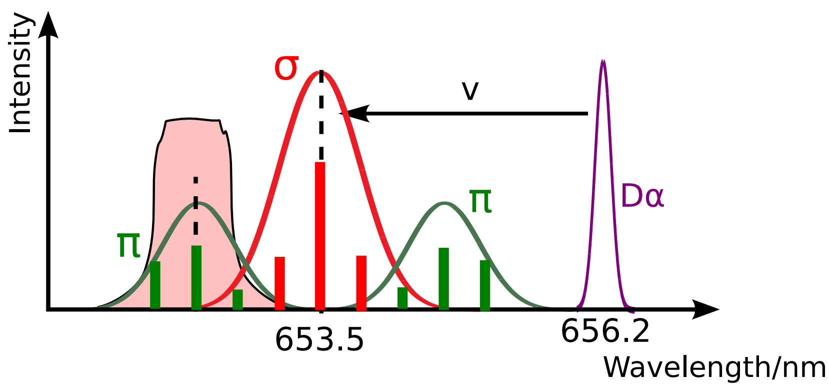
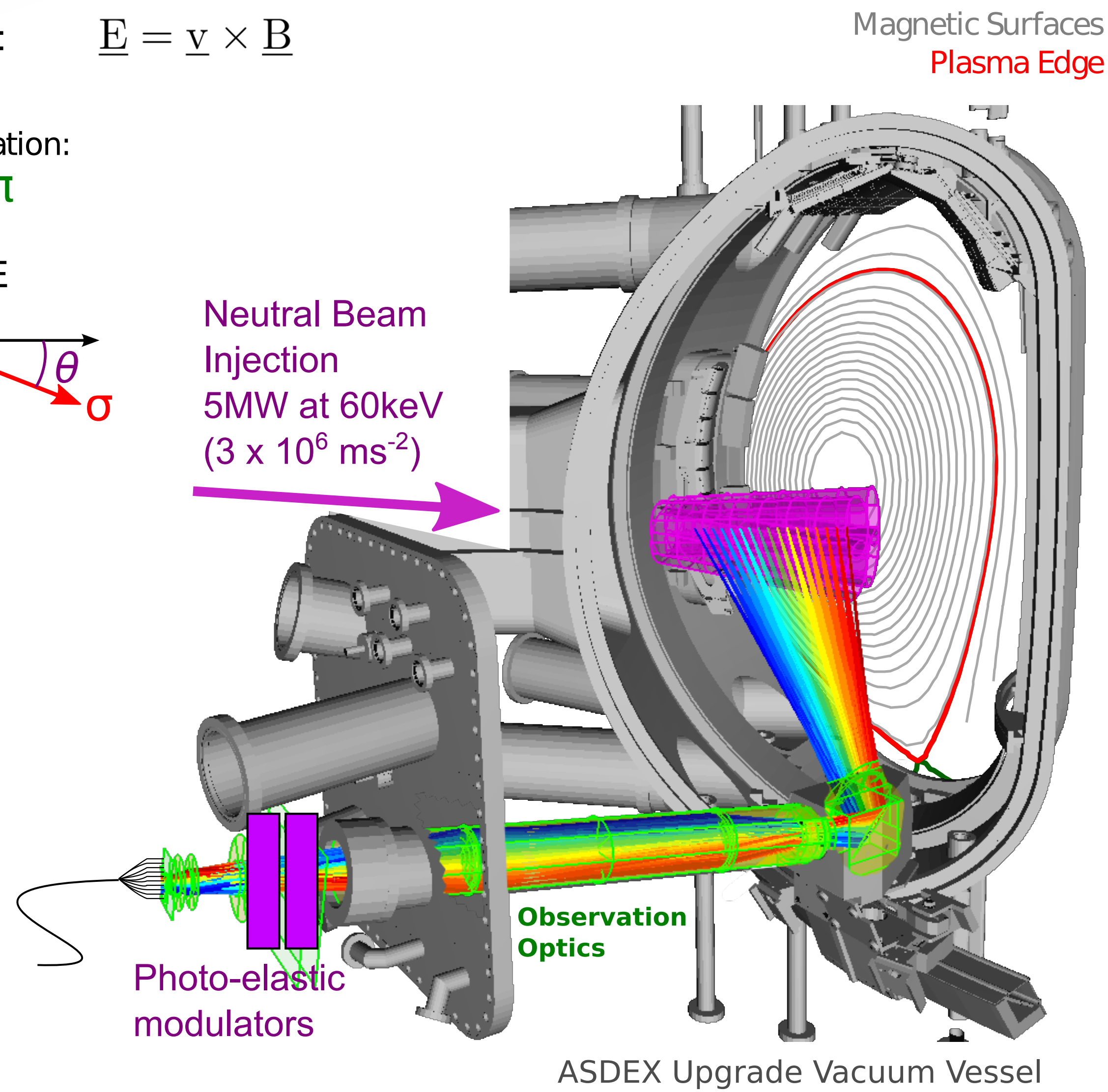




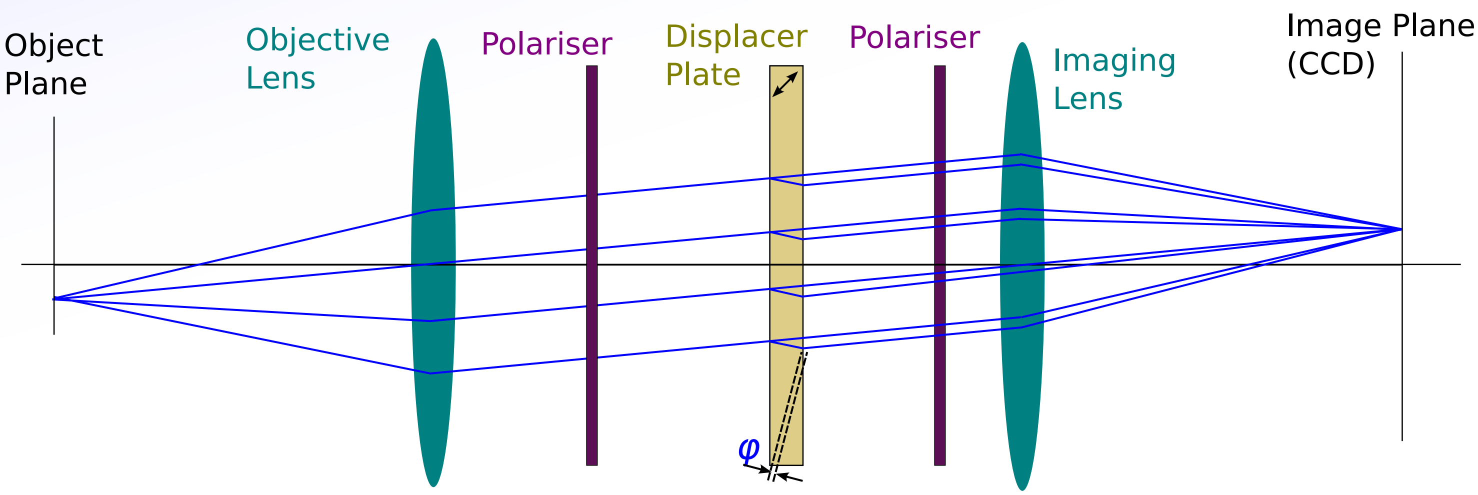




The MSE diagnostic:
Isolate a π component and measure the polarisation --> magnetic field direction.

Coherence Imaging

Coherence Imaging: Spectroscopic technique, modulated in space and imaged with a CMOS camera.
Also used for imaging spectroscopic moments



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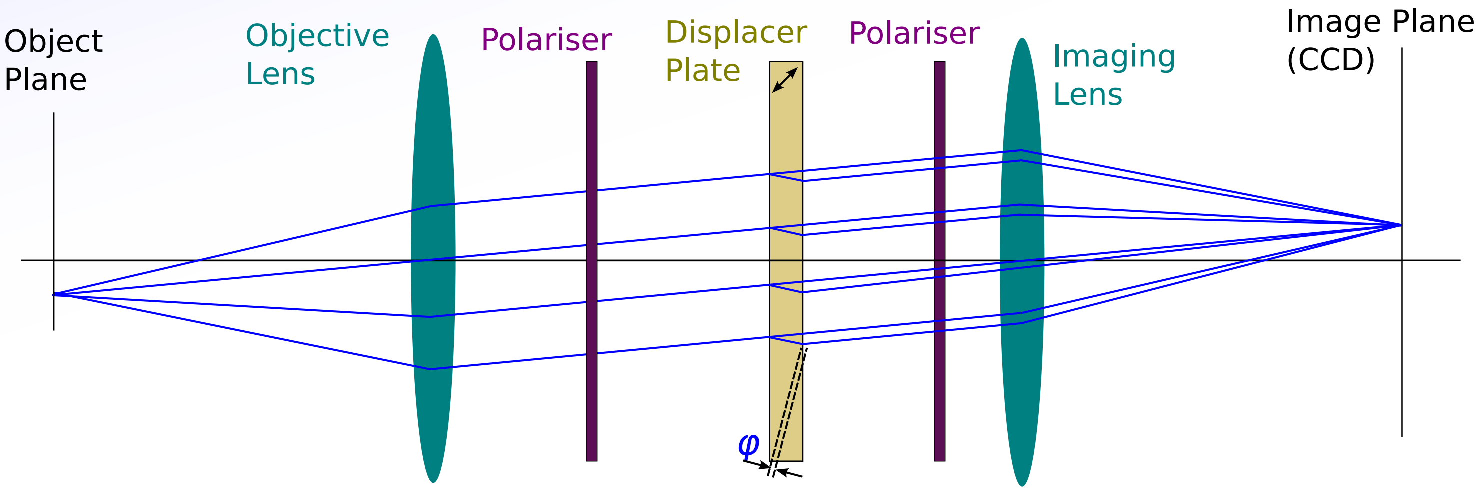

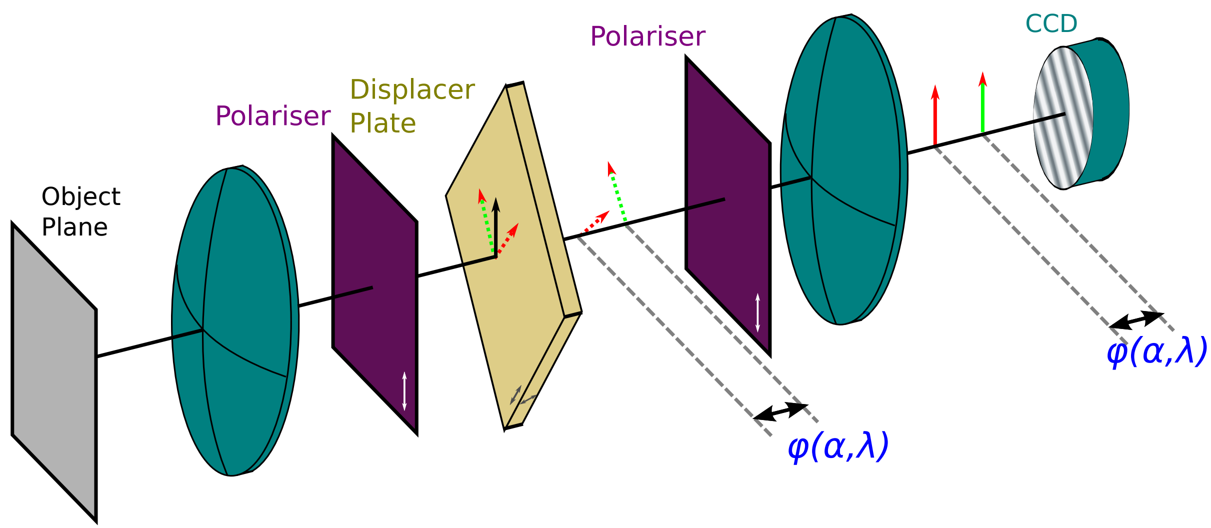

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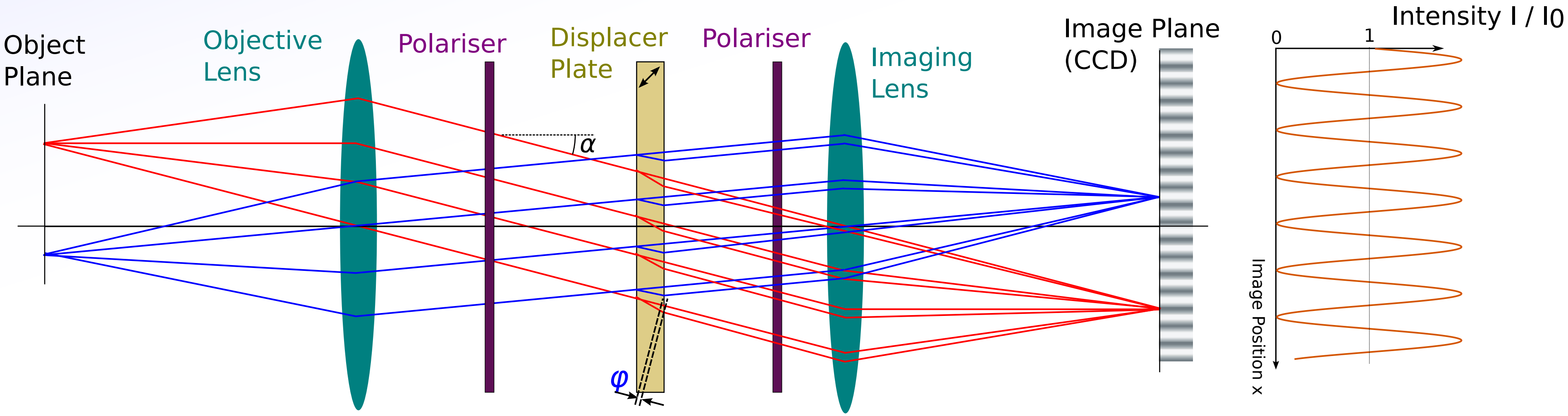

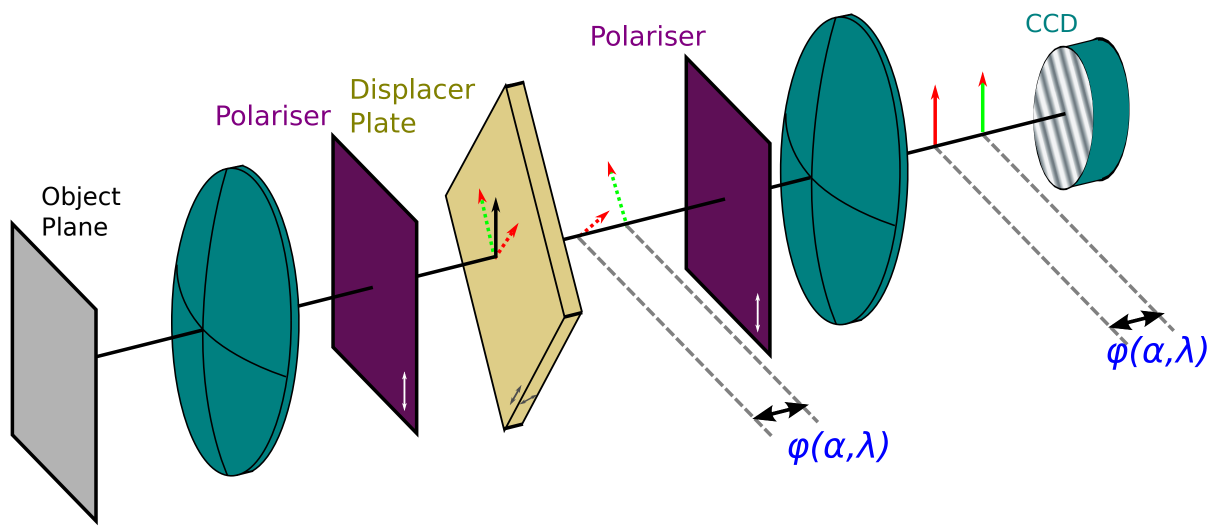

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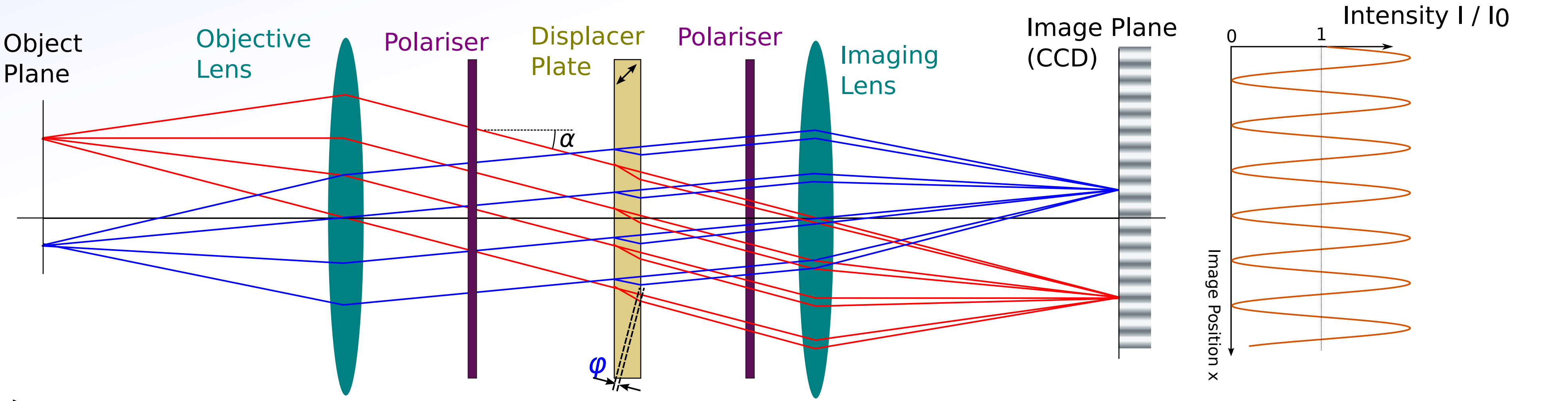

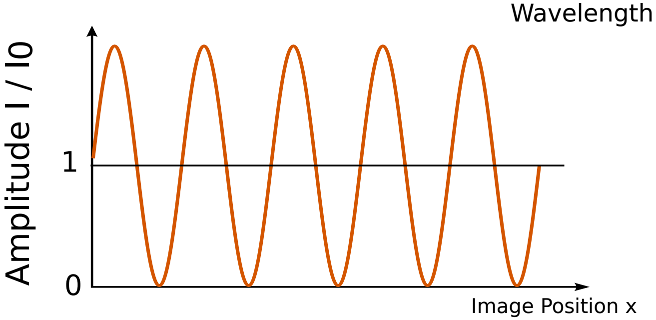

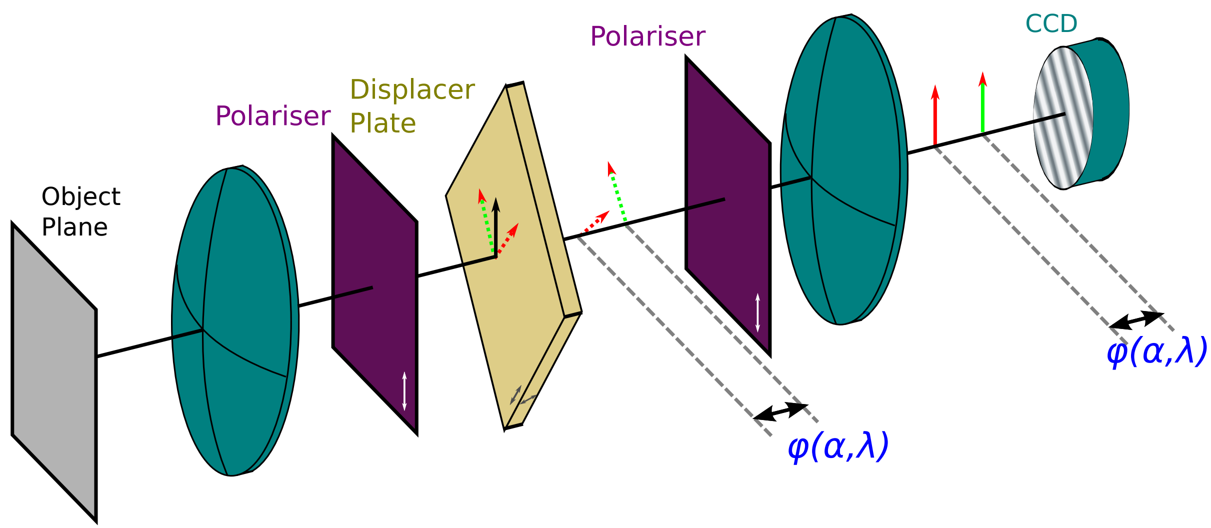

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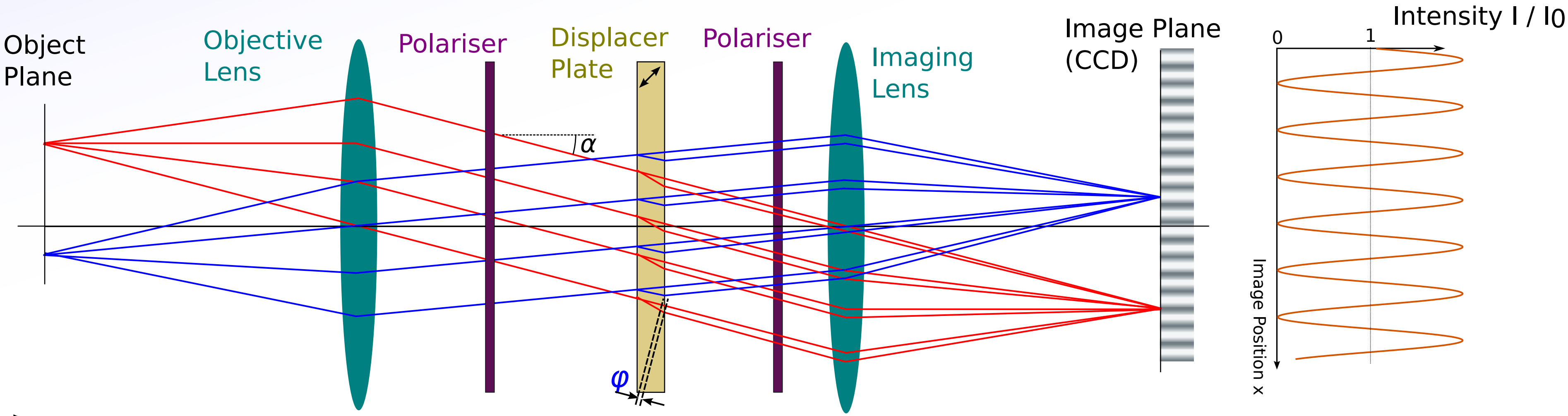
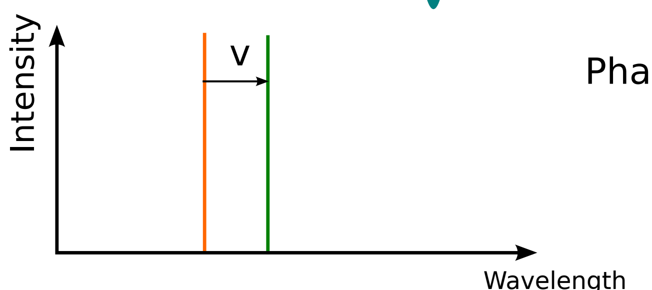
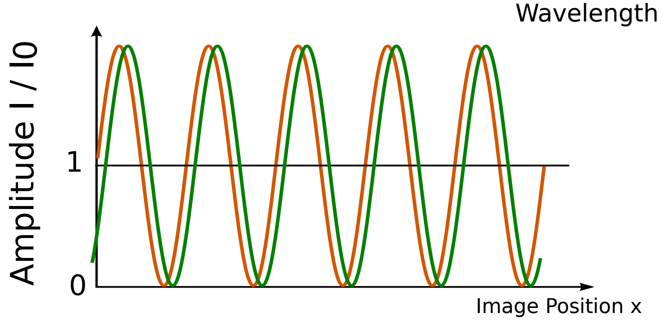
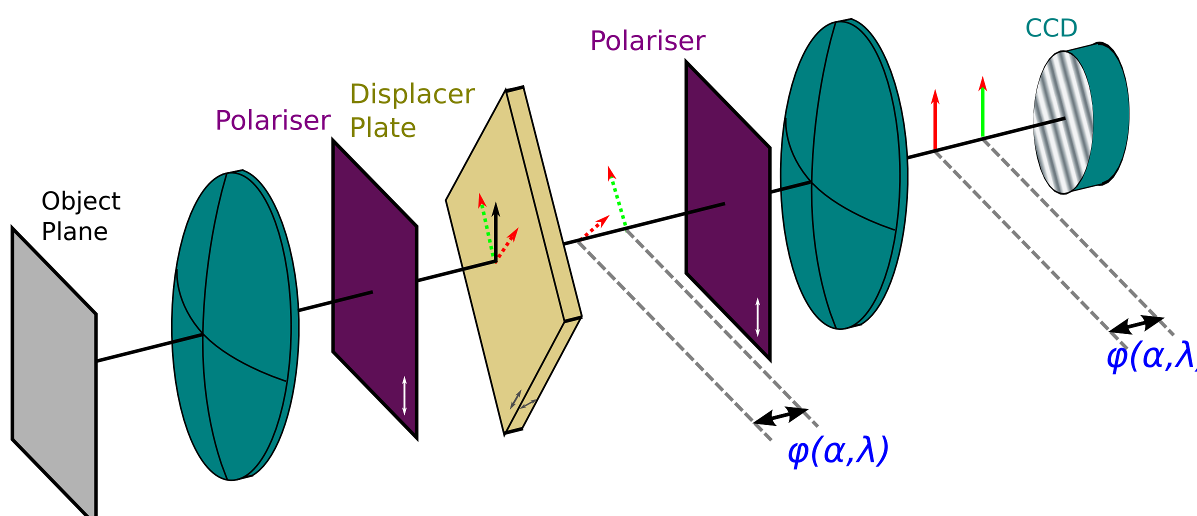




Phase ⟶ Wavelength ⟶ Velocity





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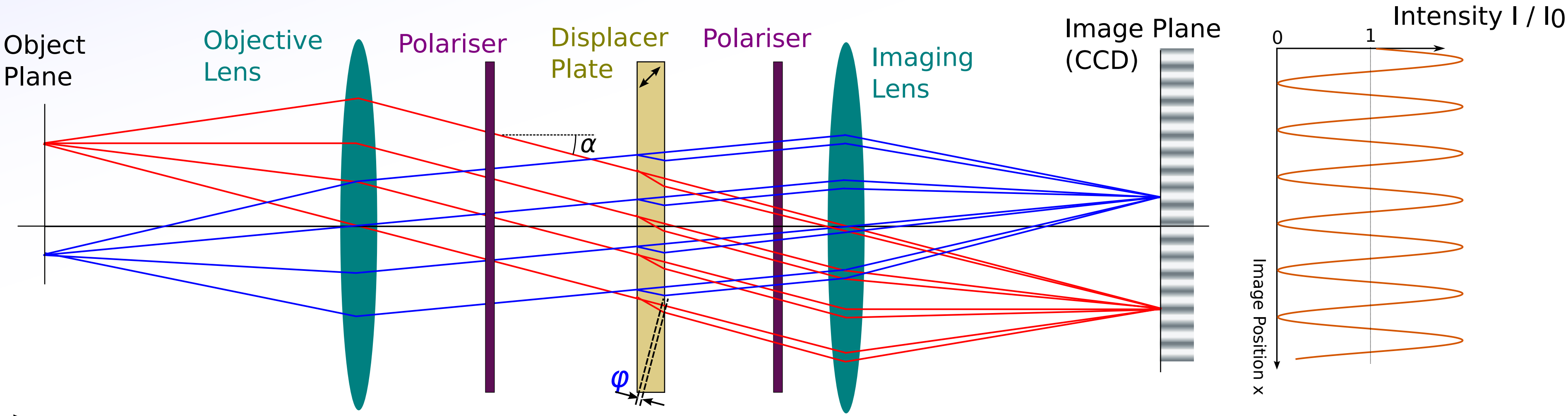
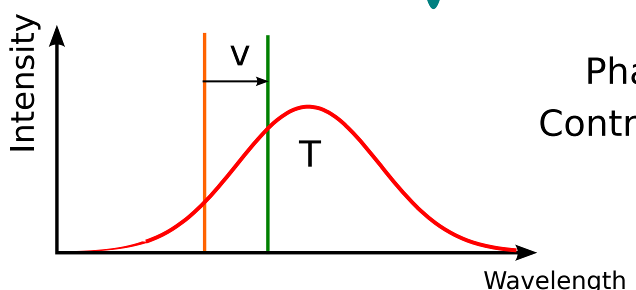
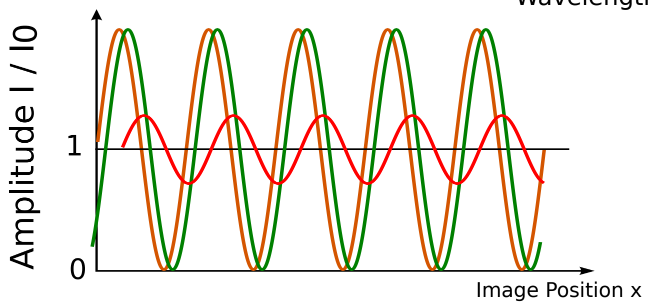
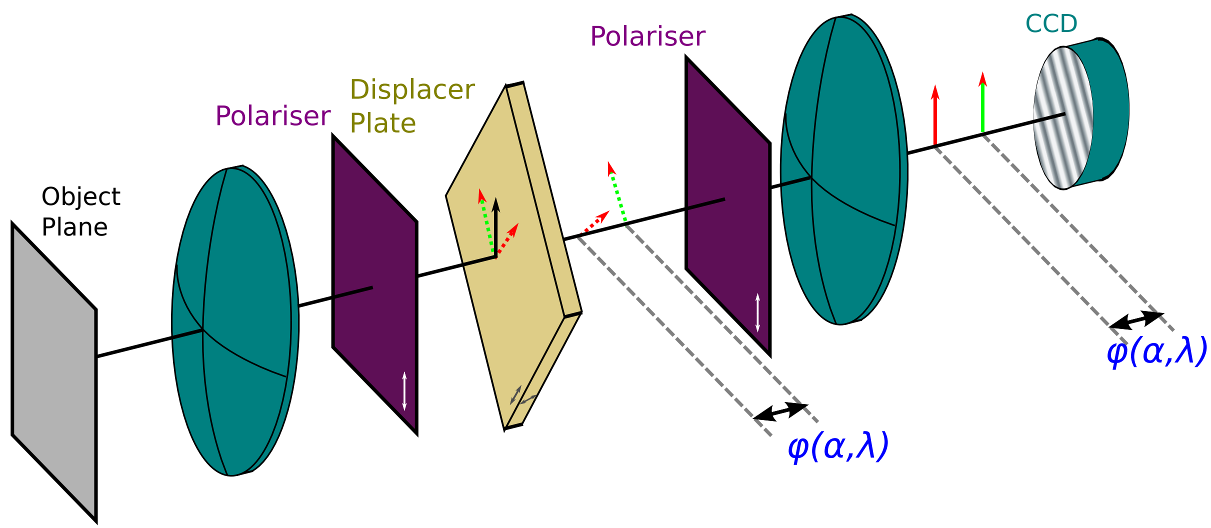




Phase → Wavelength → Velocity
Contrast → Width → Temperature





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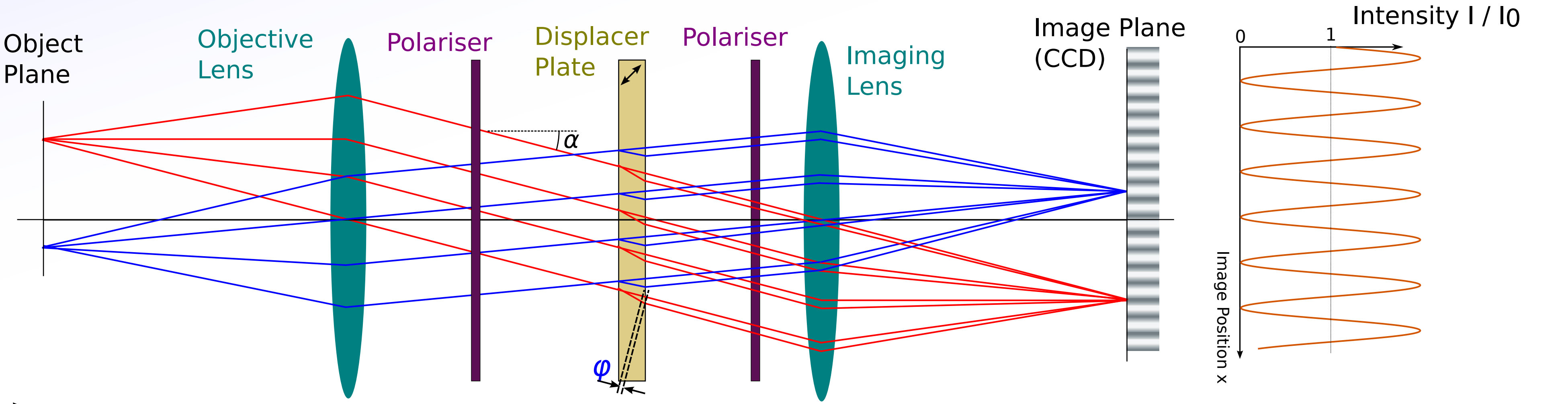
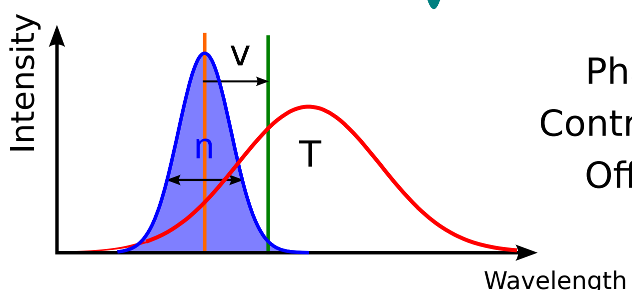
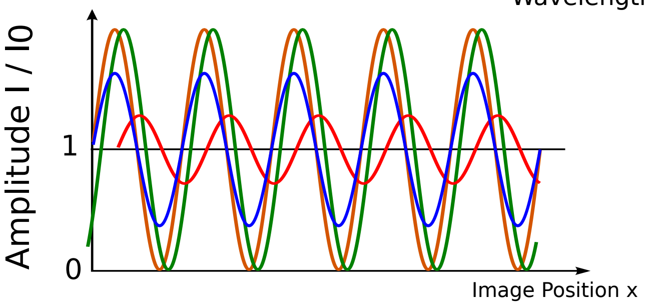
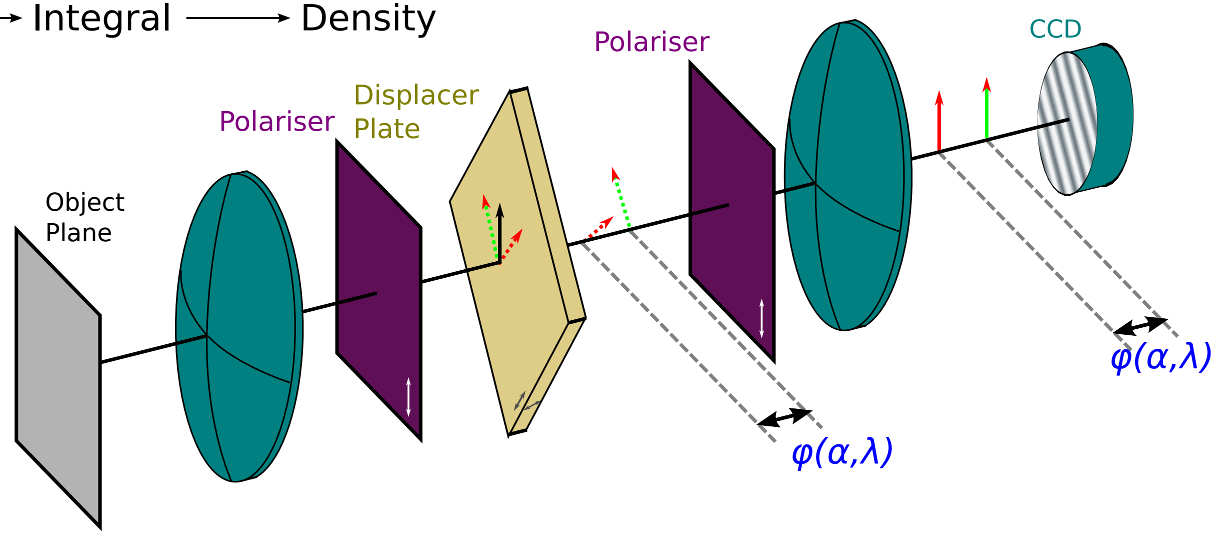




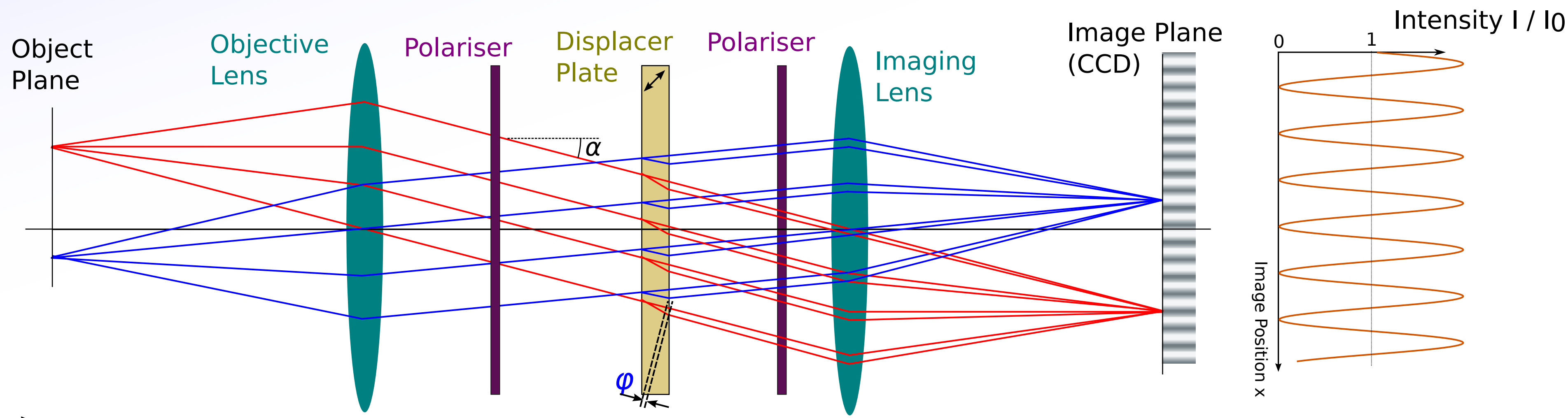
Phase → Wavelength → Velocity
Contrast → Width → Temperature
Offset → Integral → Density

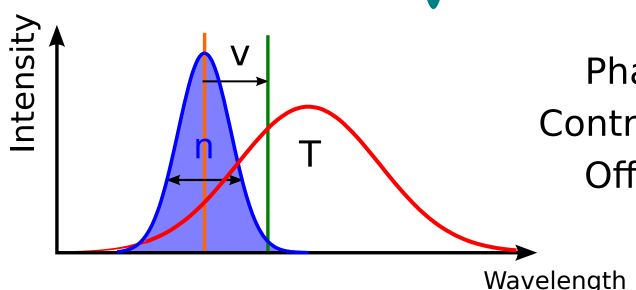




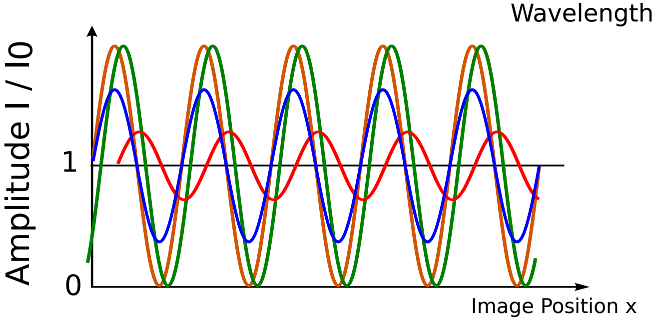
Coherence Imaging

Coherence Imaging: Spectroscopic technique, modulated in space and imaged with a CMOS camera.
Also used for imaging spectroscopic moments

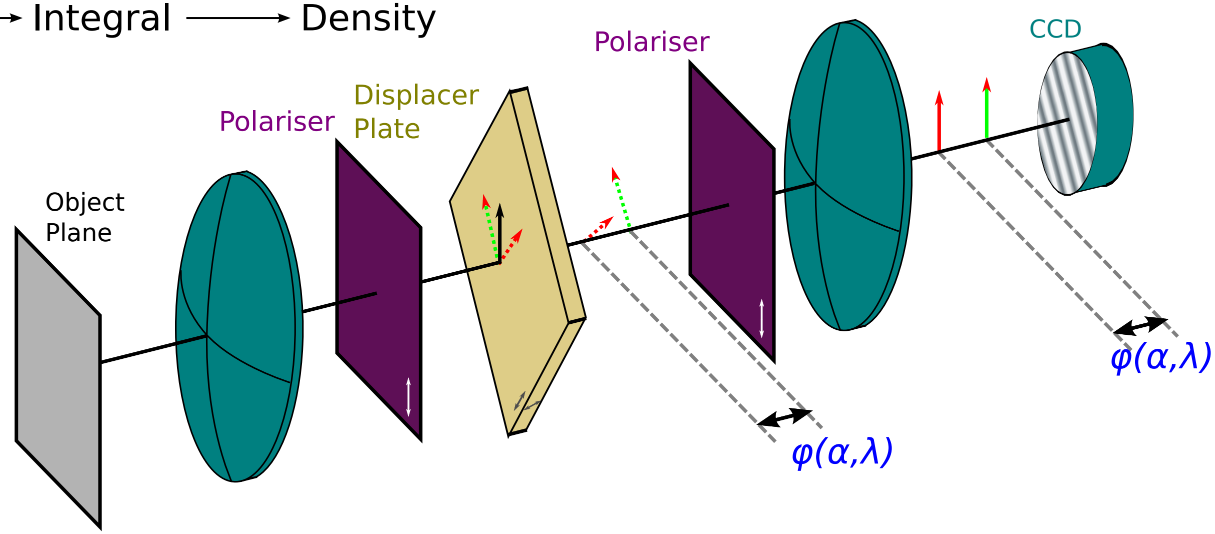




Phase → Wavelength → Velocity
Contrast → Width → Temperature
Offset → Integral → Density



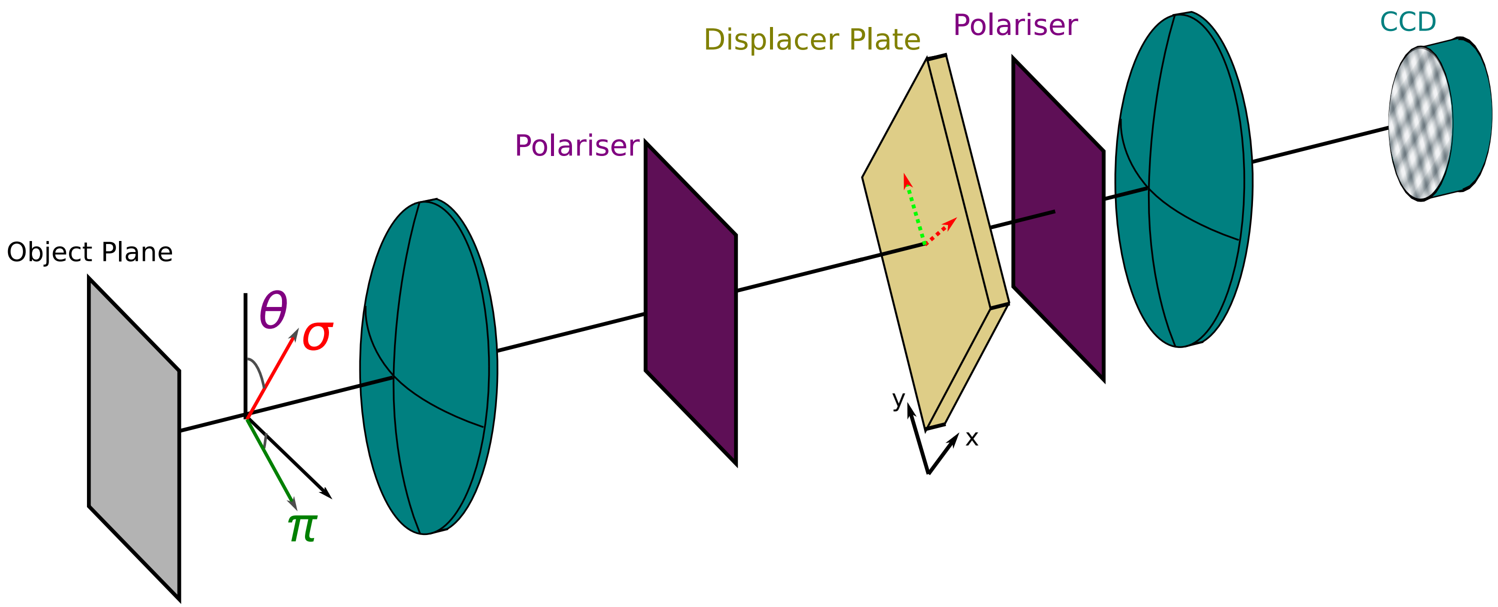
$$I \propto 1 + \zeta\, cos((\omega + \Delta\omega)x)$$



Multiplet Polarisation Coherence Imaging

Removing the first polariser gives a dependence on the initial polarisation:

$$I \propto 1 + \zeta \, cos2\theta \, cos(x)$$

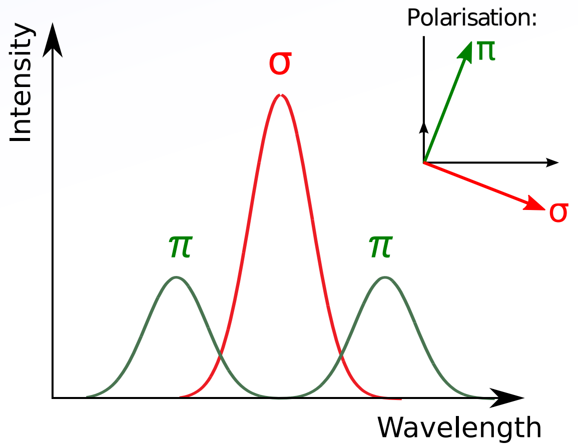


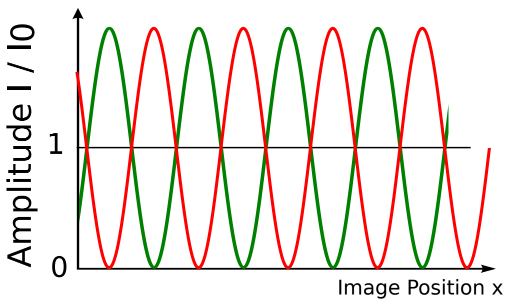
Multiplet Polarisation Coherence Imaging

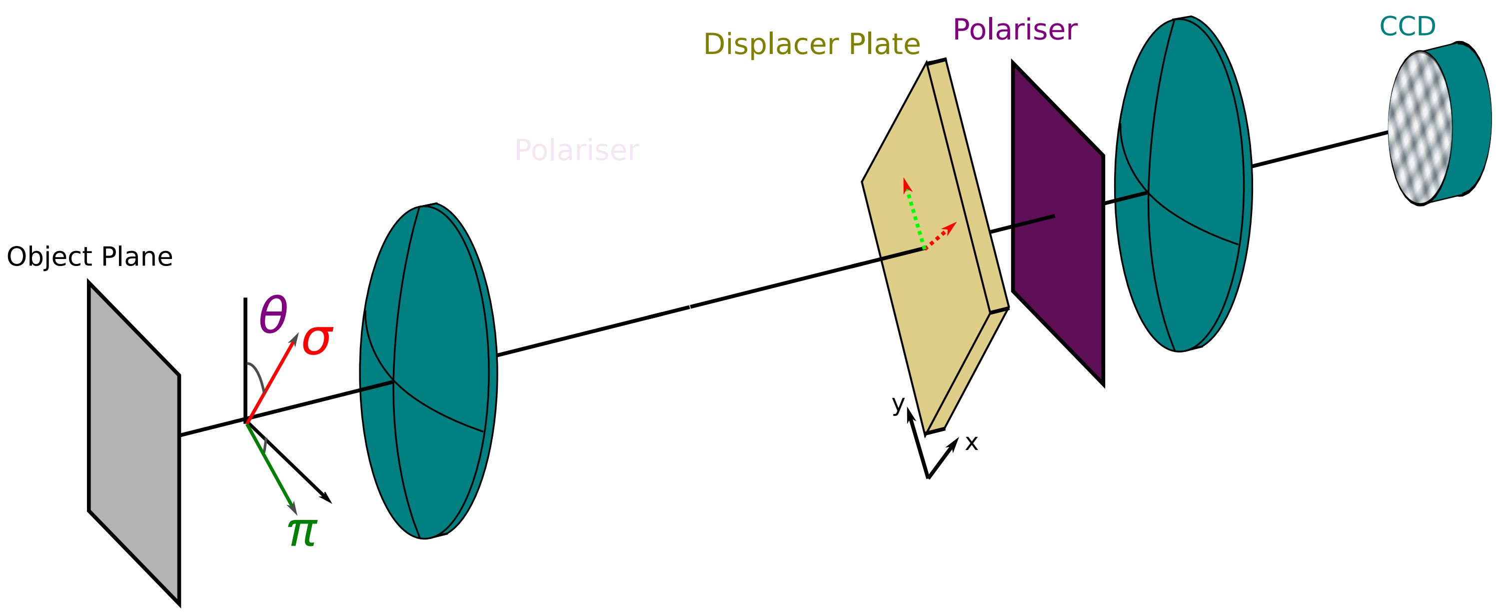
Removing the first polariser gives a dependence on the initial polarisation:

$$I \propto 1 + \zeta \, cos2\theta \, cos(x)$$

For the Stark/Zeeman spectrum, the π component is at 90° to σ, introducing a 180° phase shift, so they would cancel.







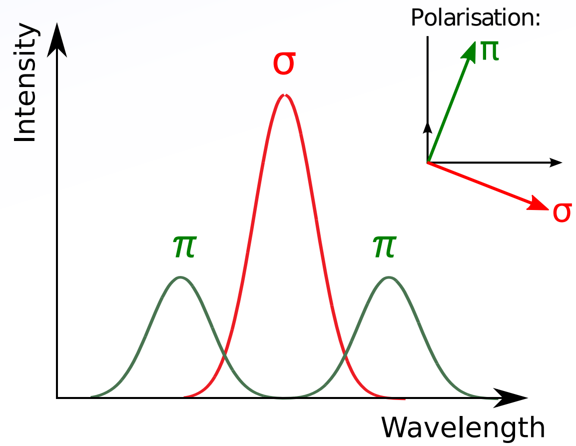
Multiplet Polarisation Coherence Imaging

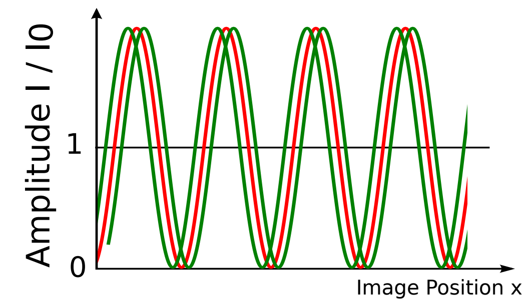
Removing the first polariser gives a dependence on the initial polarisation:

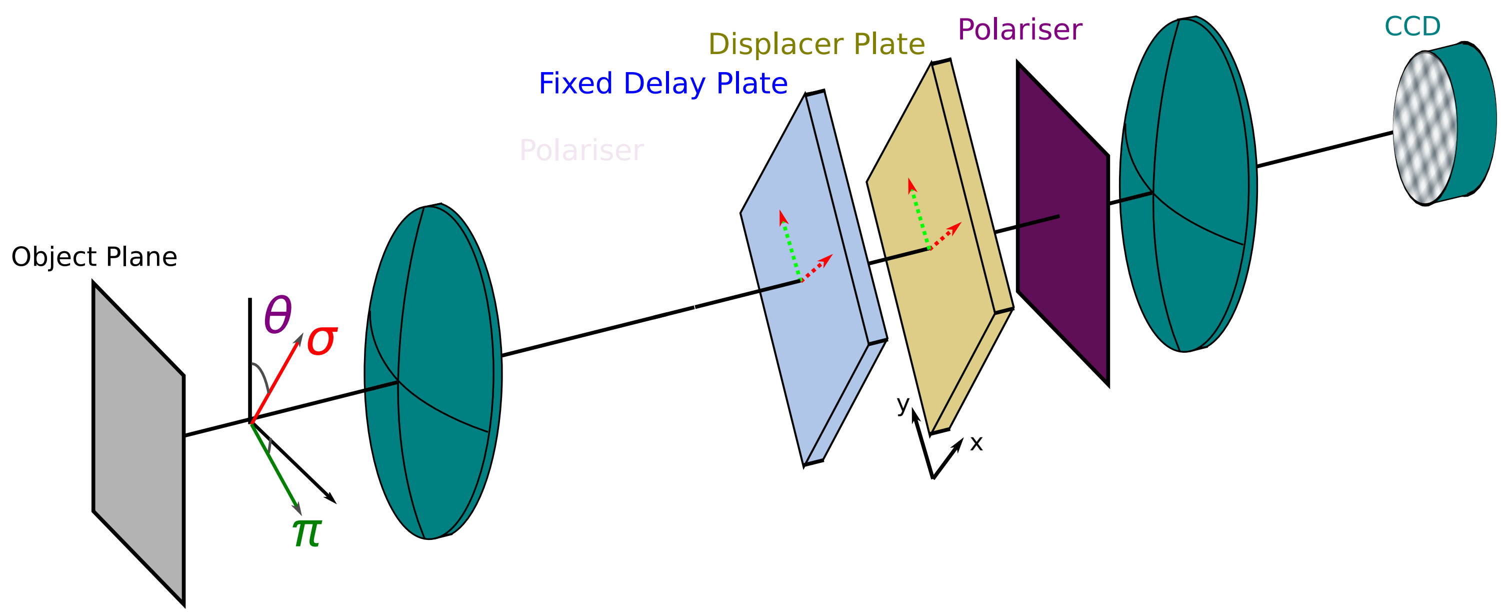
$$I \propto 1 + \zeta \cos 2\theta \cos(x)$$

For the Stark/Zeeman spectrum, the π component is at 90° to σ, introducing a 180° phase shift, so they would cancel.

At some specific plate thickness t, the phase of the π wings is 180° from σ. This cancels the 180° from the opposite polarisation, and the patterns add. We add a delay plate with the optimal τ_0.





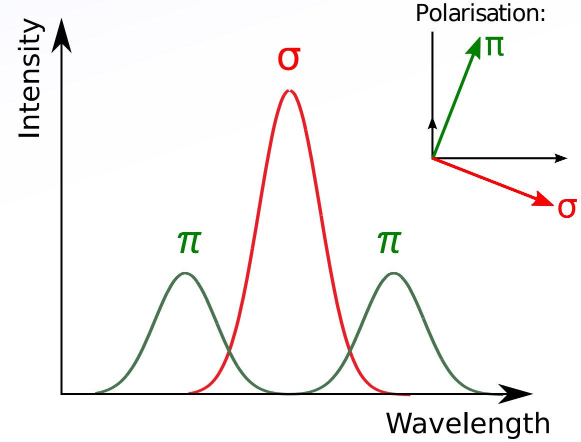


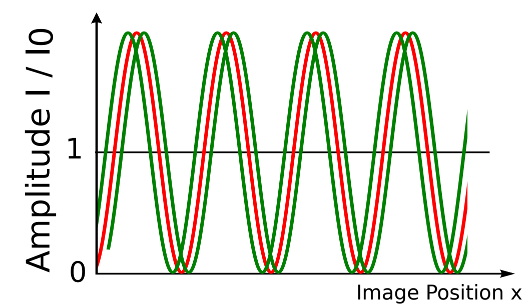
Multiplet Polarisation Coherence Imaging

Removing the first polariser gives a dependence on the initial polarisation:

$$I \propto 1 + \zeta\, cos2\theta\, cos(x)$$

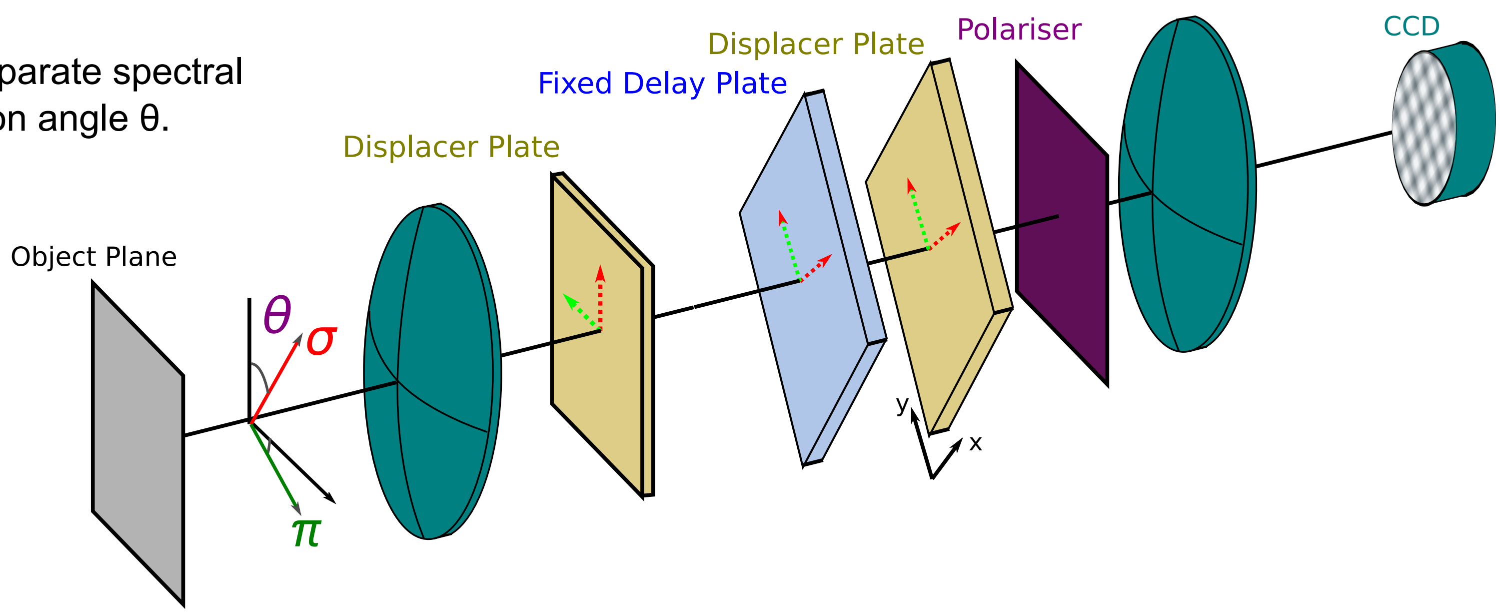
For the Stark/Zeeman spectrum, the π component is at 90° to σ, introducing a 180° phase shift, so they would cancel.





At some specific plate thickness t, the phase of the π wings is 180° from σ. This cancels the 180° from the opposite polarisation, and the patterns add. We add a delay plate with the optimal τ_0.

However, we now need to separate spectral contrast ζ from the polariastion angle θ.

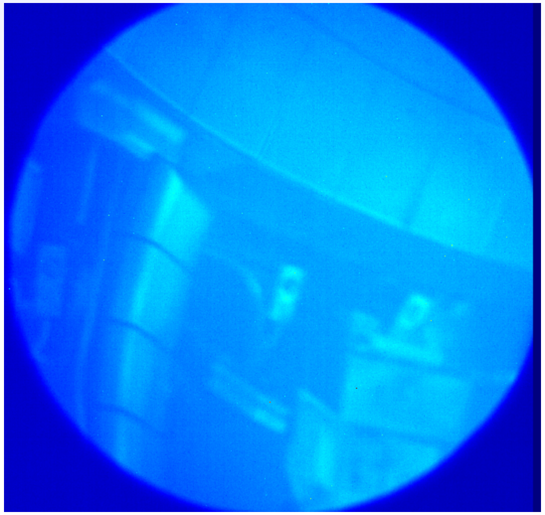


add another displacer at 45°. Combined effect adds 2 extra terms:

$$I \propto 1 + \zeta\, cos2\theta\, cos(x) + \zeta\, sin2\theta\, cos(x-y) - \zeta\, sin2\theta\, cos(x+y)$$

Imaging Motional Stark Effect results

Raw image (without neutral beam)



Imaging Motional Stark Effect results

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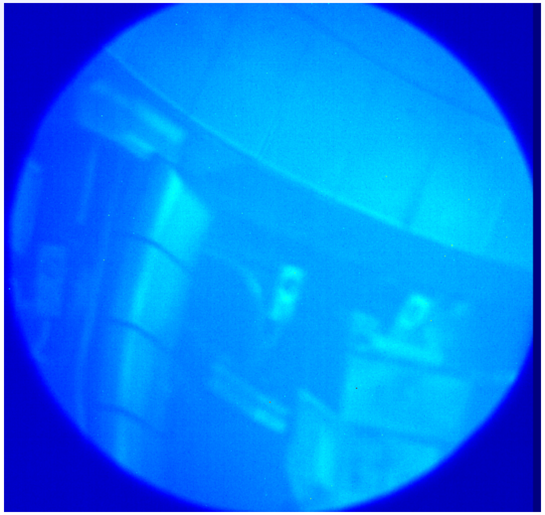
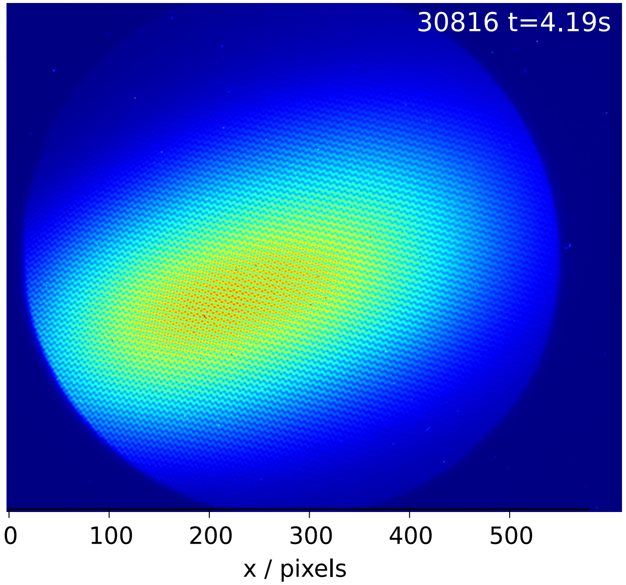


Raw image (with neutral beam)



Imaging Motional Stark Effect results

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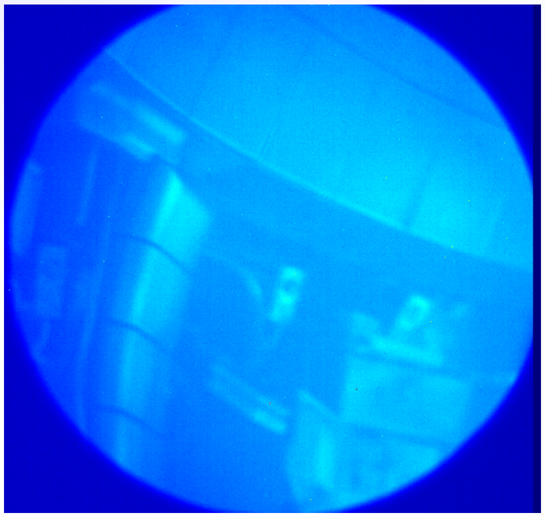
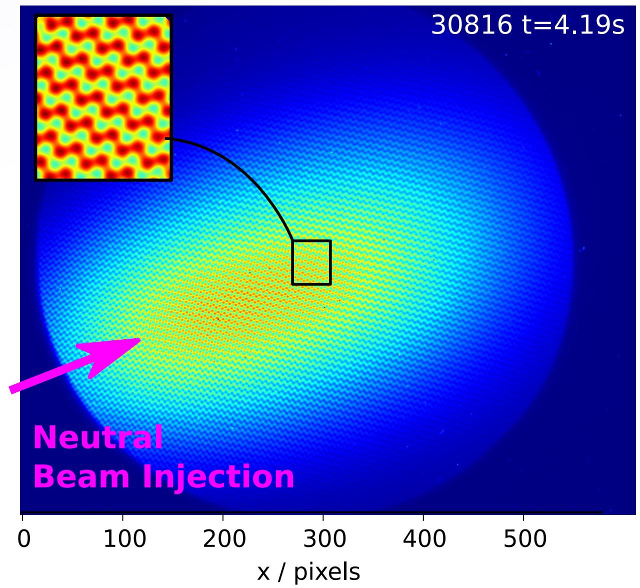


Raw image (with neutral beam)



Imaging Motional Stark Effect results

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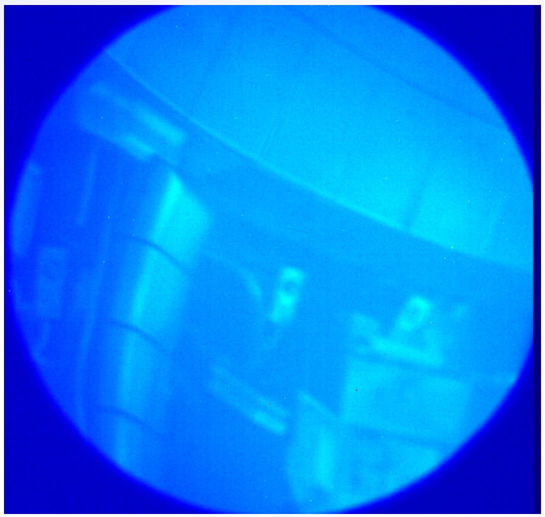
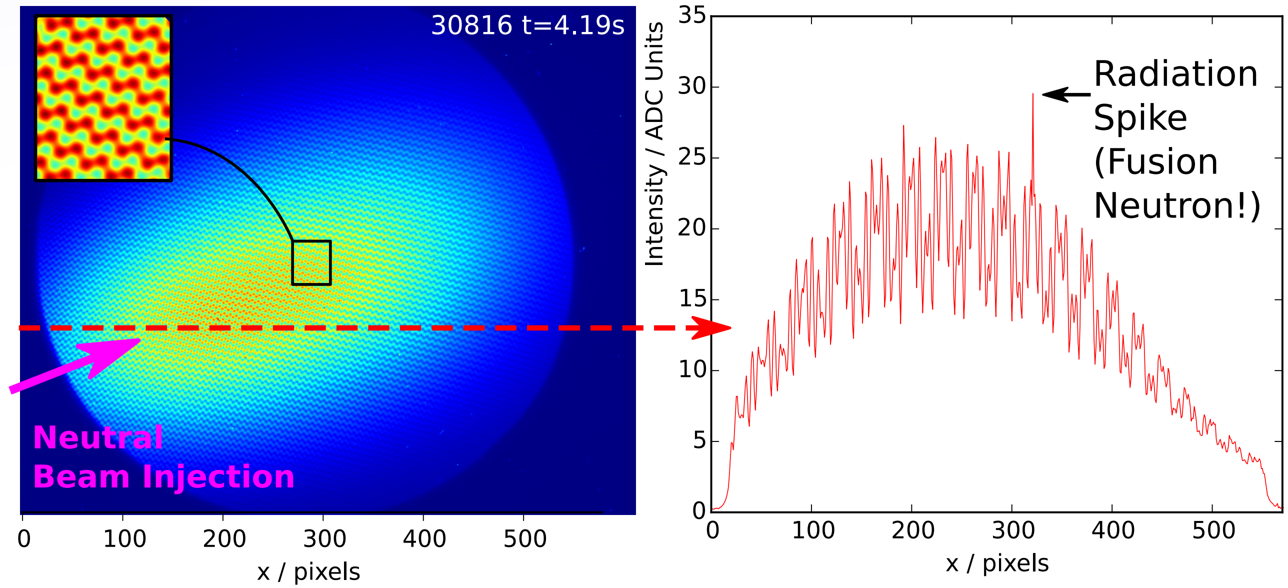
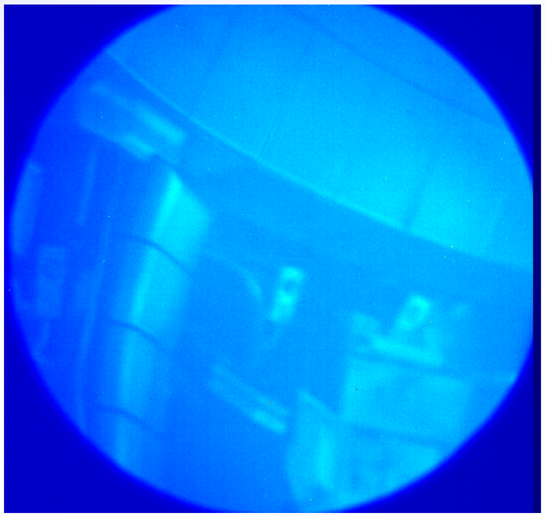


Raw image (with neutral beam)

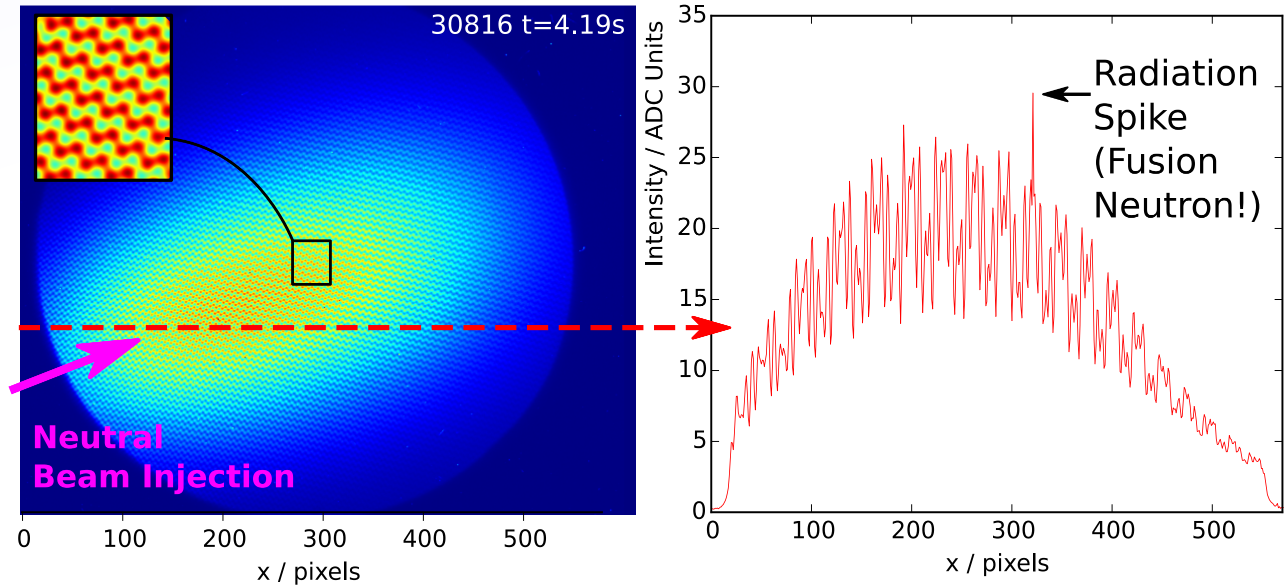


Imaging Motional Stark Effect results

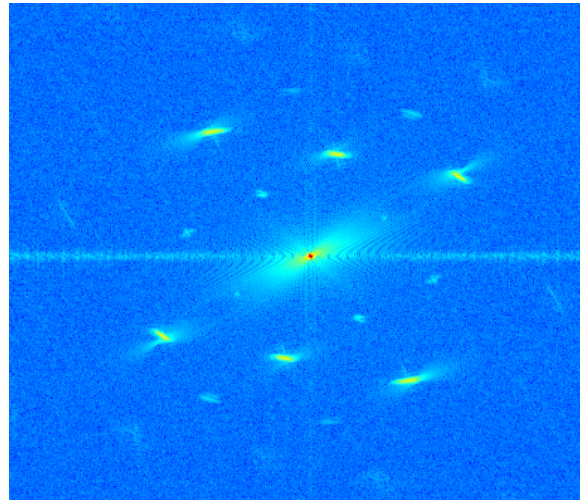
Raw image (without neutral beam)



Raw image (with neutral beam)



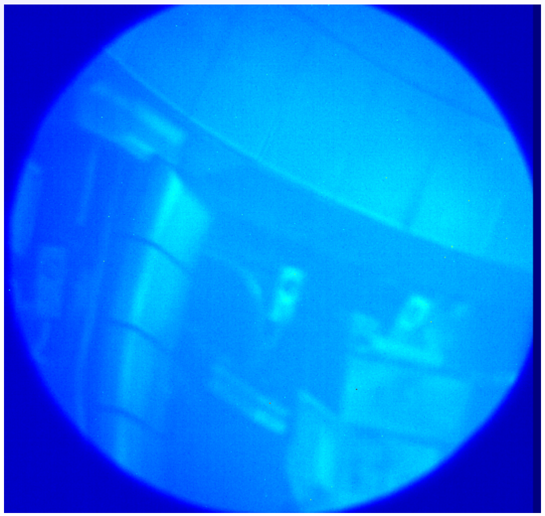
Fourier transform



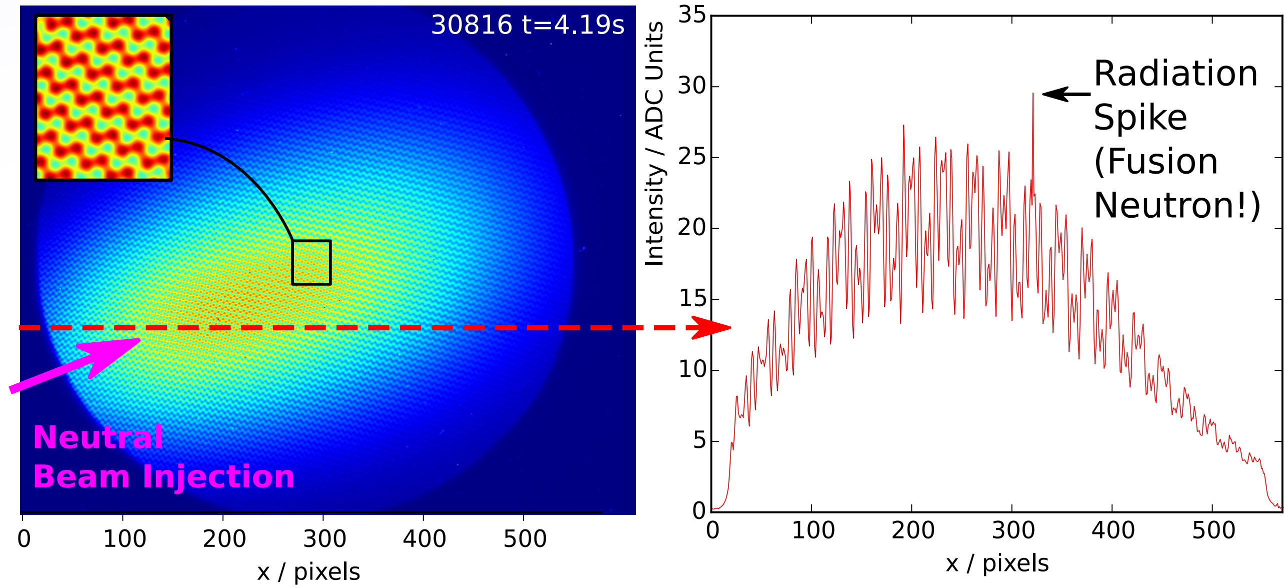
$$
\begin{aligned}
I \propto 1 &+ \zeta\, cos2\theta\, cos(x) \\
&+ \zeta\, sin2\theta\, cos(x - y) \\
&- \zeta\, sin2\theta\, cos(x + y)
\end{aligned}
$$

Imaging Motional Stark Effect results

Raw image (without neutral beam)

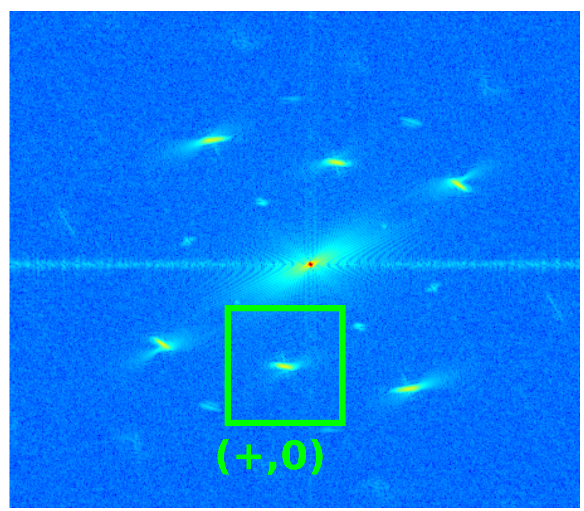


Raw image (with neutral beam)



$$I \propto 1 + \zeta\, cos2\theta\, cos(x) + \zeta\, sin2\theta\, cos(x-y) - \zeta\, sin2\theta\, cos(x+y)$$

Fourier transform



Imaging Motional Stark Effect results

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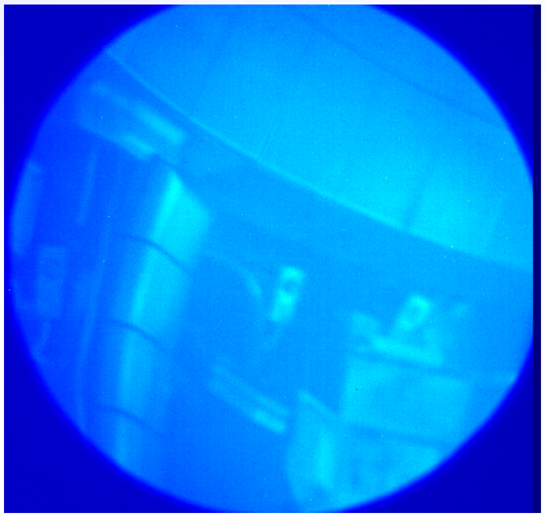
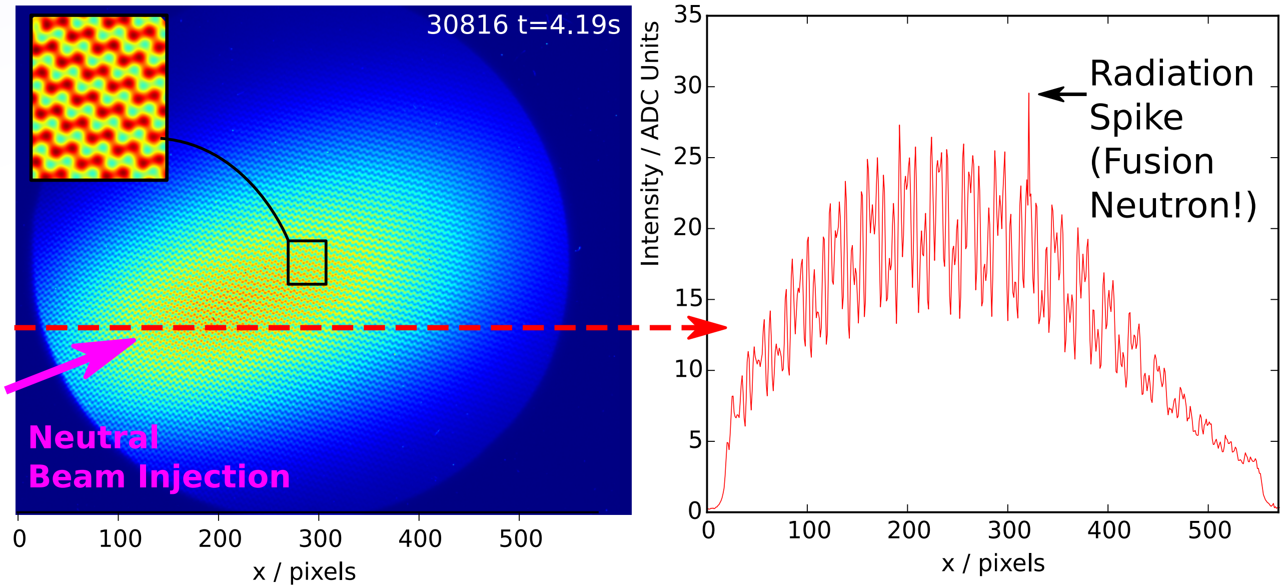
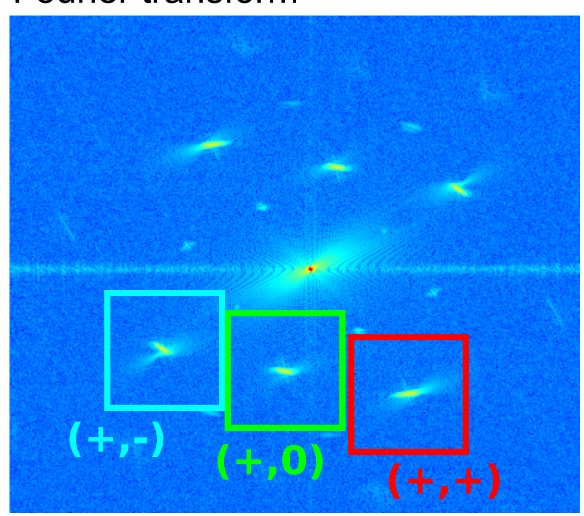


Raw image (with neutral beam)



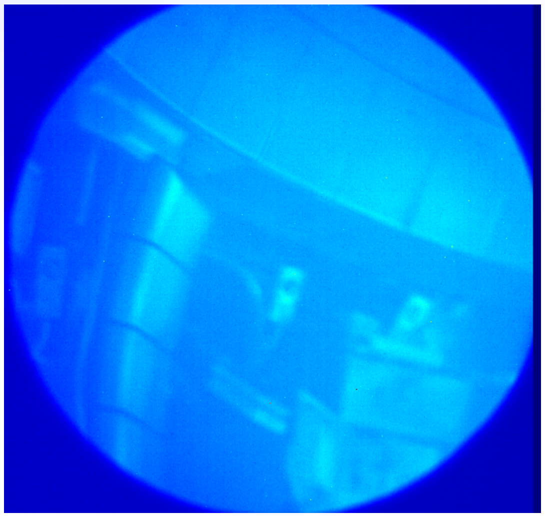
Fourier transform



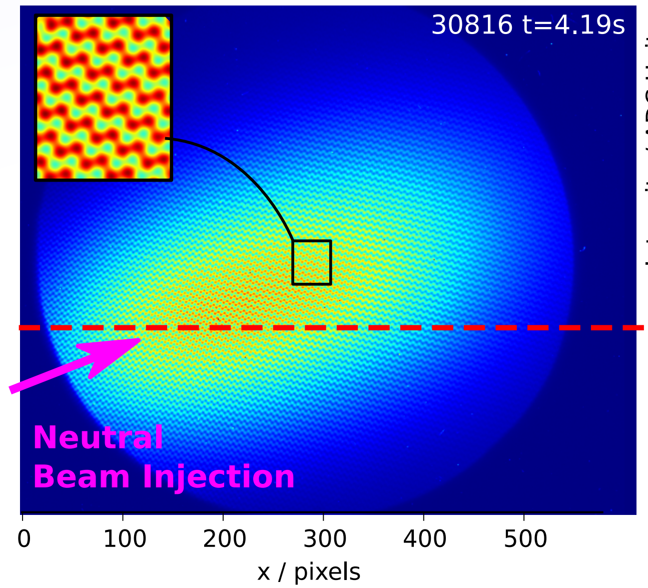
$$
\begin{aligned}
I \propto 1 &+ \zeta\, cos2\theta\, cos(x) \\
&+ \zeta\, sin2\theta\, cos(x-y) \\
&- \zeta\, sin2\theta\, cos(x+y)
\end{aligned}
$$

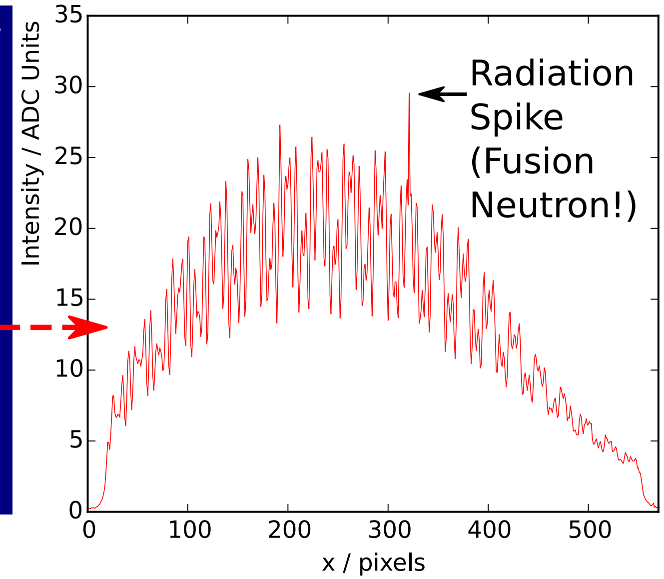
Imaging Motional Stark Effect results

Raw image (without neutral beam)



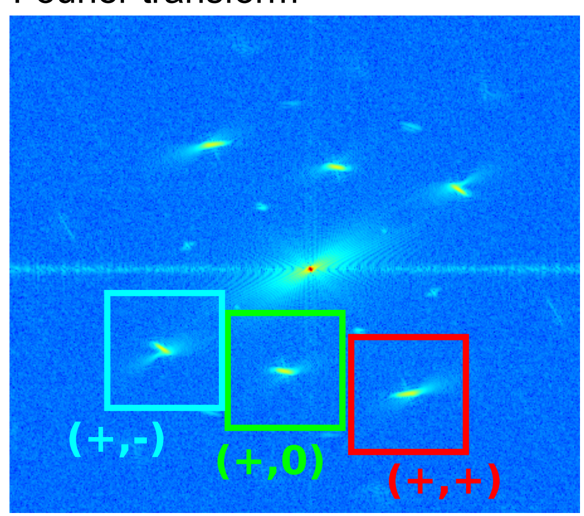
Raw image (with neutral beam)



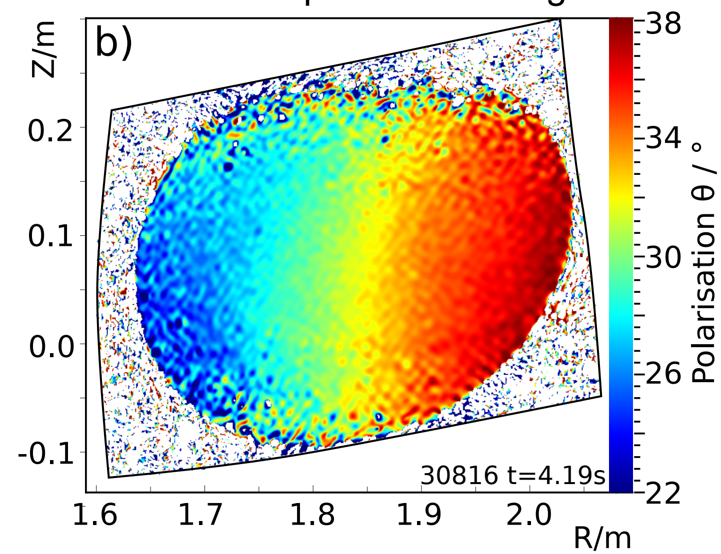


$$I \propto 1 + \zeta\, cos2\theta\, cos(x) + \zeta\, sin2\theta\, cos(x-y) - \zeta\, sin2\theta\, cos(x+y)$$

Fourier transform



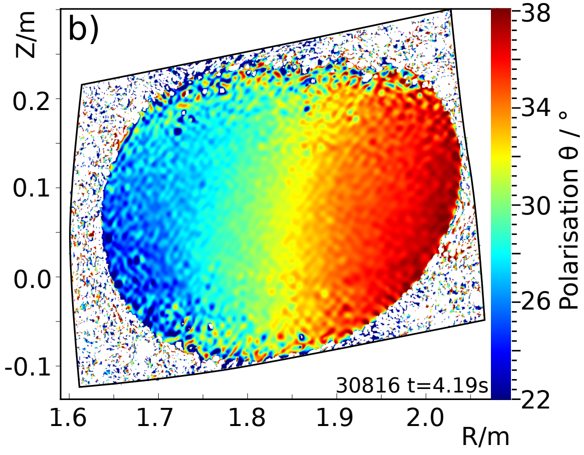
Demodulated polarisation angle



Imaging Motional Stark Effect results

What does θ tell us about j?
Maybe we have enough data now to image j directly?

Demodulated polarisation angle



Imaging Motional Stark Effect results

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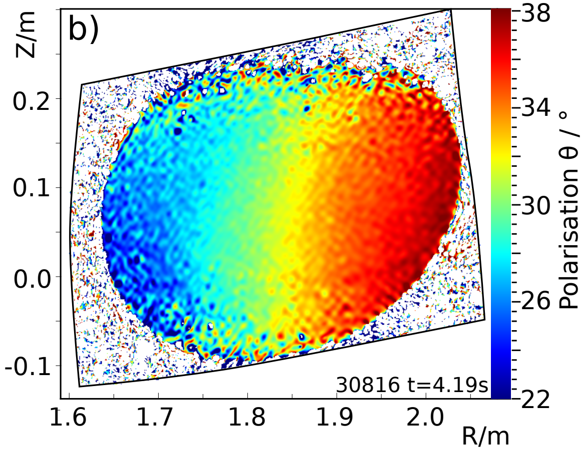
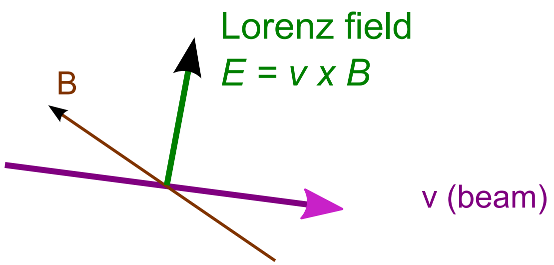
- π is in Lorenz electric field direction.

$$\underline{\mathrm{E}} = \underline{\mathrm{v}} \times \underline{\mathrm{B}}$$

Demodulated polarisation angle





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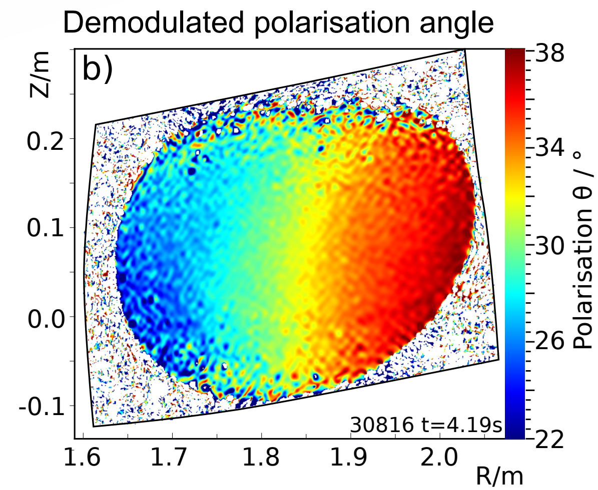
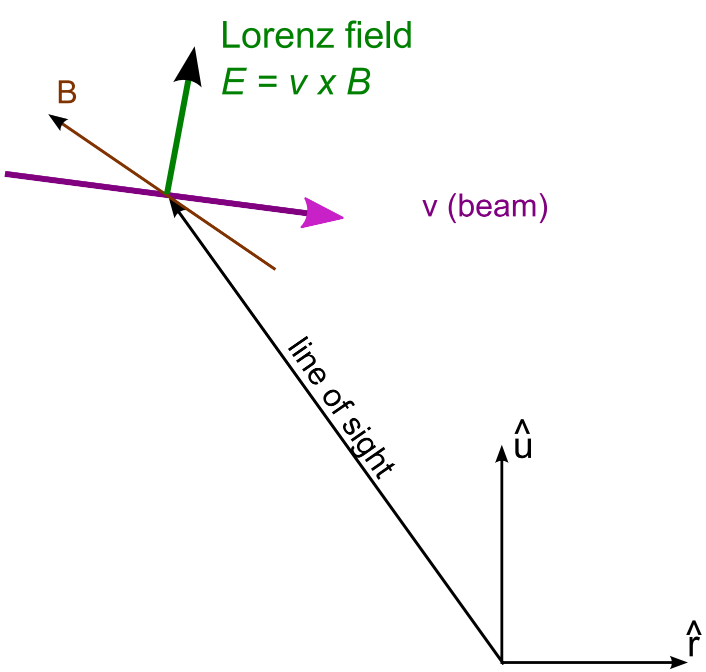
- π is in Lorenz electric field direction.

$$\underline{\mathrm{E}} = \underline{\mathrm{v}} \times \underline{\mathrm{B}}$$

- θ is the projection of this onto the measurement plane (û, r̂)

$$\tan\theta = \frac{(\underline{\mathrm{v}} \times \underline{\mathrm{B}}) \cdot \hat{\underline{\mathrm{r}}}}{(\underline{\mathrm{v}} \times \underline{\mathrm{B}}) \cdot \hat{\underline{\mathrm{u}}}}$$





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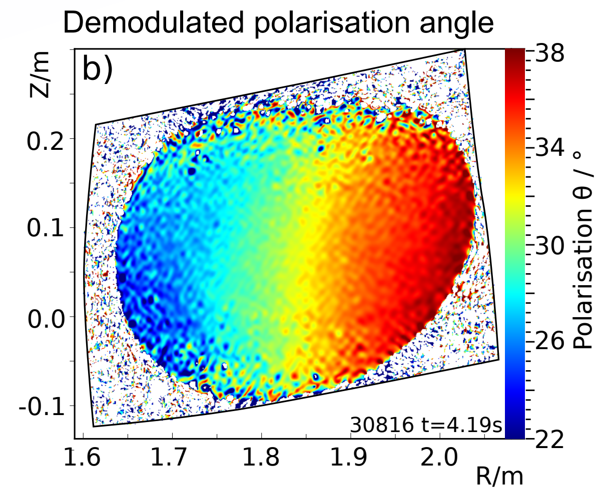

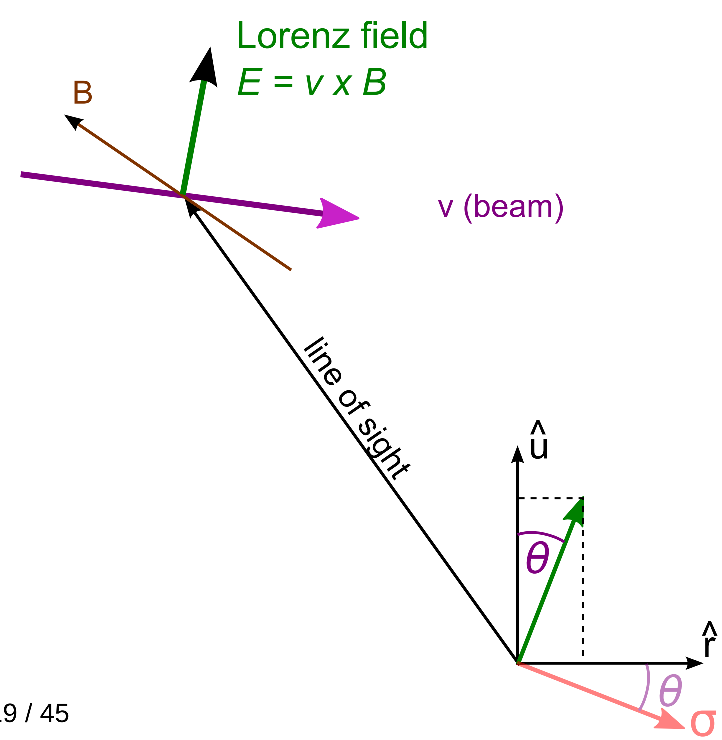

Imaging Motional Stark Effect results

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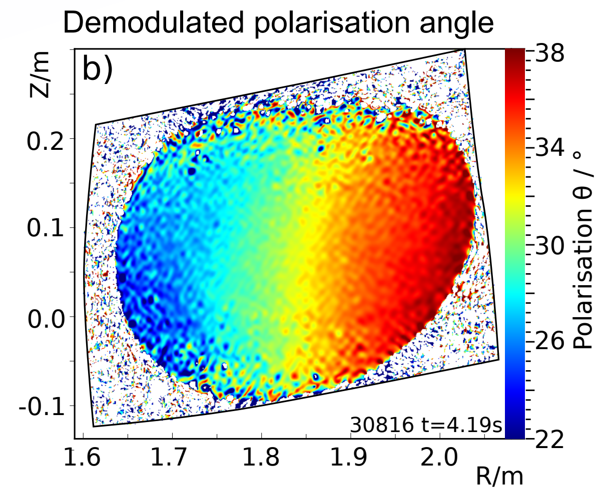
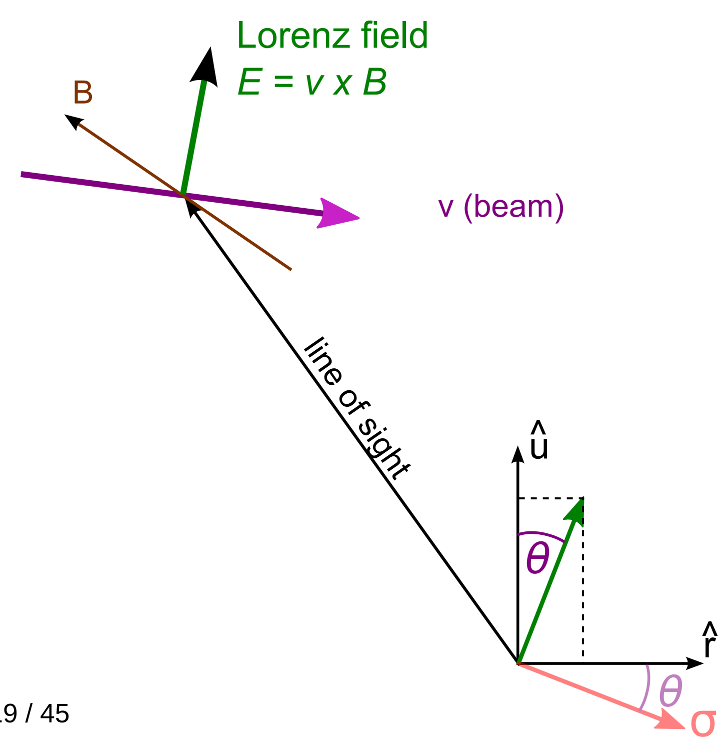


Choose offset to θ, such that θ = 0° at magnetic axis ($u = v \times \varphi$)

$$\begin{aligned}\tan\theta \;=\; & \frac{(\underline{\mathrm{v}} \times \underline{\hat{\phi}} \cdot \underline{\hat{\mathrm{r}}})}{(\underline{\mathrm{v}} \times \underline{\hat{\phi}} \cdot \underline{\hat{\mathrm{u}}})} \\ & + \left[\frac{(\underline{\mathrm{v}} \times \underline{\hat{\mathrm{R}}} \cdot \underline{\hat{\mathrm{r}}})}{(\underline{\mathrm{v}} \times \underline{\hat{\phi}} \cdot \underline{\hat{\mathrm{u}}})} - \frac{(\underline{\mathrm{v}} \times \underline{\hat{\phi}} \cdot \underline{\hat{\mathrm{r}}})}{(\underline{\mathrm{v}} \times \underline{\hat{\phi}} \cdot \underline{\hat{\mathrm{u}}})}\frac{(\underline{\mathrm{v}} \times \underline{\hat{\mathrm{R}}} \cdot \underline{\hat{\mathrm{u}}})}{(\underline{\mathrm{v}} \times \underline{\hat{\phi}} \cdot \underline{\hat{\mathrm{u}}})}\right]\frac{B_R}{B_\phi} \\ & + \left[\frac{(\underline{\mathrm{v}} \times \underline{\hat{\mathrm{Z}}} \cdot \underline{\hat{\mathrm{r}}})}{(\underline{\mathrm{v}} \times \underline{\hat{\phi}} \cdot \underline{\hat{\mathrm{u}}})} - \frac{(\underline{\mathrm{v}} \times \underline{\hat{\phi}} \cdot \underline{\hat{\mathrm{r}}})}{(\underline{\mathrm{v}} \times \underline{\hat{\phi}} \cdot \underline{\hat{\mathrm{u}}})}\frac{(\underline{\mathrm{v}} \times \underline{\hat{\mathrm{Z}}} \cdot \underline{\hat{\mathrm{u}}})}{(\underline{\mathrm{v}} \times \underline{\hat{\phi}} \cdot \underline{\hat{\mathrm{u}}})}\right]\frac{B_Z}{B_\phi}\end{aligned}$$



Imaging Motional Stark Effect results

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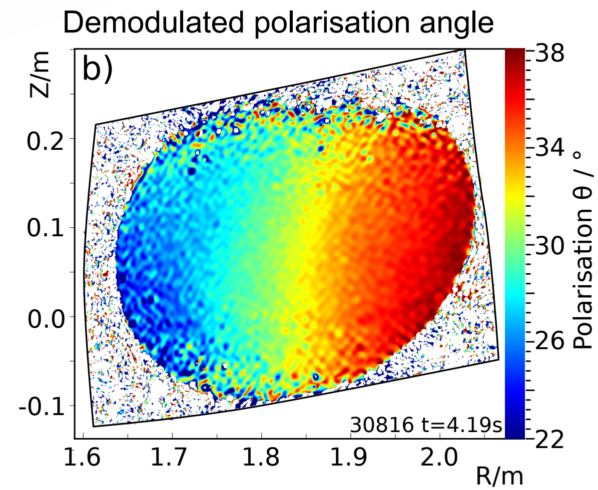
$$\tan\theta = \frac{(\underline{v} \times \underline{B}) \cdot \hat{\underline{r}}}{(\underline{v} \times \underline{B}) \cdot \hat{\underline{u}}}$$

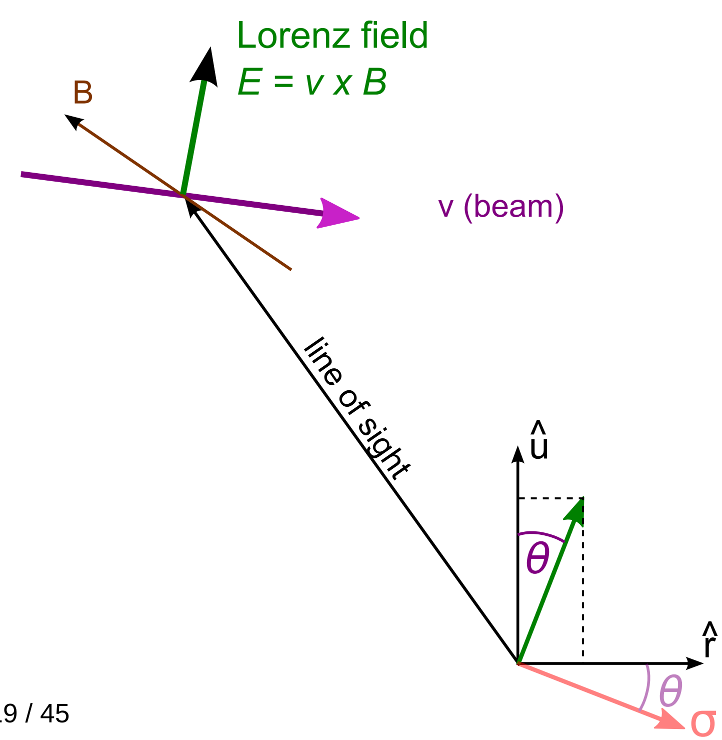


Choose offset to θ, such that θ = 0° at magnetic axis ($u = v \times \varphi$)

$$\tan\theta = \frac{(\underline{v} \times \hat{\underline{\phi}} \cdot \hat{\underline{r}})}{(\underline{v} \times \hat{\underline{\phi}} \cdot \hat{\underline{u}})} + \left[\frac{(\underline{v} \times \hat{\underline{R}} \cdot \hat{\underline{r}})}{(\underline{v} \times \hat{\underline{\phi}} \cdot \hat{\underline{u}})} - \frac{(\underline{v} \times \hat{\underline{\phi}} \cdot \hat{\underline{r}})}{(\underline{v} \times \hat{\underline{\phi}} \cdot \hat{\underline{u}})} \frac{(\underline{v} \times \hat{\underline{R}} \cdot \hat{\underline{u}})}{(\underline{v} \times \hat{\underline{\phi}} \cdot \hat{\underline{u}})} \right] \frac{B_R}{B_\phi} + \left[\frac{(\underline{v} \times \hat{\underline{Z}} \cdot \hat{\underline{r}})}{(\underline{v} \times \hat{\underline{\phi}} \cdot \hat{\underline{u}})} - \frac{(\underline{v} \times \hat{\underline{\phi}} \cdot \hat{\underline{r}})}{(\underline{v} \times \hat{\underline{\phi}} \cdot \hat{\underline{u}})} \frac{(\underline{v} \times \hat{\underline{Z}} \cdot \hat{\underline{u}})}{(\underline{v} \times \hat{\underline{\phi}} \cdot \hat{\underline{u}})} \right] \frac{B_Z}{B_\phi}$$

$$\tan\theta = 0 + [\,0.05\,] \frac{B_R}{B_\phi} + [\,0.6\,] \frac{B_Z}{B_\phi}$$

$$\tan\theta = 0.6 \frac{B_Z}{B_\phi}$$



Imaging Motional Stark Effect results

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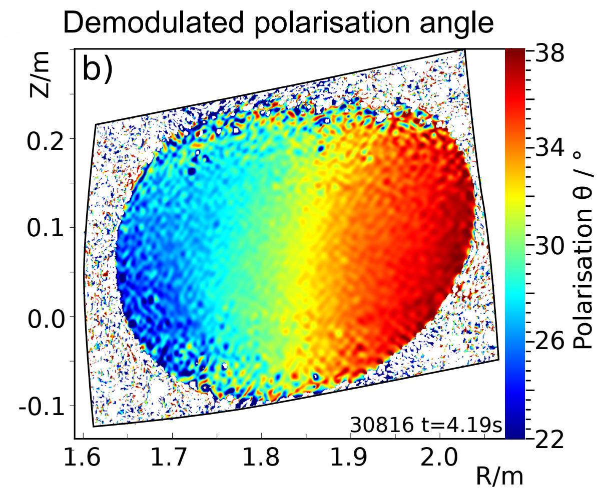
$$\underline{E} = \underline{v} \times \underline{B}$$

- θ is the projection of this onto the measurement plane (û, r̂)

$$\tan\theta = \frac{(\underline{v} \times \underline{B}) \cdot \hat{\underline{r}}}{(\underline{v} \times \underline{B}) \cdot \hat{\underline{u}}}$$



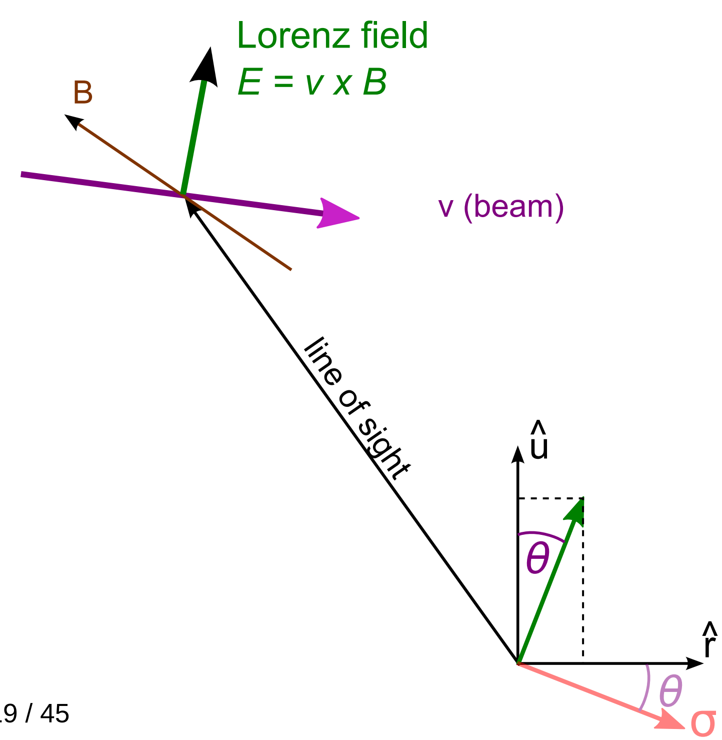
Choose offset to θ, such that θ = 0° at magnetic axis ($u = v \times \varphi$)

$$\tan\theta = \frac{(\underline{v} \times \hat{\phi} \cdot \hat{\underline{r}})}{(\underline{v} \times \hat{\phi} \cdot \hat{\underline{u}})} + \left[\frac{(\underline{v} \times \hat{R} \cdot \hat{\underline{r}})}{(\underline{v} \times \hat{\phi} \cdot \hat{\underline{u}})} - \frac{(\underline{v} \times \hat{\phi} \cdot \hat{\underline{r}})}{(\underline{v} \times \hat{\phi} \cdot \hat{\underline{u}})}\frac{(\underline{v} \times \hat{R} \cdot \hat{\underline{u}})}{(\underline{v} \times \hat{\phi} \cdot \hat{\underline{u}})}\right]\frac{B_R}{B_\phi} + \left[\frac{(\underline{v} \times \hat{Z} \cdot \hat{\underline{r}})}{(\underline{v} \times \hat{\phi} \cdot \hat{\underline{u}})} - \frac{(\underline{v} \times \hat{\phi} \cdot \hat{\underline{r}})}{(\underline{v} \times \hat{\phi} \cdot \hat{\underline{u}})}\frac{(\underline{v} \times \hat{Z} \cdot \hat{\underline{u}})}{(\underline{v} \times \hat{\phi} \cdot \hat{\underline{u}})}\right]\frac{B_Z}{B_\phi}$$

0   0.05   0.6

$$\tan\theta = 0.6\frac{B_Z}{B_\phi}$$

Magnetic field pitch angle.
Gives B_z since B_φ is well known.



Imaging Motional Stark Effect results

What does θ tell us about *j*?
Maybe we have enough data now to image *j* directly?

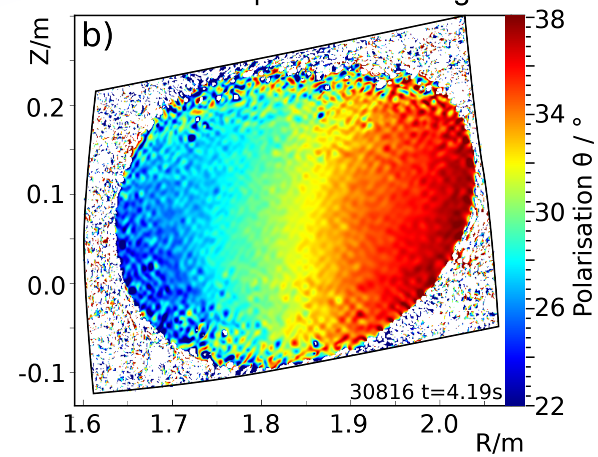
- π is in Lorenz electric field direction.

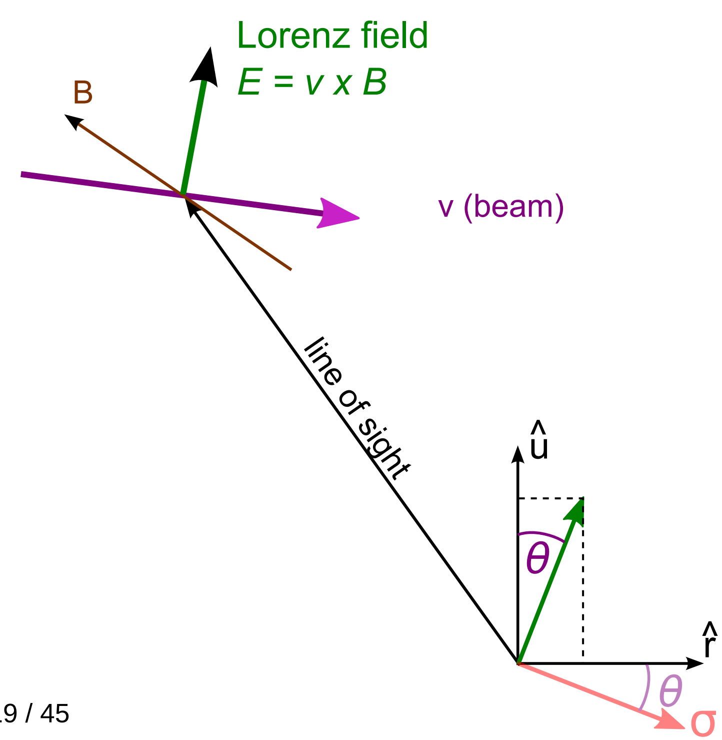
$$\underline{\mathrm{E}} = \underline{\mathrm{v}} \times \underline{\mathrm{B}}$$

- θ is the projection of this onto the measurement plane (û, r̂)

$$\tan\theta = \frac{(\underline{\mathrm{v}} \times \underline{\mathrm{B}}) \cdot \hat{\underline{\mathrm{r}}}}{(\underline{\mathrm{v}} \times \underline{\mathrm{B}}) \cdot \hat{\underline{\mathrm{u}}}}$$

Demodulated polarisation angle





Choose offset to θ, such that θ = 0° at magnetic axis (*u = v x φ*)

$$\tan\theta = \frac{(\underline{\mathrm{v}} \times \hat{\underline{\phi}} \cdot \hat{\underline{\mathrm{r}}})}{(\underline{\mathrm{v}} \times \hat{\underline{\phi}} \cdot \hat{\underline{\mathrm{u}}})} \;\; [0]$$

$$+ \left[\frac{(\underline{\mathrm{v}} \times \hat{\underline{\mathrm{R}}} \cdot \hat{\underline{\mathrm{r}}})}{(\underline{\mathrm{v}} \times \hat{\underline{\phi}} \cdot \hat{\underline{\mathrm{u}}})} - \frac{(\underline{\mathrm{v}} \times \hat{\underline{\phi}} \cdot \hat{\underline{\mathrm{r}}})}{(\underline{\mathrm{v}} \times \hat{\underline{\phi}} \cdot \hat{\underline{\mathrm{u}}})} \frac{(\underline{\mathrm{v}} \times \hat{\underline{\mathrm{R}}} \cdot \hat{\underline{\mathrm{u}}})}{(\underline{\mathrm{v}} \times \hat{\underline{\phi}} \cdot \hat{\underline{\mathrm{u}}})} \right] \frac{B_R}{B_\phi} \;\; [0.05]$$

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$$\tan\theta = 0.6 \frac{B_Z}{B_\phi}$$

Magnetic field pitch angle.
Gives B_z since B_φ is well known.

We can now make a B_z image:

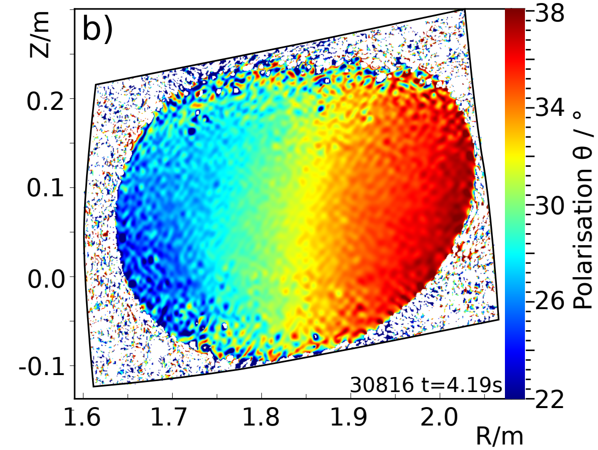
$$B_Z \approx \frac{1.7 B_0}{R} \theta$$

$B_0 = B_\varphi$ at R=1m

Imaging Motional Stark Effect results

What does B_z tell us about j in the core?

Demodulated polarisation angle



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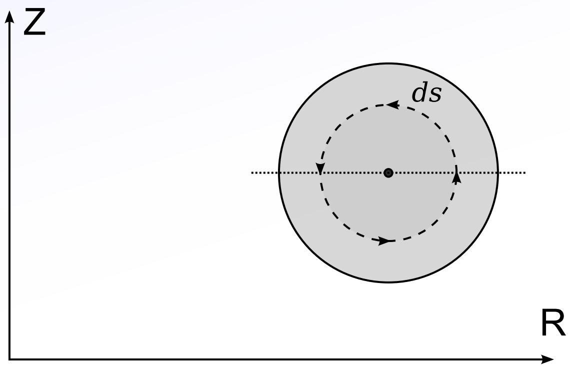
Imaging Motional Stark Effect results

What does B_z tell us about j in the core?

Large aspect ratio approximation (assume core is a cylinder)



$$\int B \cdot ds = \mu_0 \int j \, dA$$

$$B = \frac{1}{2}\mu_0 j r$$

$$\frac{dB_Z}{dR} = \frac{1}{2}\mu_0 j$$

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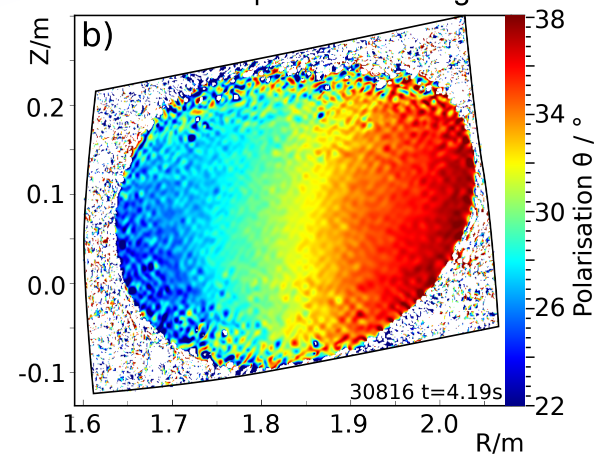

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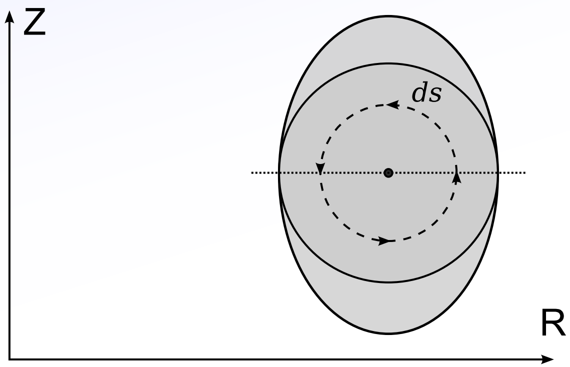

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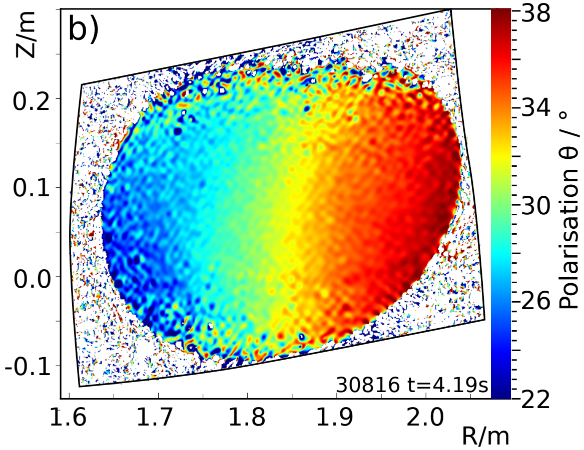
More generally elongation is important:

[CC.Petty Nucl. Fus. 2002]

$$\mu_0 j \approx -\left(1 + \tfrac{1}{\kappa^2}\right)\frac{dB_Z}{dR}$$

Demodulated polarisation angle



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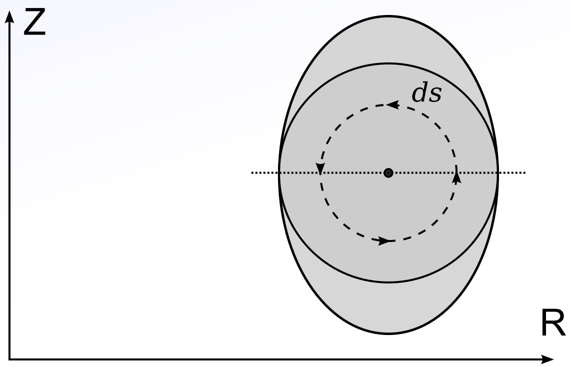

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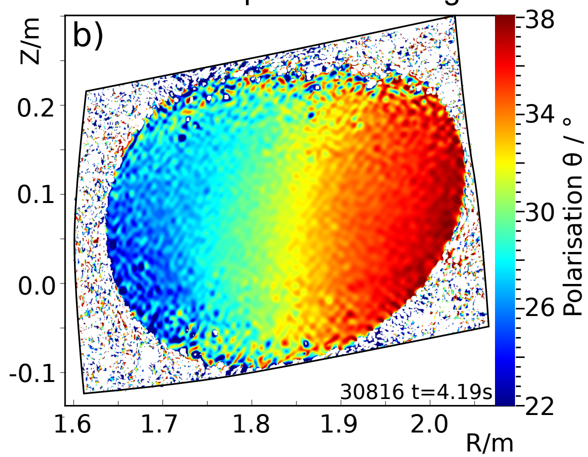

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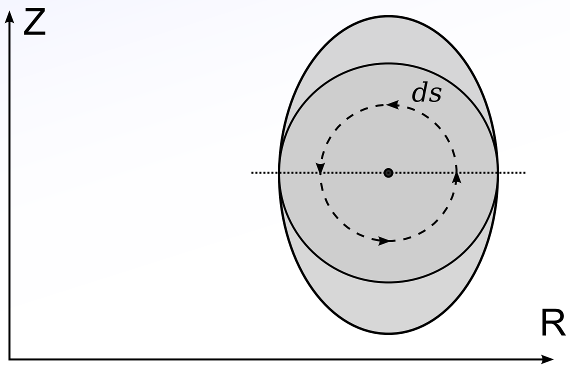

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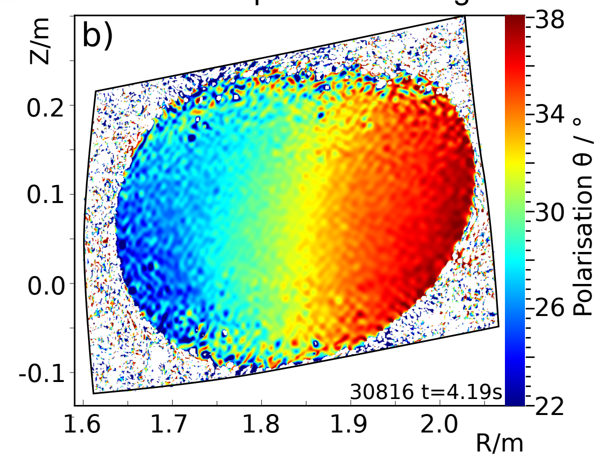

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Typically, in core:
R ~ 1.6m
θ < 5°
dθ/dR ~ 35°m^{-1}.
dθ/dR >> (θ/R)

To first order, local *j* relates to local derivative of measurement

This is only an approximation! ... but we now understand that *dθ/dR* holds the information about *j*.
What can we see in *dθ/dR* at the axis?

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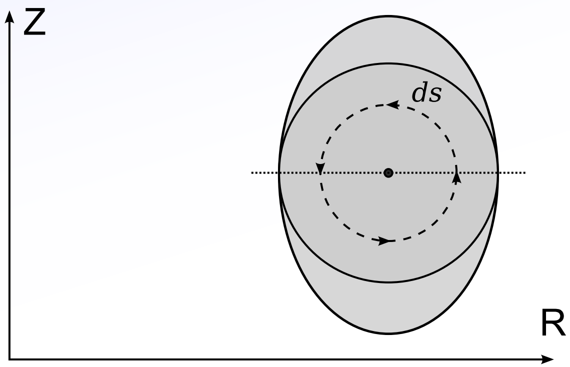

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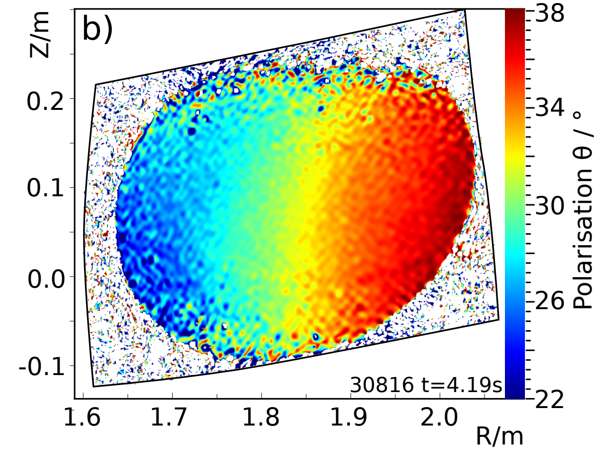

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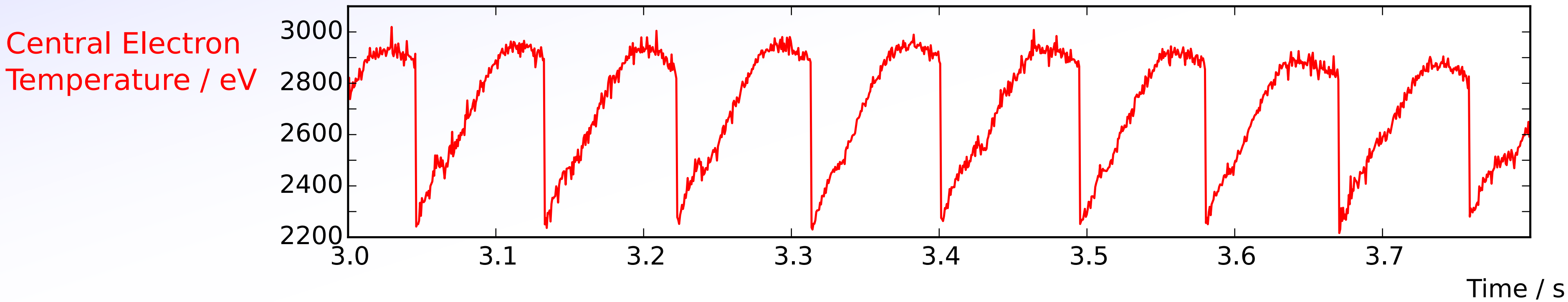
This is only an approximation! ... but we now understand that *dθ/dR* holds the information about *j*.
What can we see in *dθ/dR* at the axis?

Central safety factor also requires location of centre:

$$q_0 \approx \frac{2B_\phi}{\mu_0 j_0 R} \approx \frac{2B_\phi^{1m}}{\mu_0 j_0 R_0^2}$$

Sawteeth - Magnetic Reconnection



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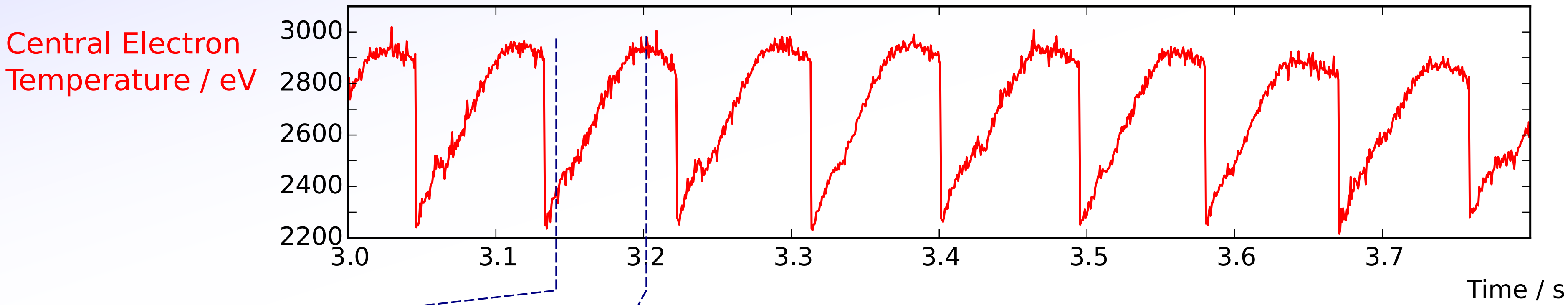
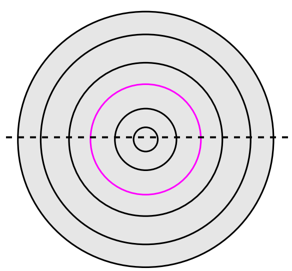
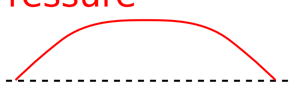


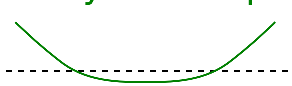
Slow build-up of
pressure and current.



Pressure



Safety Factor q



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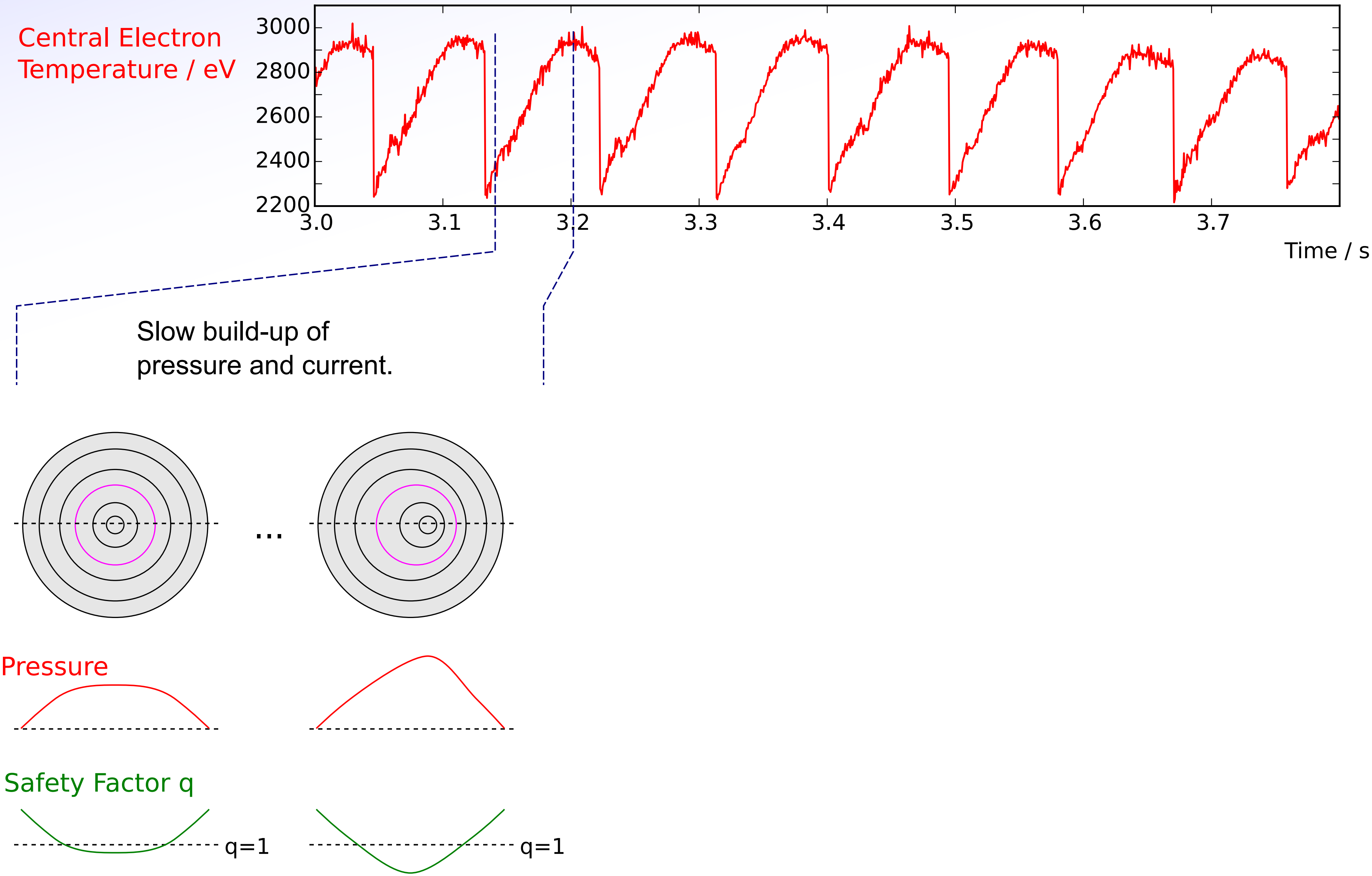

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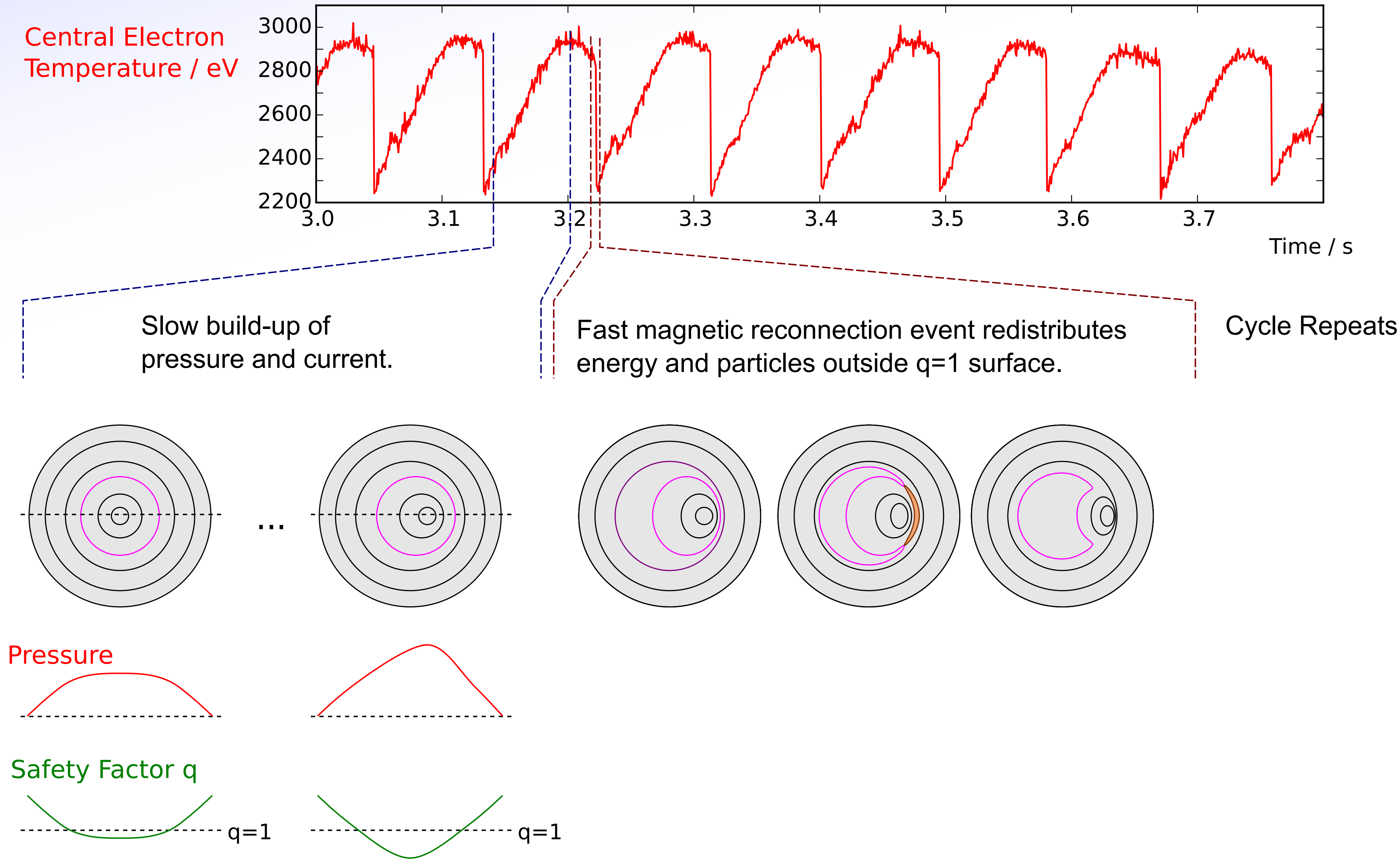

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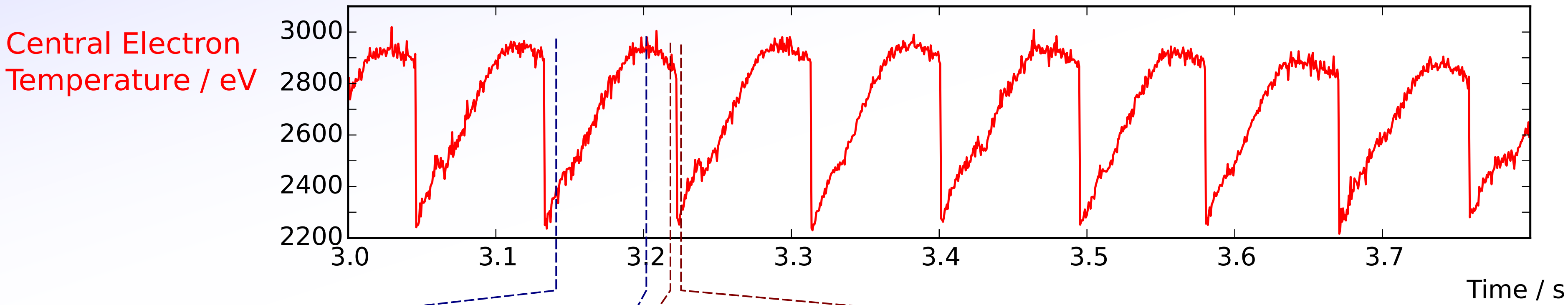
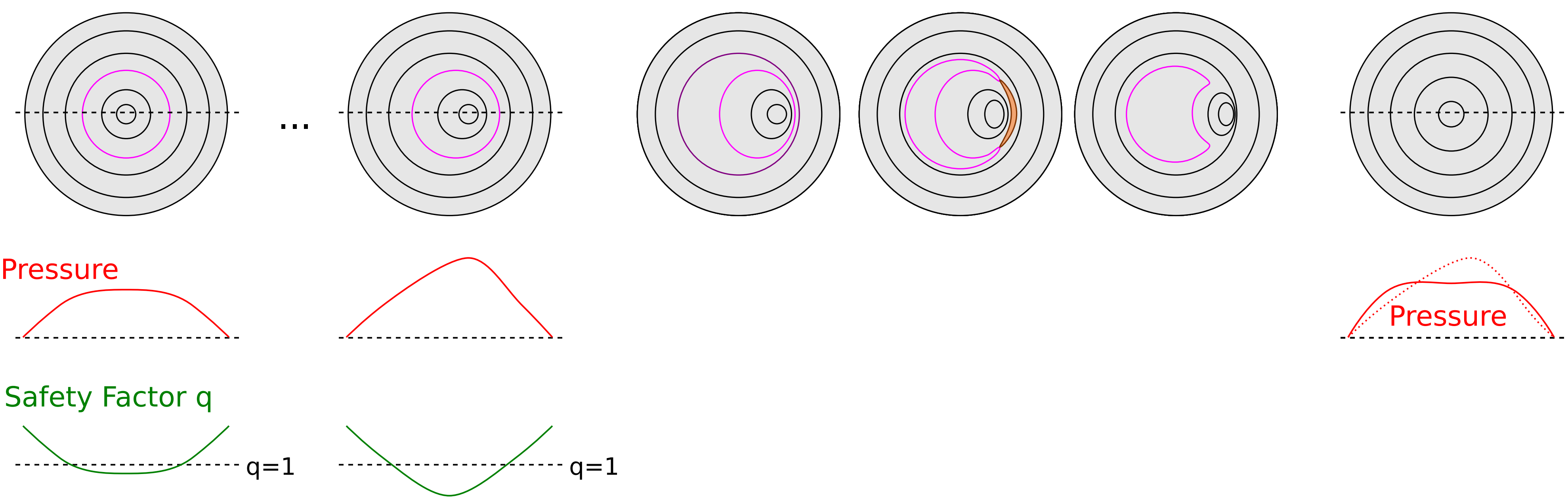


Slow build-up of pressure and current.

Fast magnetic reconnection event redistributes energy and particles outside q=1 surface.

Cycle Repeats



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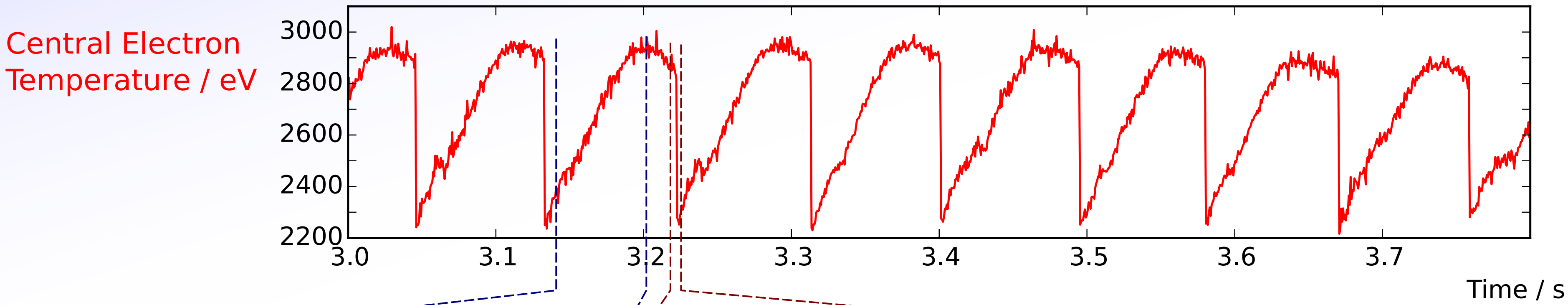

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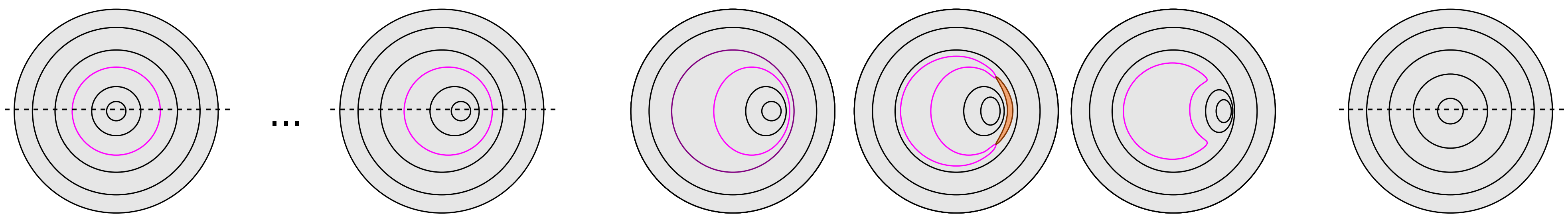
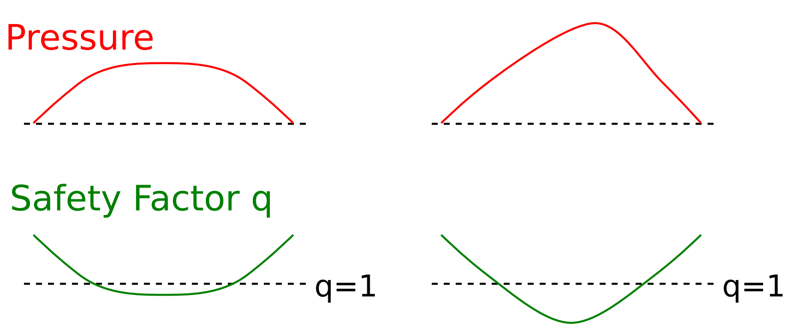
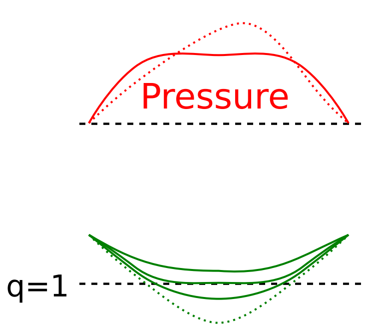




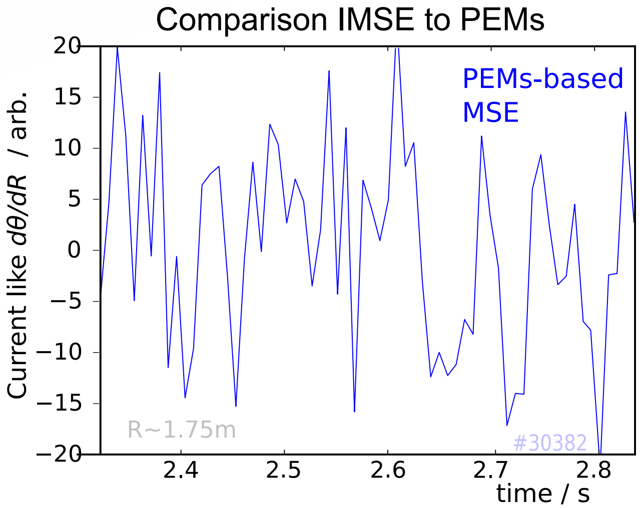
Reconnection observed is much faster than normal models allow (single-fluid MHD).

Many new models proposed, e.g. stochastic reconnection.



Sawteeth - Magnetic Reconnection

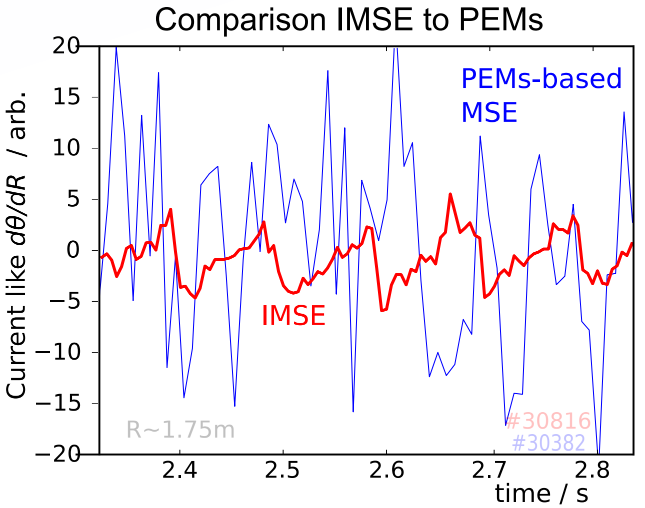
What do we see in the IMSE data?



Sawteeth - Magnetic Reconnection

What do we see in the IMSE data?

- Sawtooth changes are ***very*** small - need good statistics.



Sawteeth - Magnetic Reconnection

What do we see in the IMSE data?

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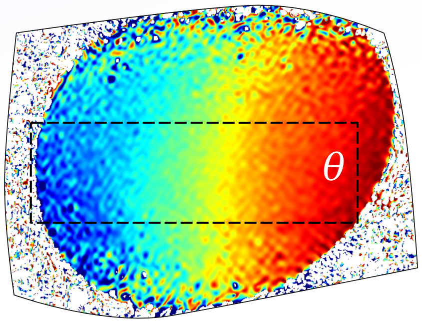
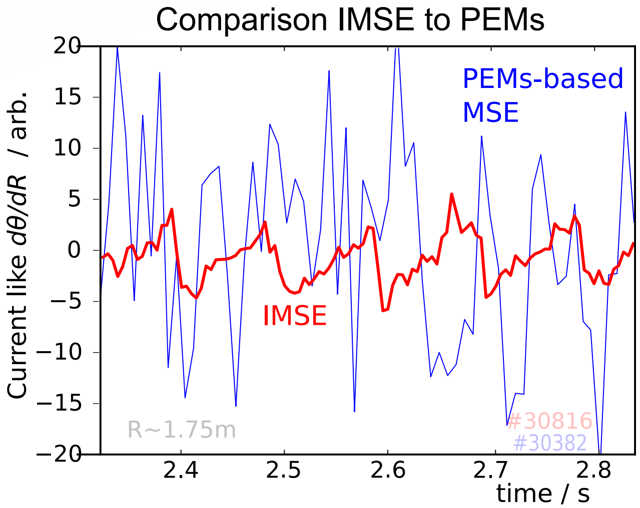
- Average over Z near axis





Sawteeth - Magnetic Reconnection

What do we see in the IMSE data?

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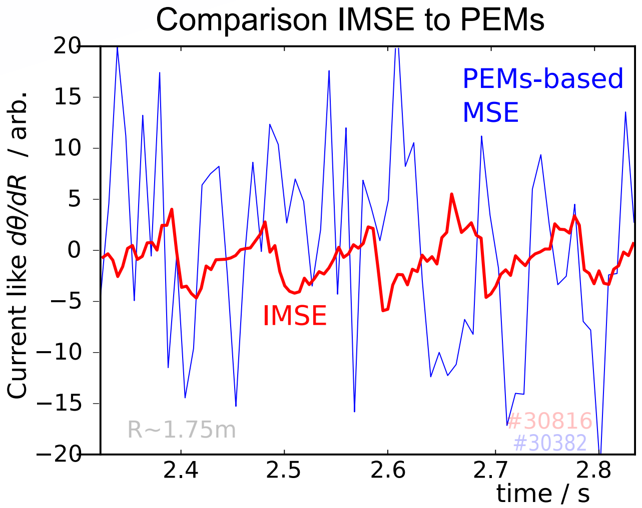
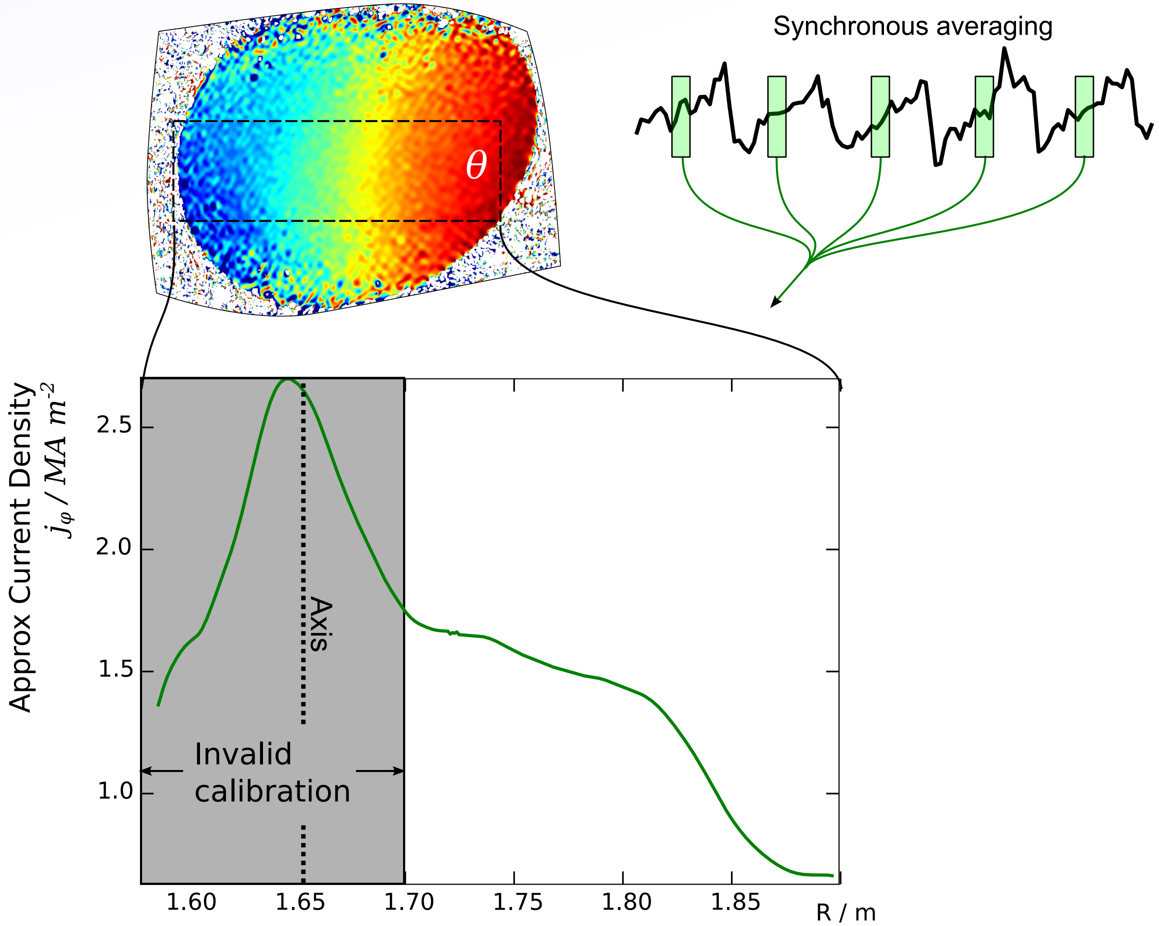
- Average over Z near axis
- Synchronous average over many sawteeth in time.





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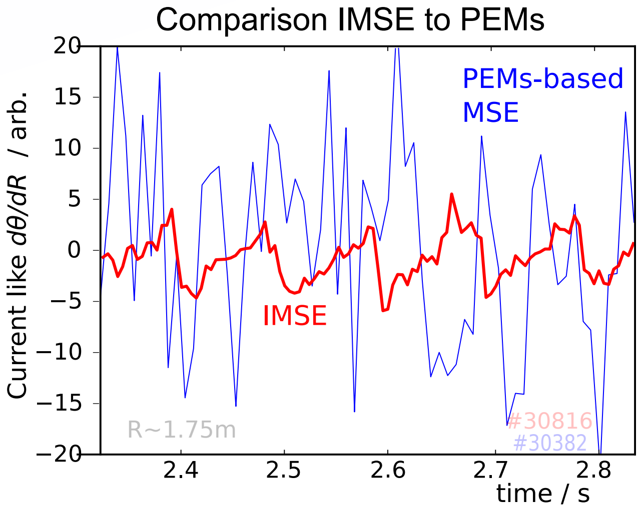

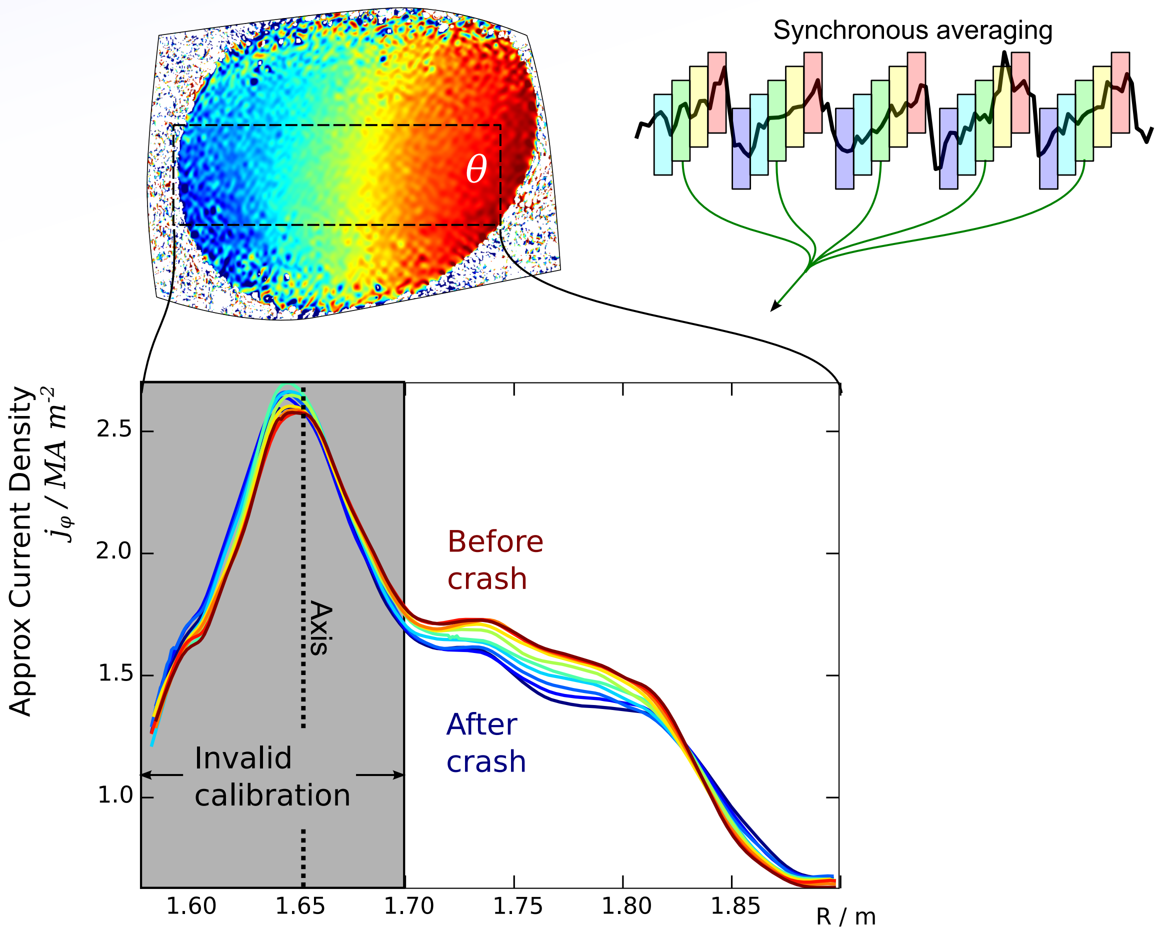

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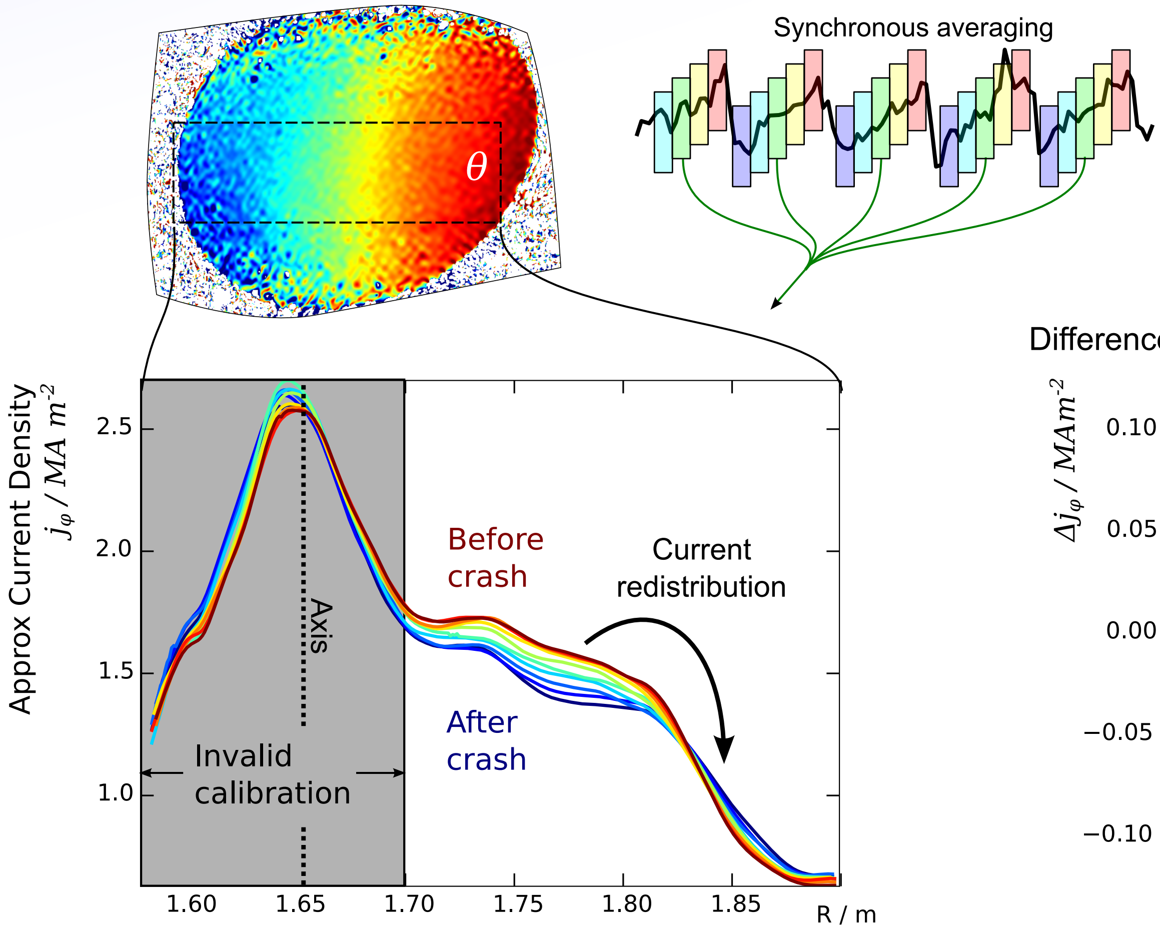
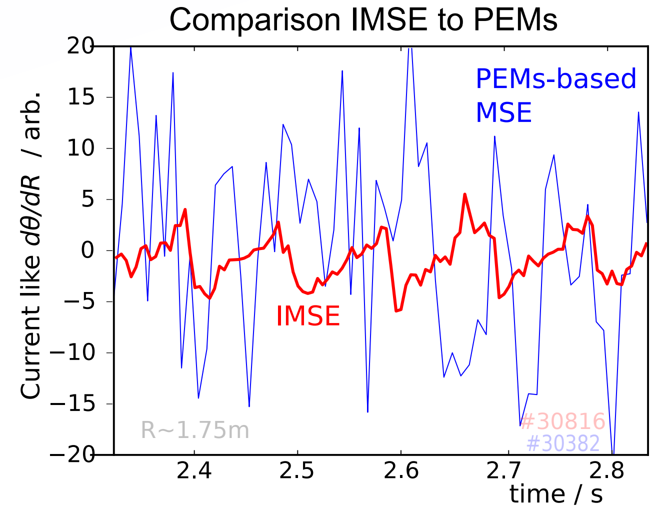
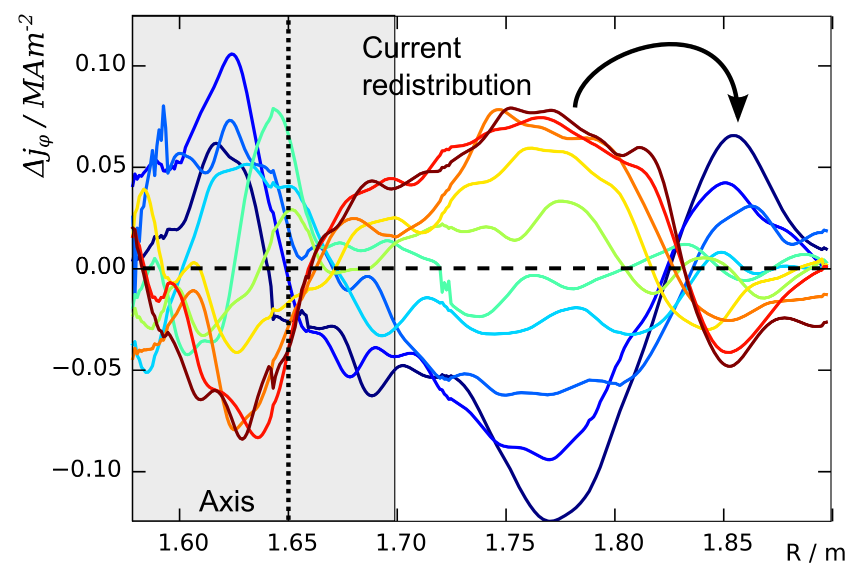




Difference from average profile:



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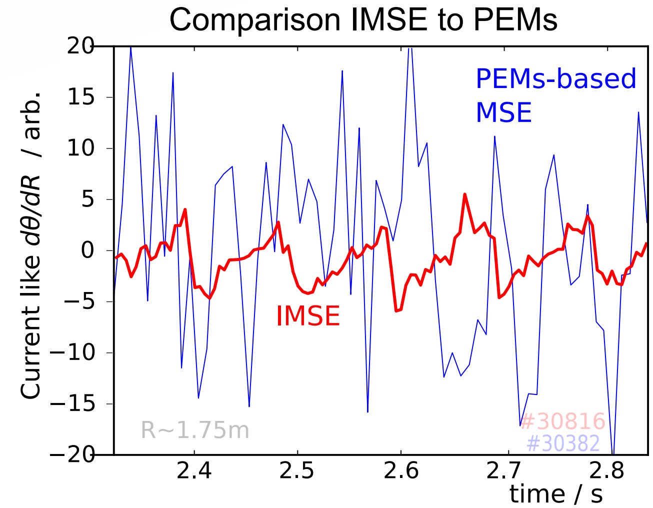

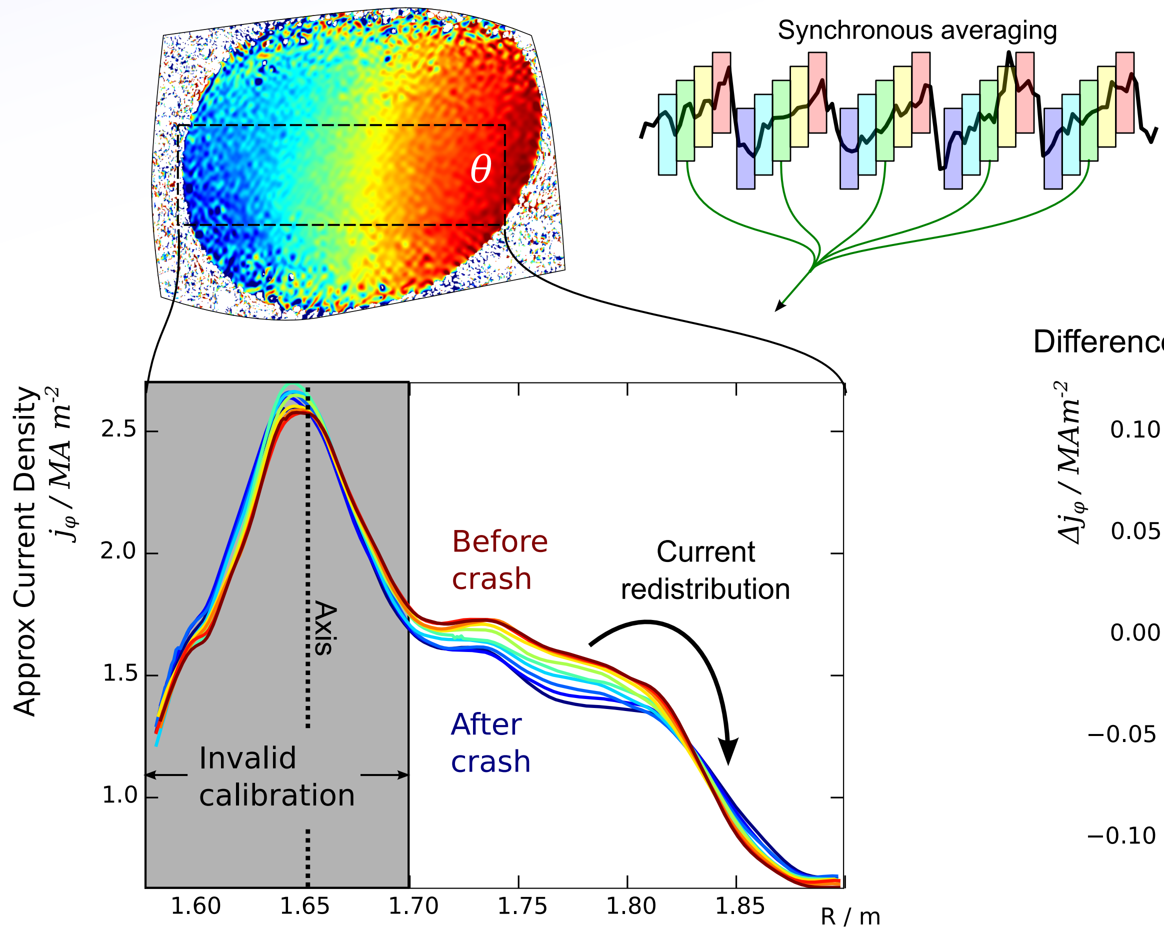

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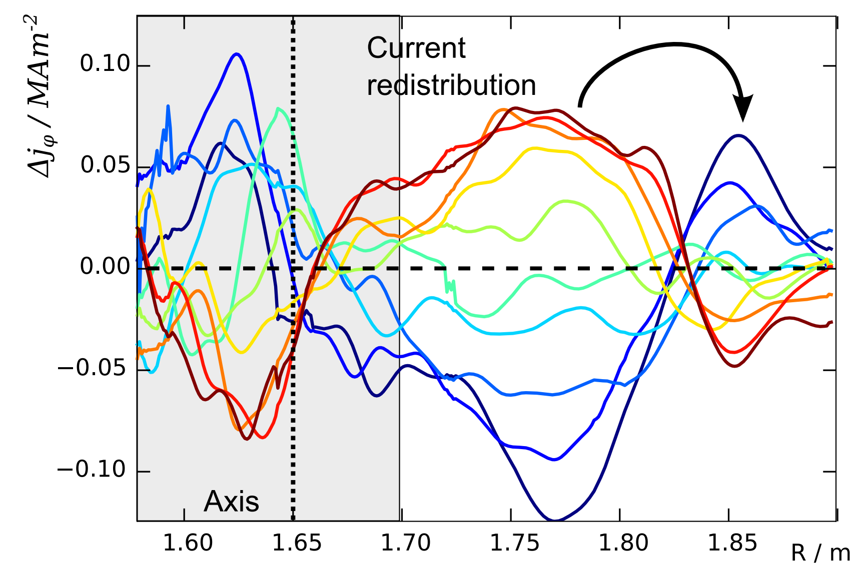

Current redistribution: $\Delta j \sim 0.050\ MA\ m^{-2}$

Measurements every ~3cm (resolution): $\Delta(d\theta/dR) \sim 0.7° m^{-1}$ --> $\boldsymbol{\Delta\theta \pm 0.02°}$ ***required for*** $\boldsymbol{\Delta R = 3cm}$

Outline

- Introduction
  - Flux surfaces and current profiles
  - Magnetic equilibrium
  - Bayesian analysis

- Bayesian equilibrium
  - Current-tomography
  - Current tomography + Grad-Shafranov
  - L-Mode reconstructions
  - H-Mode results

- Rigorous determination of uncertainty
- Too computationally intensive for H-mode
- Need internal measurements!

- Internal measurements
  - Motional Stark effect.
  - Coherence Imaging
  - Imaging MSE
  - Direct j_φ imaging.

-Excellent internal measurements.
- Good dynamics from very approximate derivation of Δj_φ
- Calibration very difficult to required accuracy.
- Need to include in equilibrium

- Integrated Data Analysis
  - Current diffusion
  - IMSE results in comparison
  - Sawtooth models

[R. Fischer et. al.]

Integrated Data Analysis - Equilibrium

New approach to equilibrium at ASDEX Upgrade:

- Grad-Shafranov solver, but with rigorous treatment of errors
- Try to mitigate effect of nonphysical regularisation with as much realistic information as possible:
  - Pressure constraints: n_e, T_e, T_i, Z_{eff}, fast-ions (from modeling)
  - Geometric information (Inboard/outboard agreement of diagnostics)

[R. Fischer et. al.]

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  - Current diffusion

Current Diffusion Equation (CDE):

$$\sigma_{\parallel} \frac{\partial \psi}{\partial t} = \frac{R_0 J^2}{\mu_0 \rho} \frac{\partial}{\partial \rho} \left(\frac{G_2}{J} \frac{\partial \psi}{\partial \rho} \right) - \frac{V'}{2\pi\rho} (j_{\mathrm{bs}} + j_{\mathrm{cd}})$$

[R. Fischer et. al.]

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  - Pressure constraints: n_e, T_e, T_i, Z_{eff}, fast-ions (from modeling)
  - Geometric information (Inboard/outboard agreement of diagnostics)
  - Current diffusion:
  - Modeled current 'sources': ECCD, bootstrap, NBI.

Current Diffusion Equation (CDE):

$$\sigma_{\parallel} \frac{\partial \psi}{\partial t} = \frac{R_0 J^2}{\mu_0 \rho} \frac{\partial}{\partial \rho} \left(\frac{G_2}{J} \frac{\partial \psi}{\partial \rho} \right) - \frac{V'}{2\pi\rho} (j_{bs} + j_{cd})$$

Bootstrap current (j_{bs})

Current drive (ECCD, NBI etc) (j_{cd})

[R. Fischer et. al.]

Integrated Data Analysis - Equilibrium

New approach to equilibrium at ASDEX Upgrade:

- Grad-Shafranov solver, but with rigorous treatment of errors
- Try to mitigate effect of nonphysical regularisation with as much realistic information as possible:
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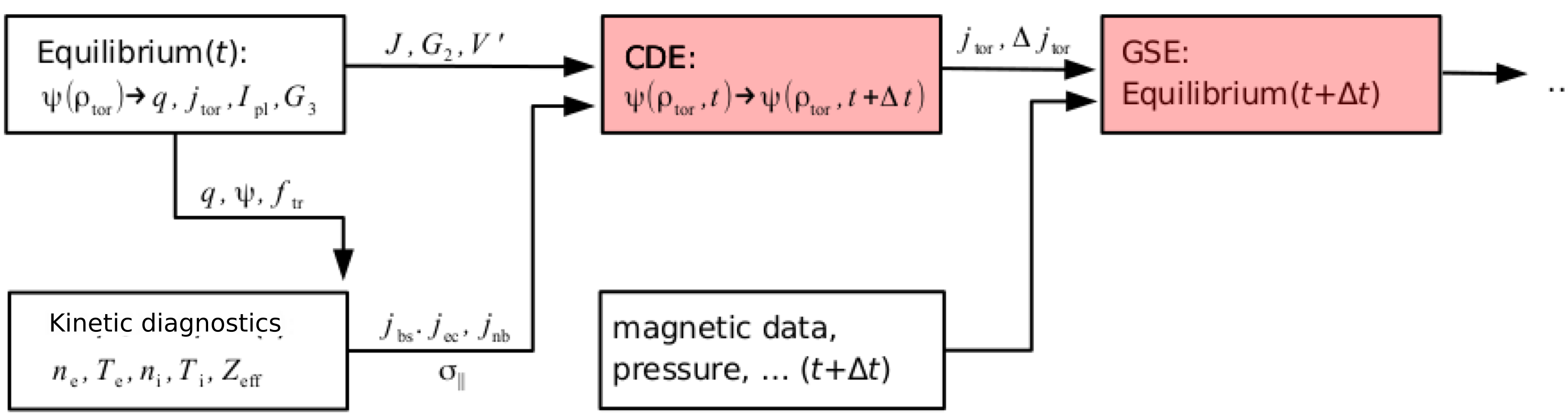
Bootstrap current (j_{bs})
Current drive (ECCD, NBI etc) (j_{cd})

Provides a weak constraint on j_φ from expected evolution from previous time-points.
- i.e. physically realistic (and informative) prior information.



[R. Fischer et. al.]

Integrated Data Analysis Equilibrium

Example: Counter-current Electron Cyclotron Current Drive (ECCD)

ECCD drives localised on-axis current
- Not seen by magnetics (small due to low area of centre)
- No effect on pressure profile = not seen by kinetic inputs

[R. Fischer et. al.]

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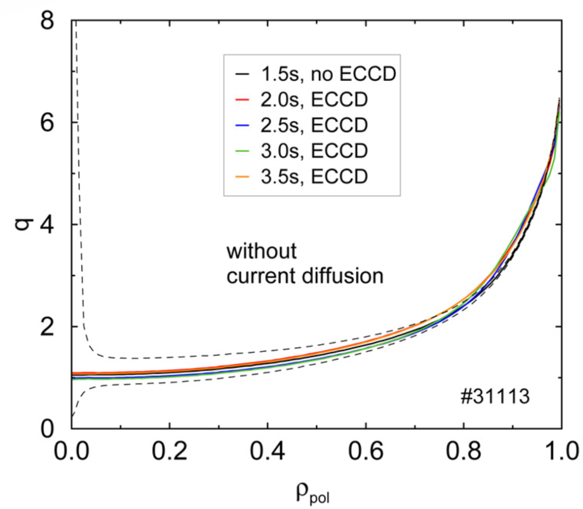

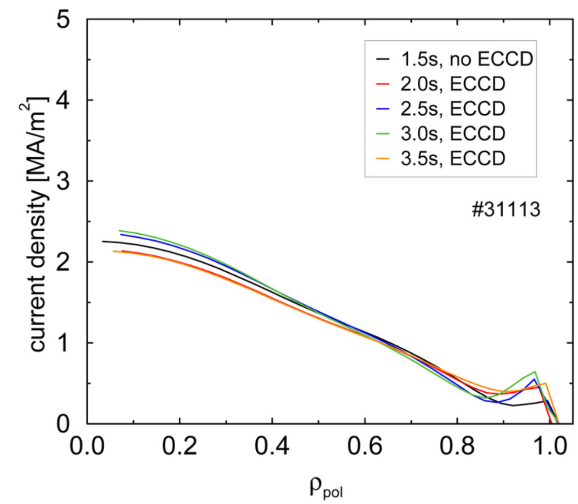

[R. Fischer et. al.]

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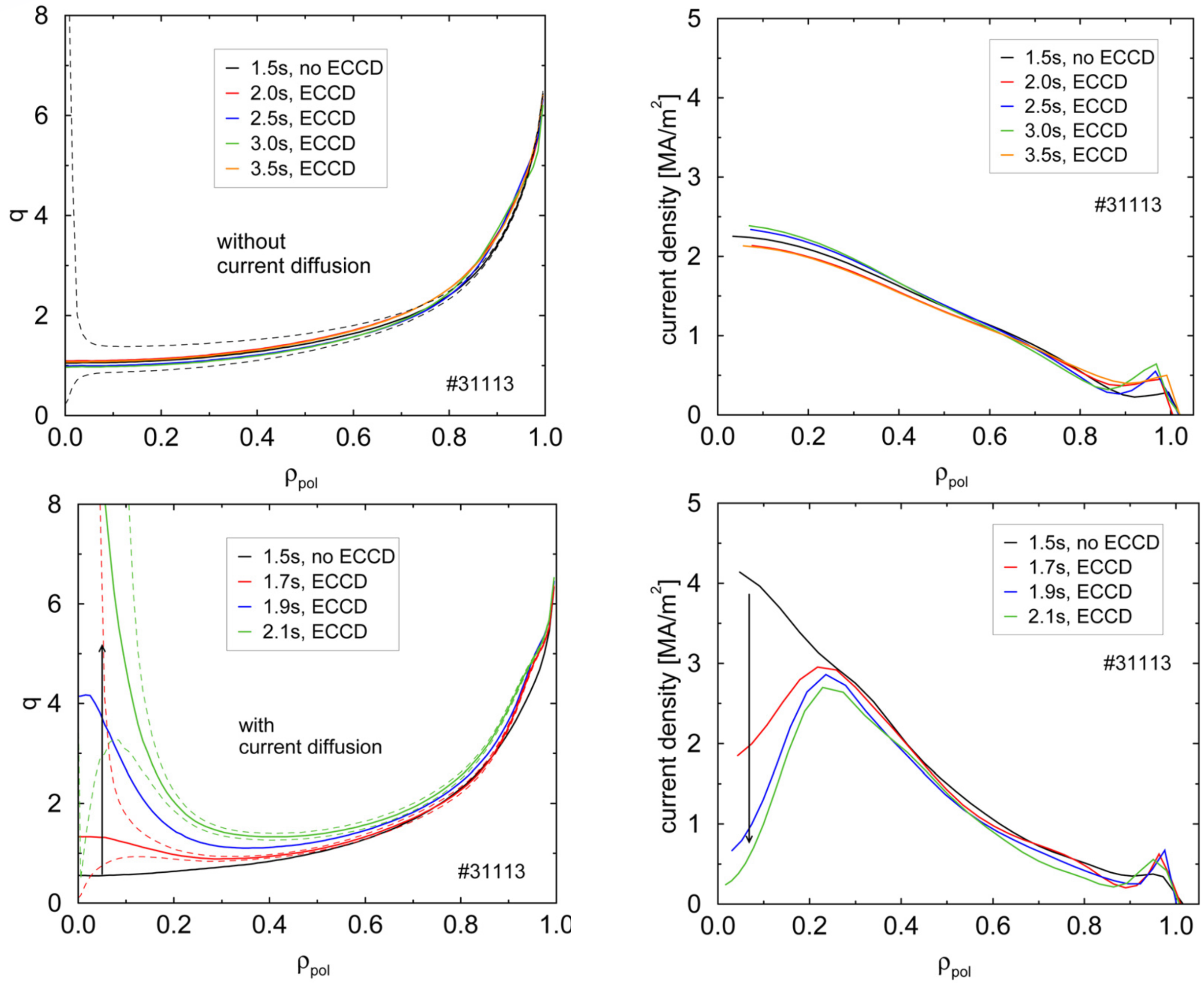
Regularised GS solution:

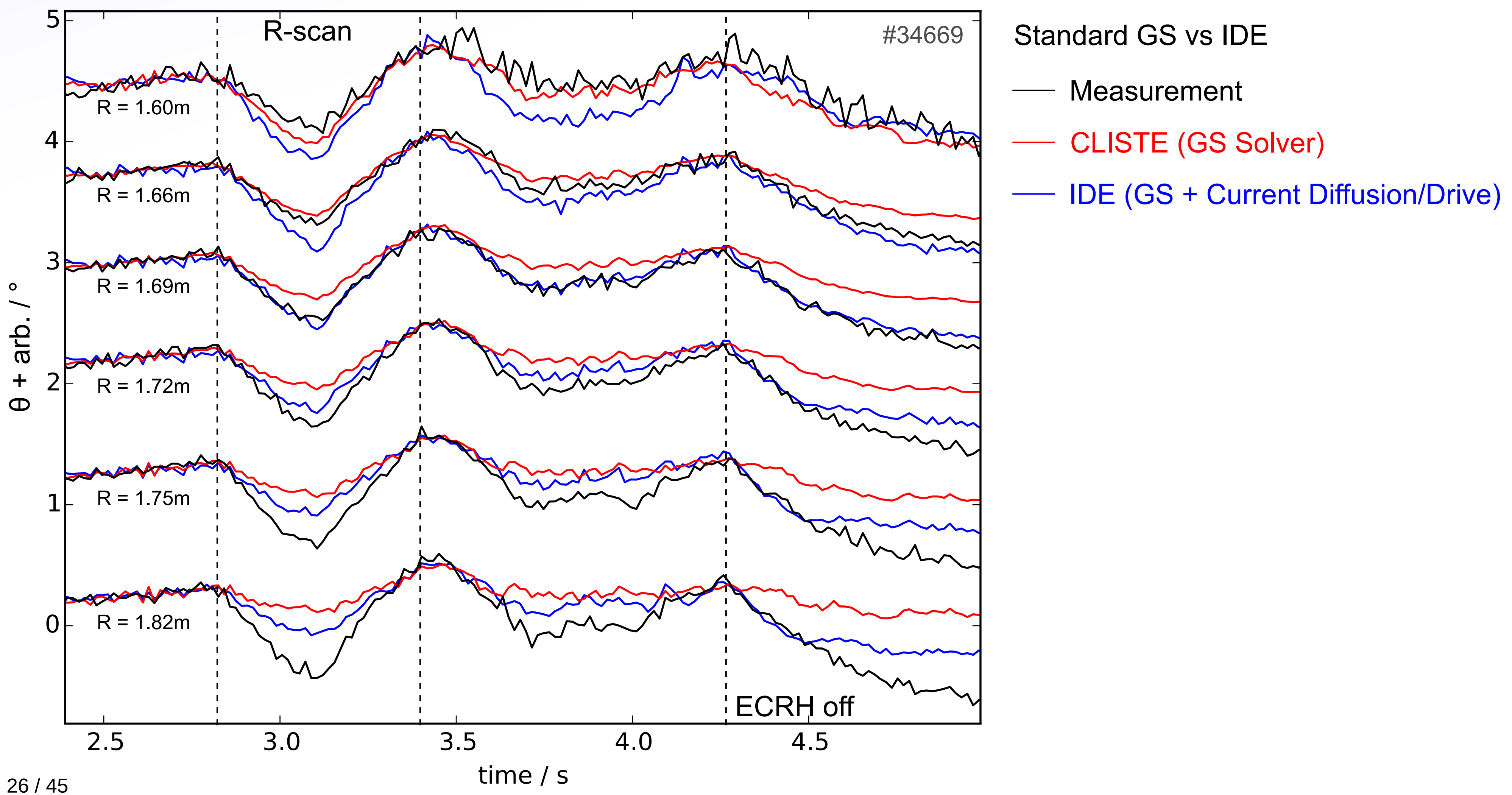
With CDE + Current Drive:



[R. Fischer et. al.]

Integrated Equilibrium vs IMSE

- By comparing by IMSE, can see where IDE predicts more physics than the 'standard' GS solver:
  1) During R-scan
  2) ECRH switch-off



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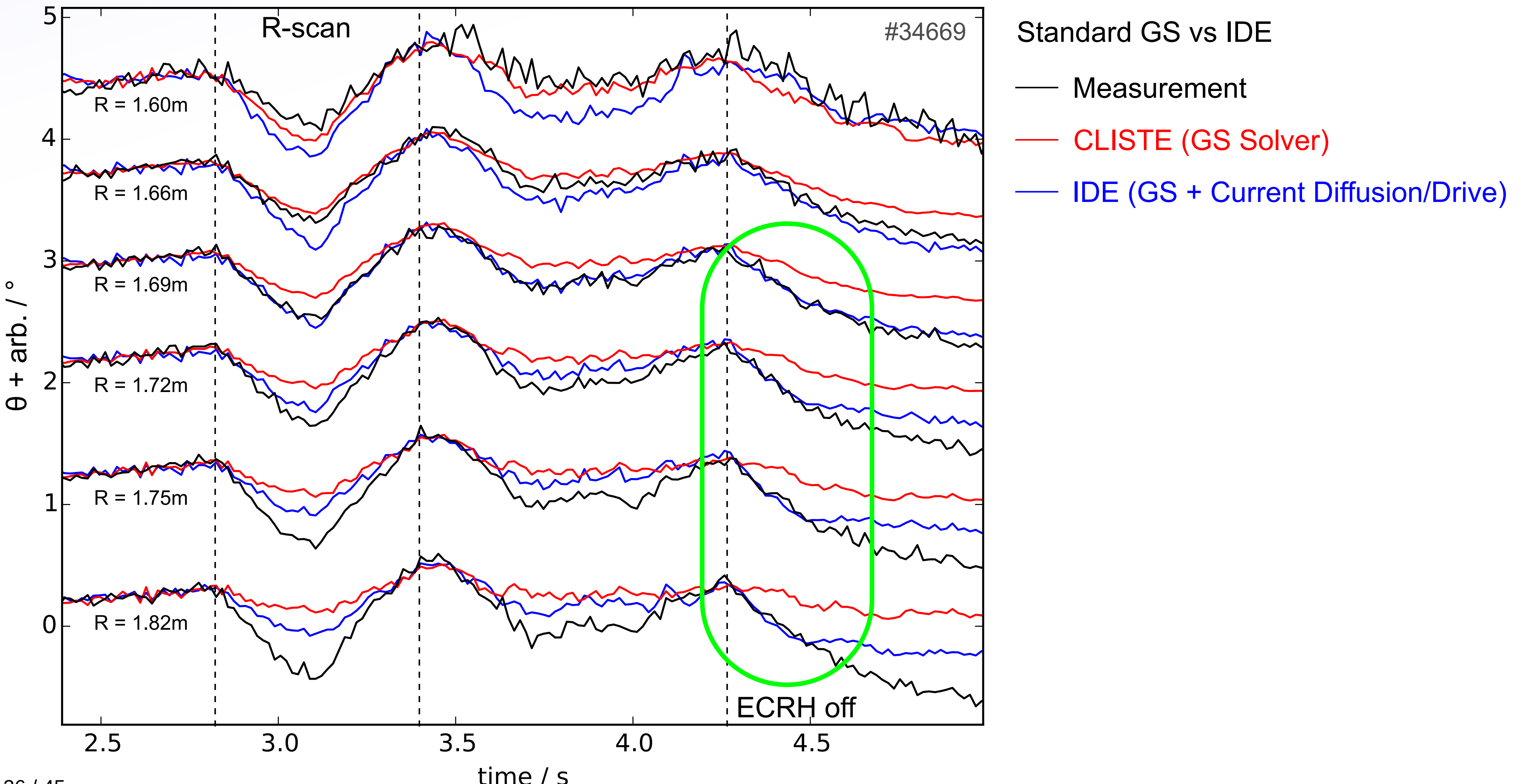

[R. Fischer et. al.]

Integrated Equilibrium vs IMSE

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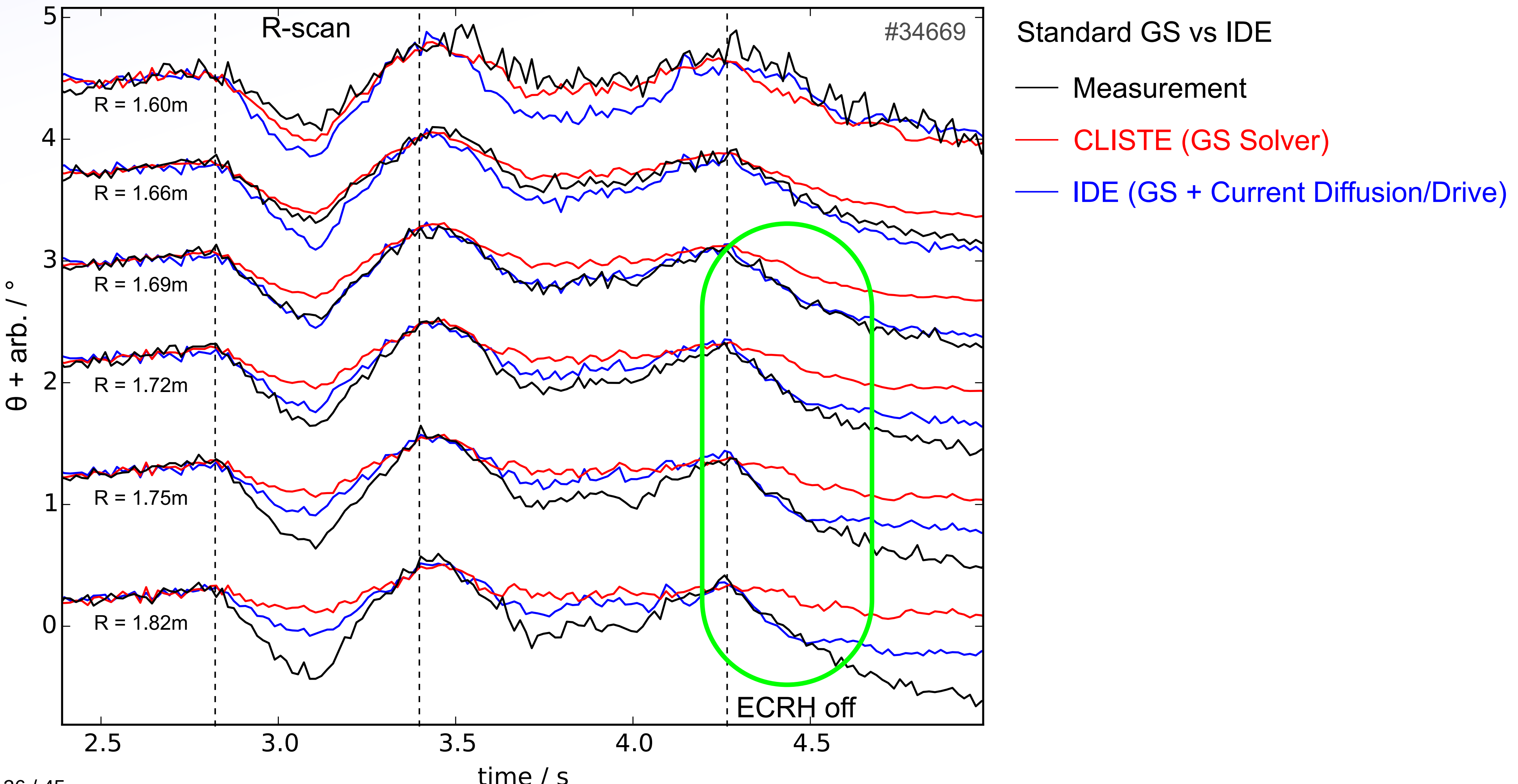
However, there is still physics only seen by diagnostic!



[R. Fischer et. al.]

Integrated Equilibrium - Sawteeth

- During sawteeth (reconnection), current diffusion not applicable.

[R. Fischer et. al.]

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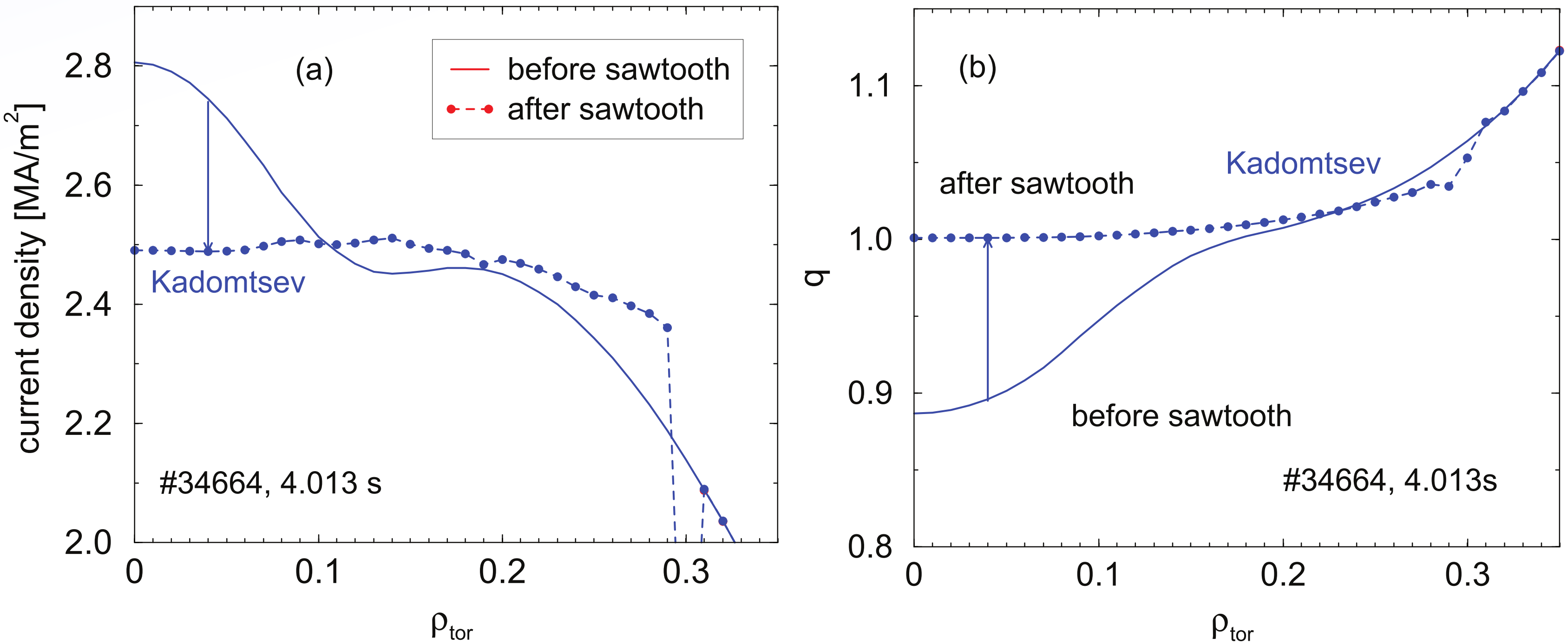
- Include different sawtooth models in equilibrium code and compare IMSE predictions to measurements.
  - Kadomtsev: Complete reconnection. q_0 --> 1. Current outside q=1 surface.
  - Flat-current model (FCM): Current conserved outside q=1, flat current density inside.



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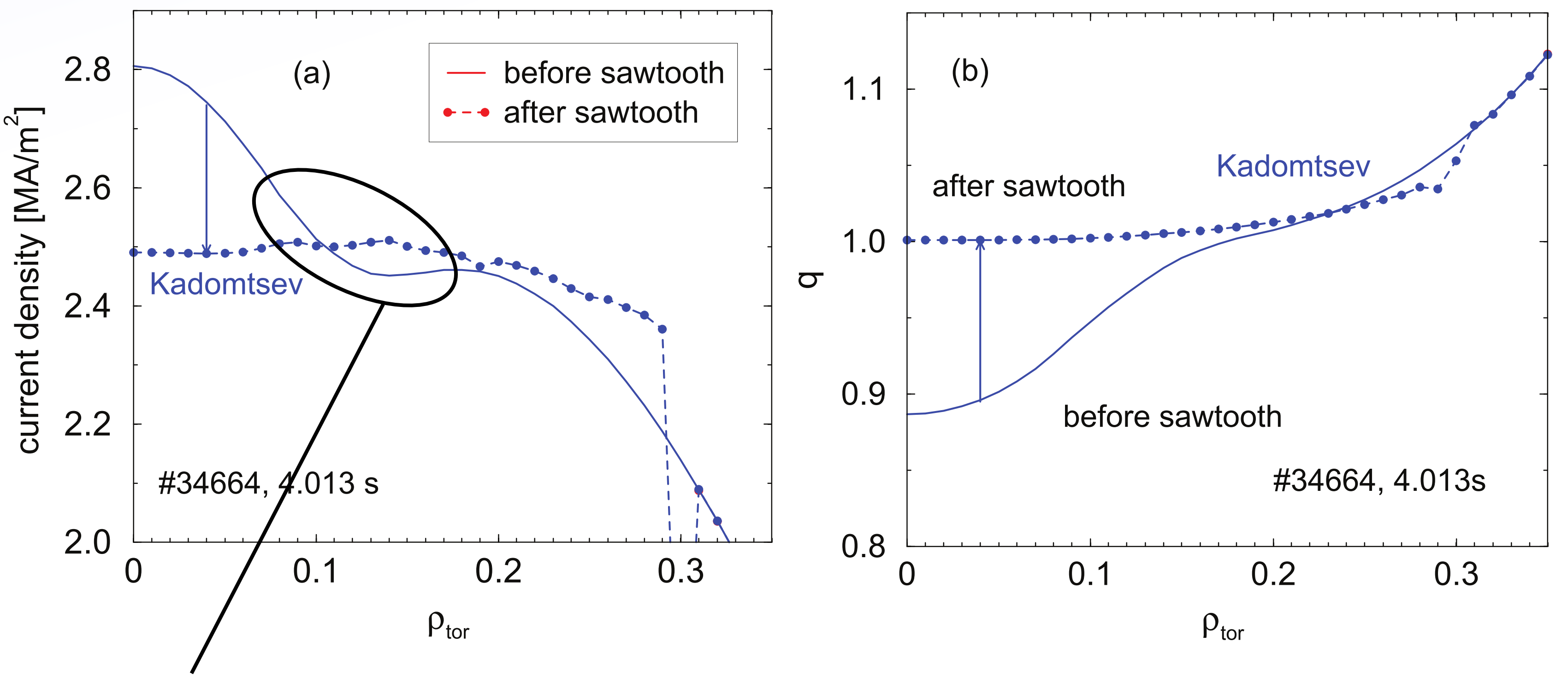

Current redistribution similar to seen in Δj_φ images.

[R. Fischer et. al.]

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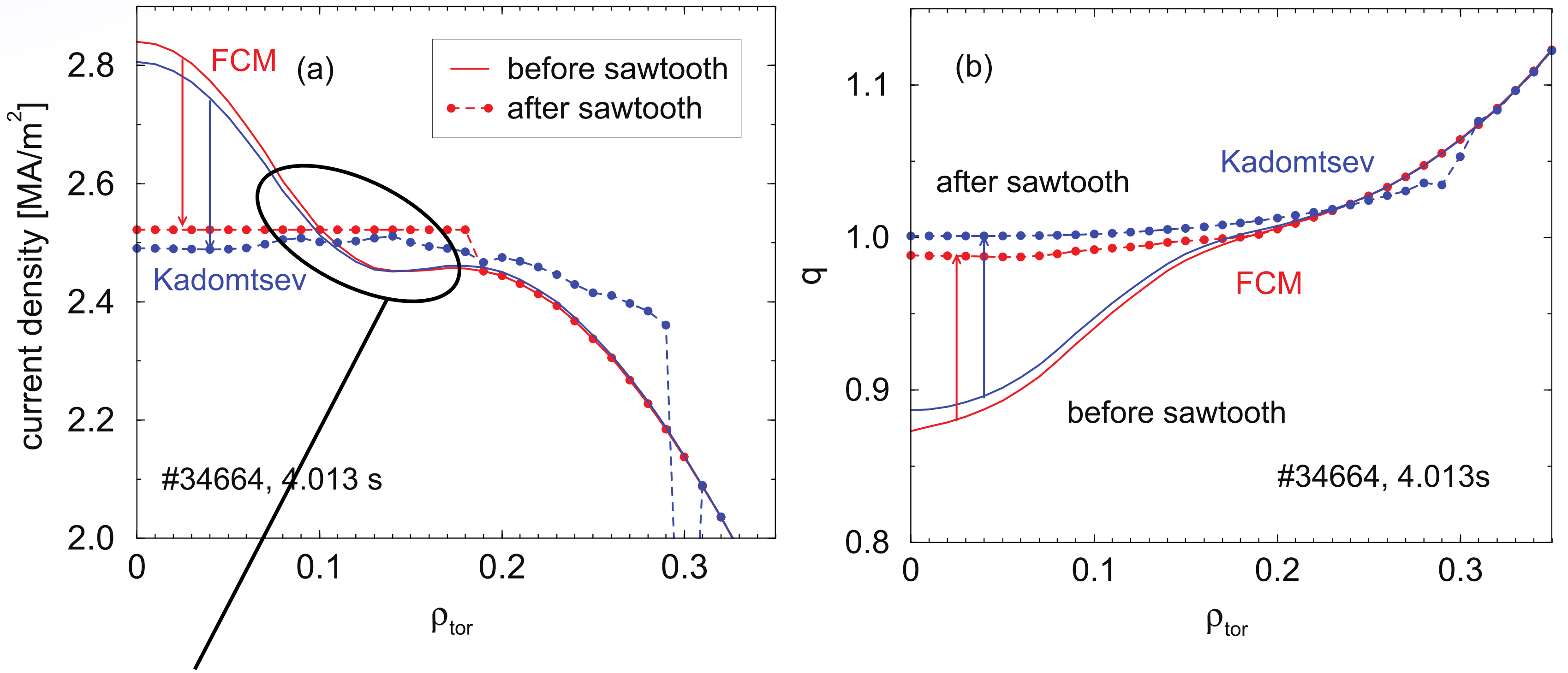

Current redistribution similar to seen in Δj_φ images.

Difference between models requires absolute $j_\varphi \sim 0.02\ MA\ m^{-2}$ --> $d\theta/dR \sim 0.01^o$ $(3\ cm^{-3})$

[R. Fischer et. al.]

Integrated Equilibrium vs IMSE - Sawteeth

Required precision is so high, many other factors become important:

[R. Fischer et. al.]

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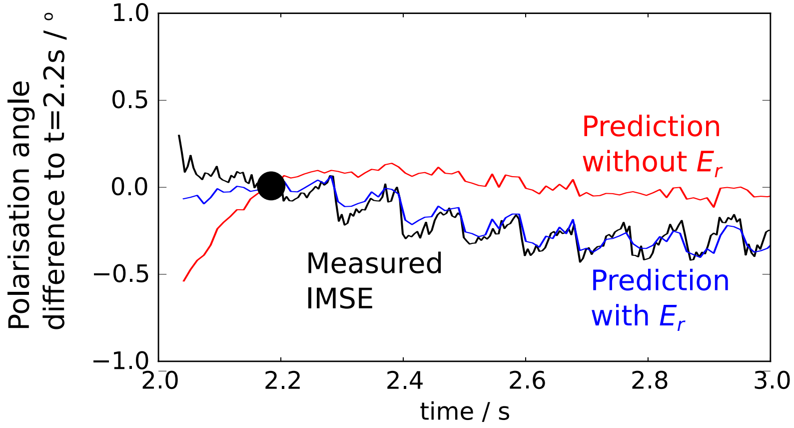
Plasma radial electric field:

$$E = v \times B + E_r$$

At some locations, ΔE_r during sawtooth dominates measurement:



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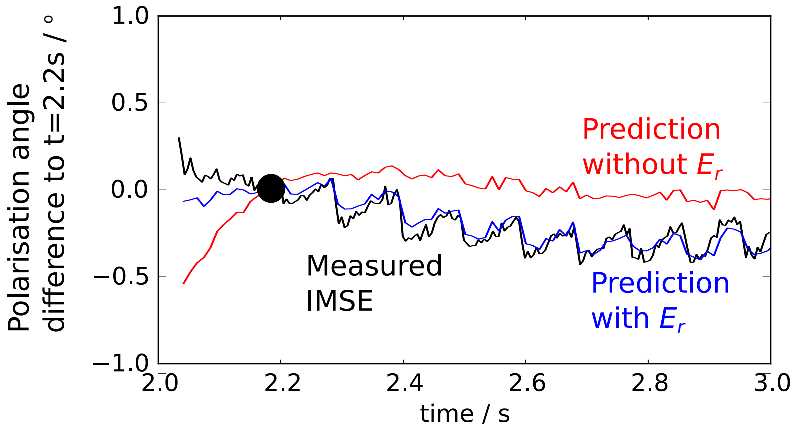
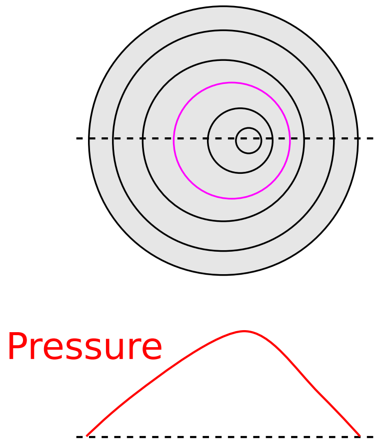


Shafranov shift:

Movement of plasma axis with pressure.



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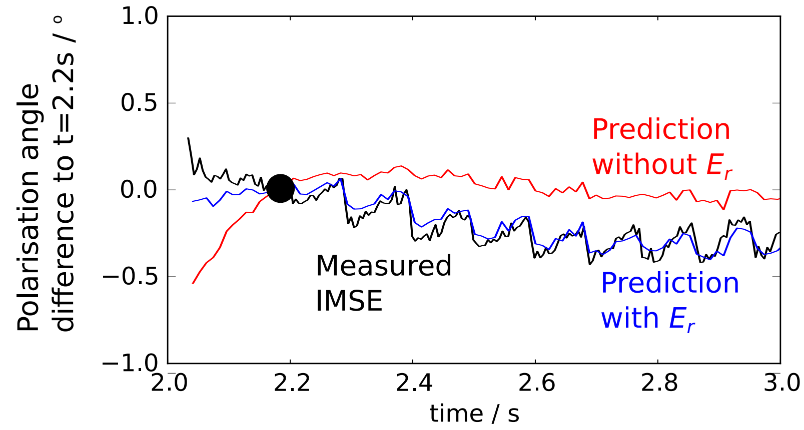
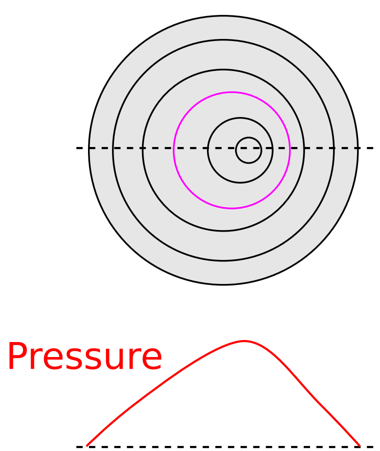
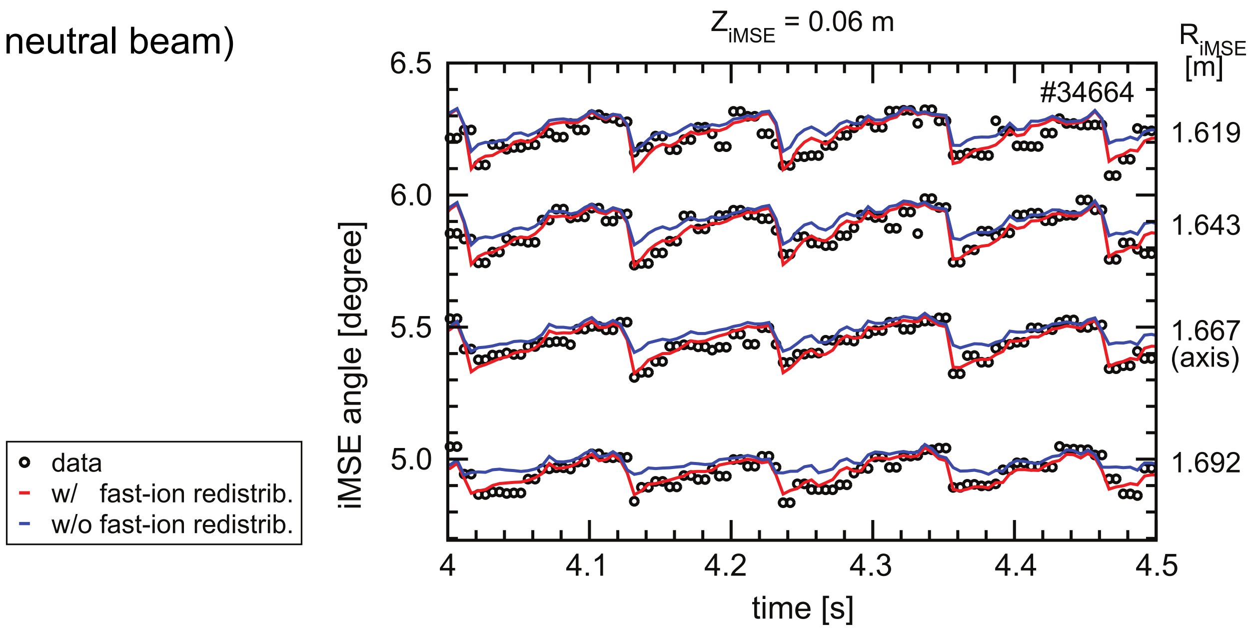


Shafranov shift:

Movement of plasma axis with pressure.

(including redistribution of fast-ions from neutral beam)





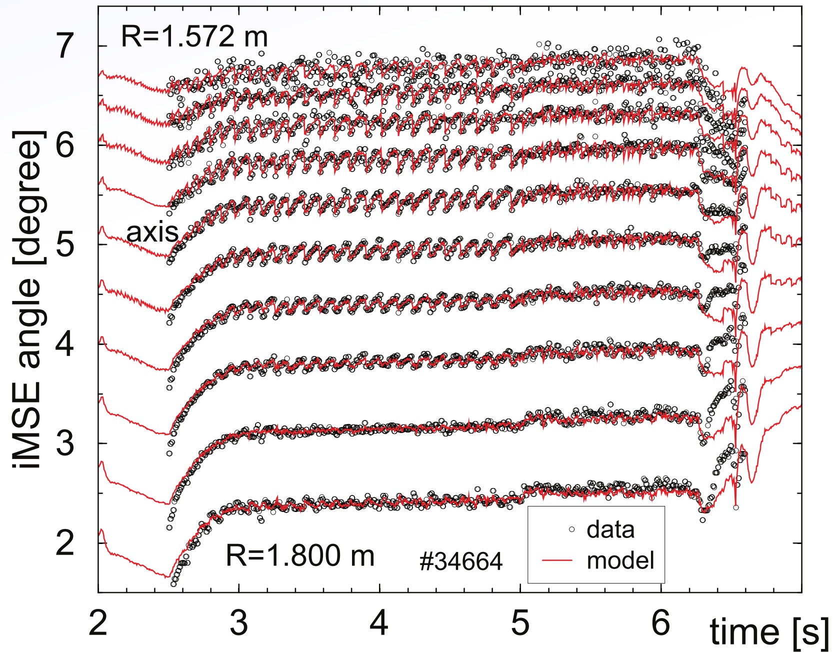
[R. Fischer et. al.]

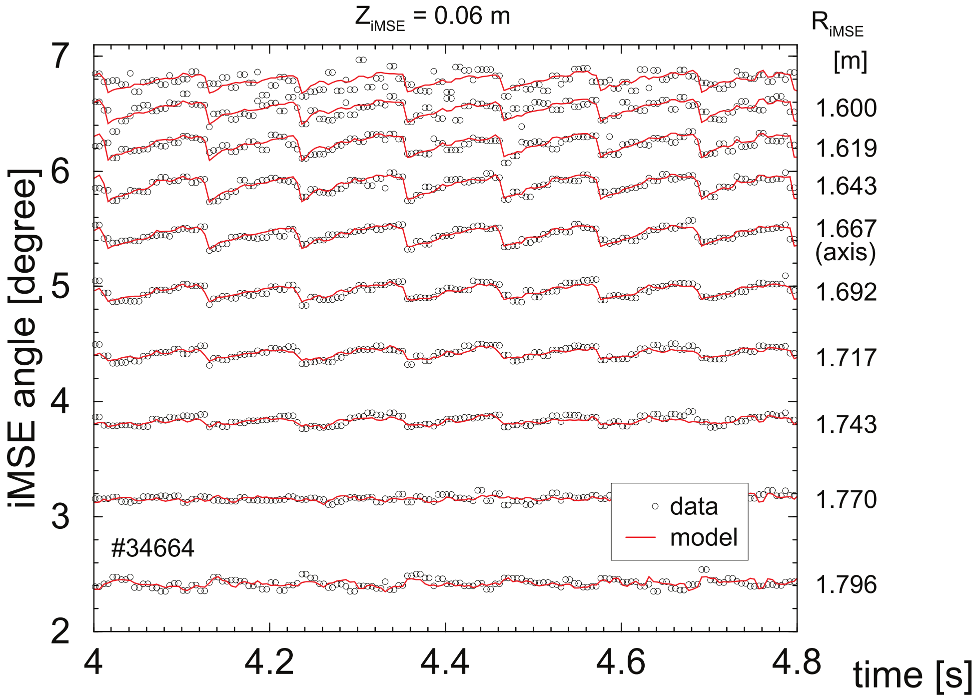
Integrated Equilibrium vs IMSE - Sawteeth

Required precision is so high, many other factors become important:

but...

we now have good agreement between full integrated model and IMSE measurements for sawtooth evolution in θ.





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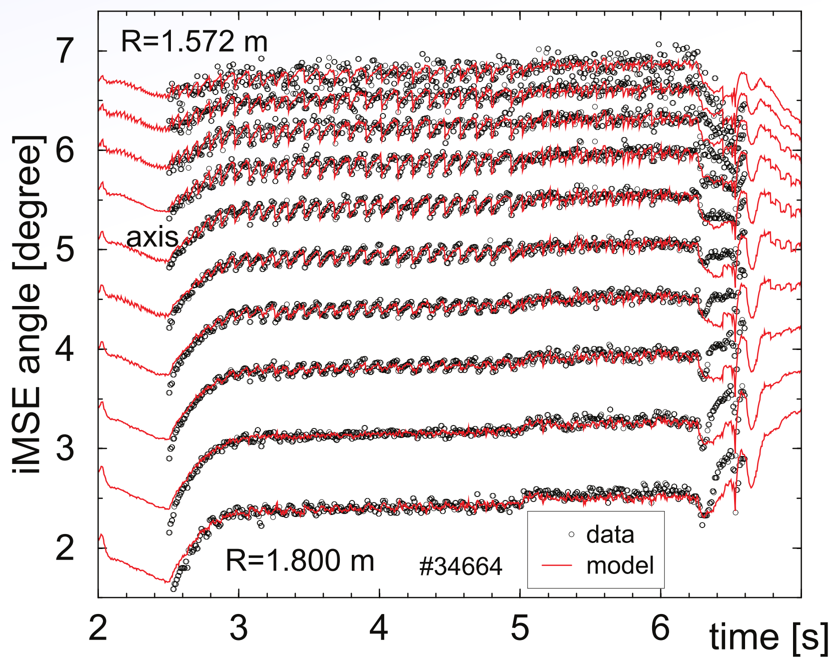

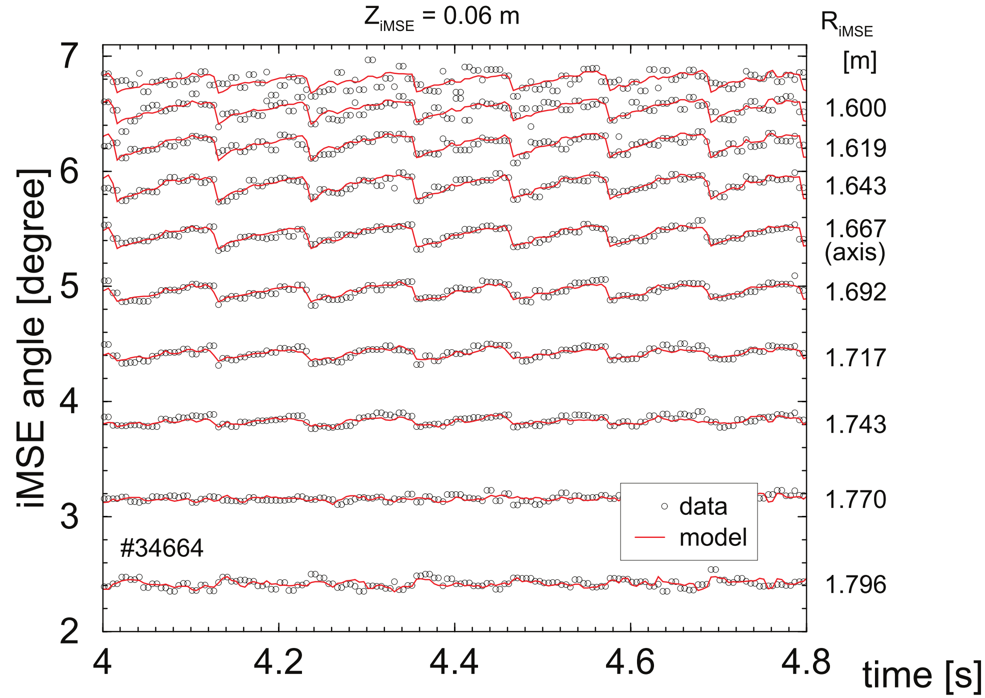

- This is where we are - 'the state of the ~~art~~ ... science'

What next?

IMSE:
- Improve calibration systematics,.

IDE:
- Modeling of effects.
- Tolerance to calibration systematics.

[R. Fischer et. al.]

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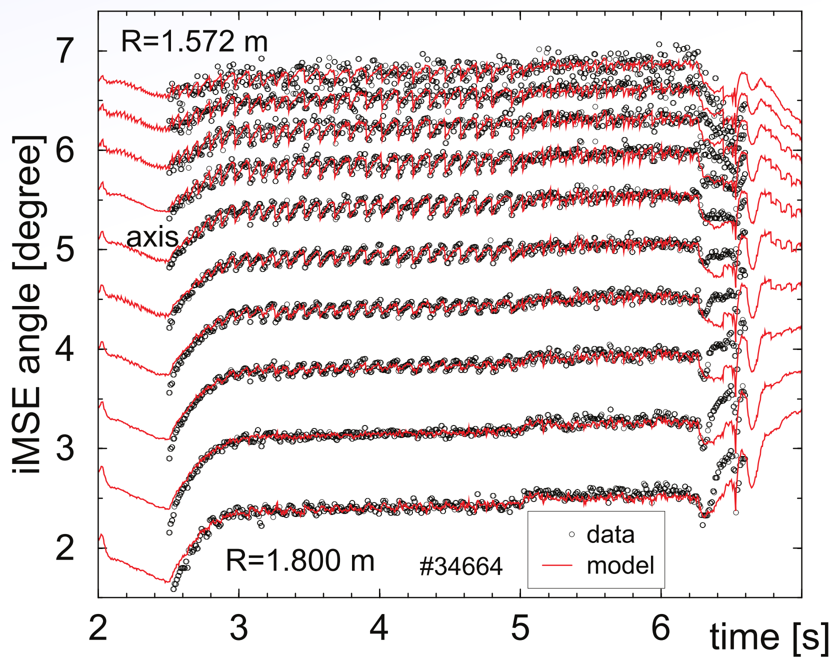

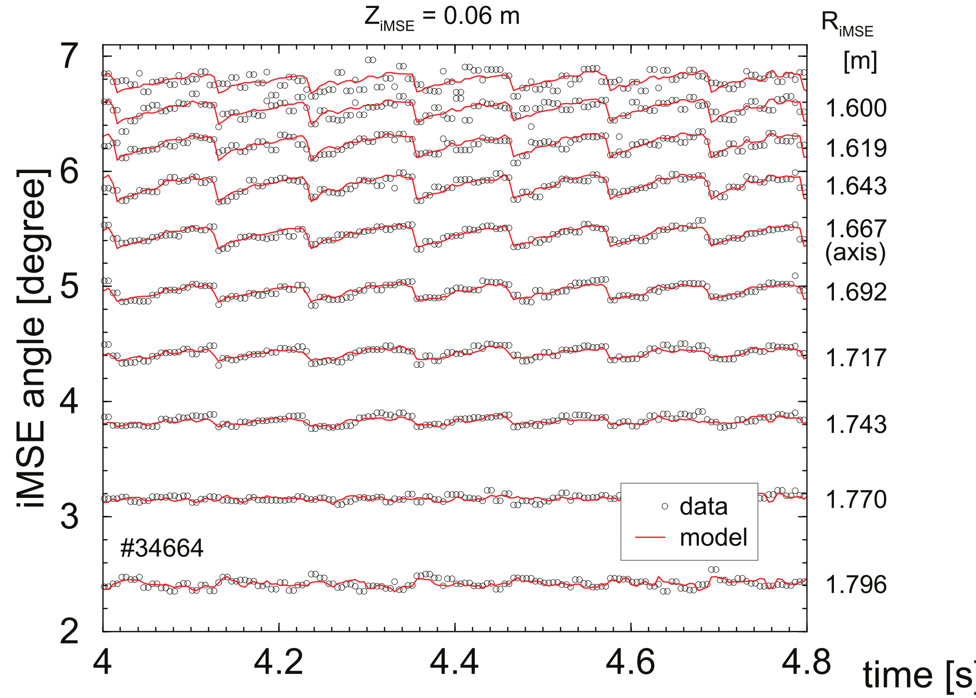

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What next?

IMSE:
- Improve calibration systematics,.

Converge → ←

IDE:
- Modeling of effects.
- Tolerance to calibration systematics.

Summary

- Introduction
  - Flux surfaces and current profiles
  - Magnetic equilibrium
  - Bayesian analysis

- Bayesian equilibrium
  - Current-tomography
  - Current tomography + Grad-Shafranov
  - L-Mode reconstructions
  - H-Mode results

- Rigorous determination of uncertainty
- Too computationally intensive for H-mode
- Need internal measurements!

- Internal measurements
  - Motional Stark effect.
  - Coherence Imaging
  - Imaging MSE
  - Direct j_φ imaging.

-Excellent internal measurements.
- Good dynamics from approximate derivation of $\Delta j\varphi$
- Calibration very difficult to required accuracy.
- Need to include in equilibrium

- Integrated Data Analysis
  - Current diffusion
  - Imaging MSE comparison
  - Sawtooth models

- Excellent tool for practical analysis with available data.
- Current diffusion provides realistic model of missing information when data incomplete.
- Sawtooth models in good agreement with IMSE evolution.
- Still need to converge IDE+IMSE to arrive at an absolute *q*.

Bayesian Inference

A simple example with electron density:

Physics model:

$$n_e(r) = n_0(1-r^2)^q$$

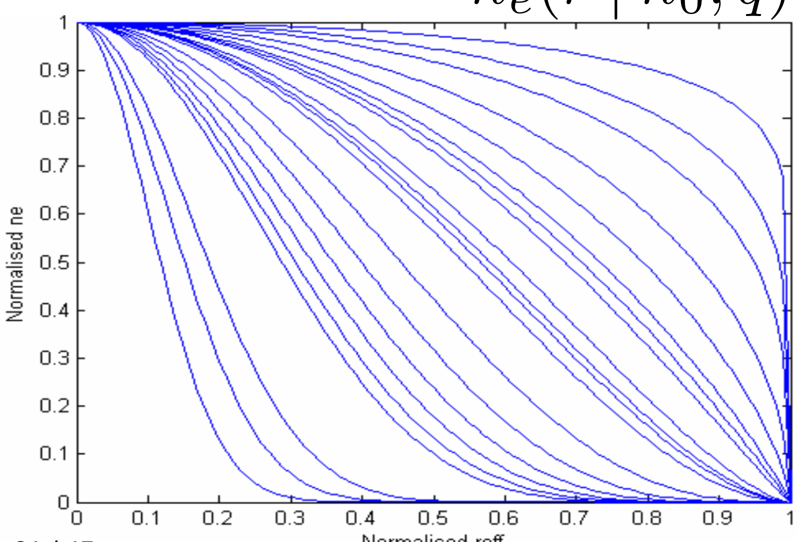
Parameters:

$$\mu = (n_0, q)$$

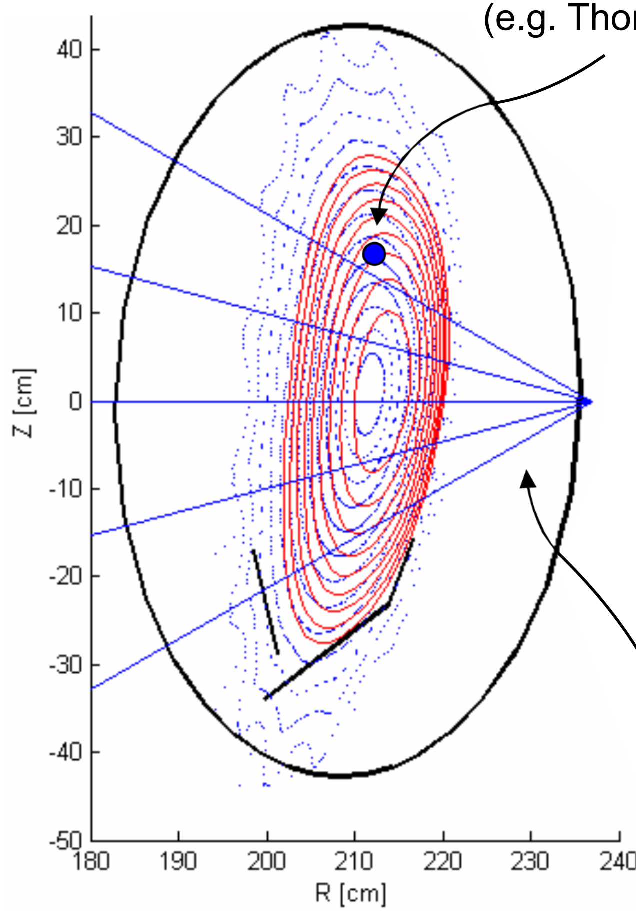
Forward model:

$$f(\mu) = \int n_e dl$$

$$n_e(r \mid n_0, q)$$



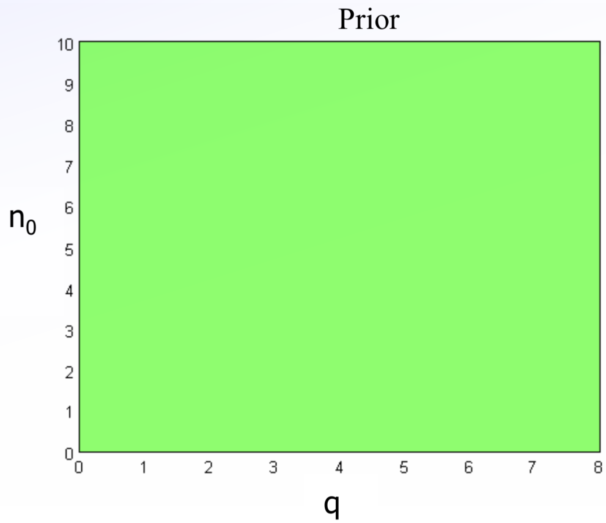
Point measurement D_p
(e.g. Thomon Scattering)



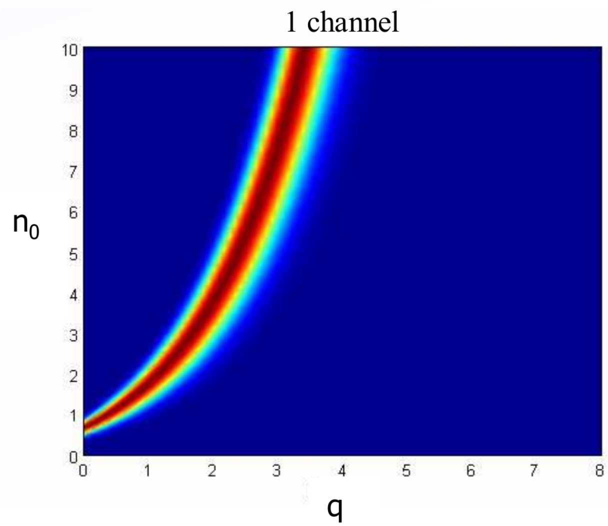
Line measurement D_i
(e.g. Interferometry)

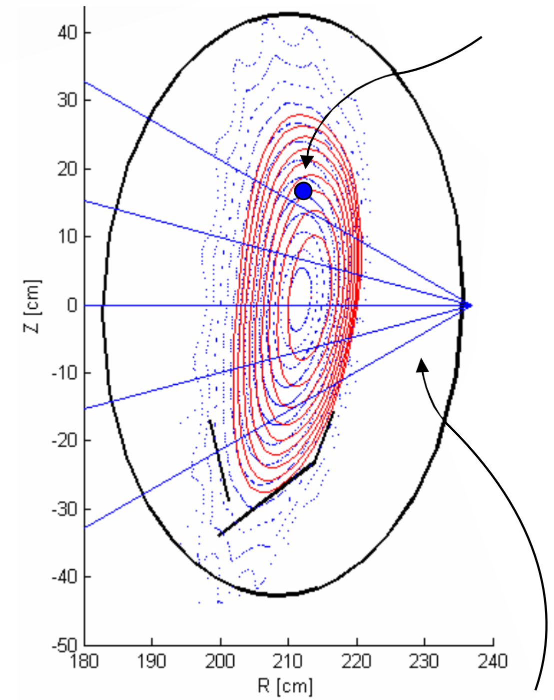
Bayesian Inference

What should we take for a prior:
- Any n0 and q is equally likely:

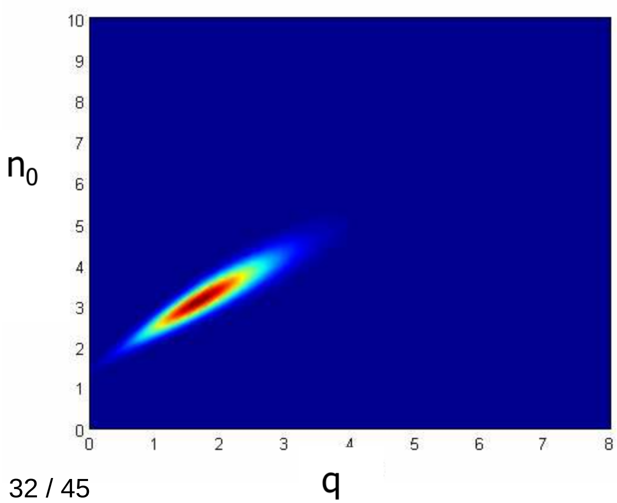


P(D | μ) for a single lie integral:

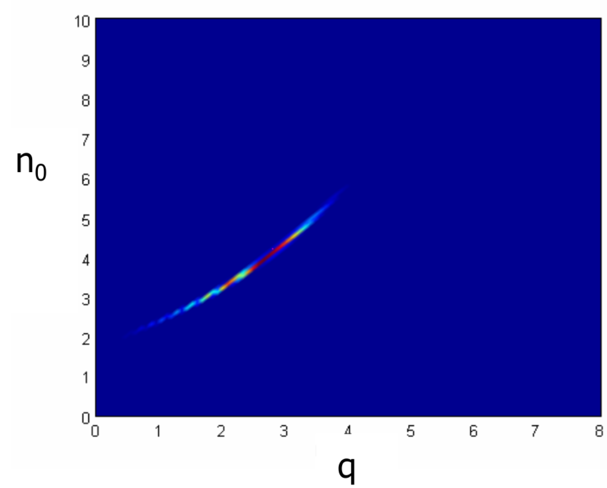




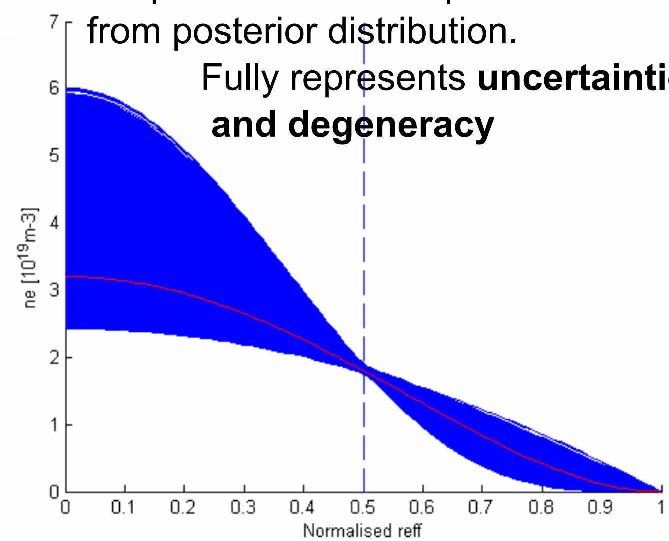
P(D | μ) for 5 line integrals:



P(D | μ) for 5 line integrals
and 1 point measurement:

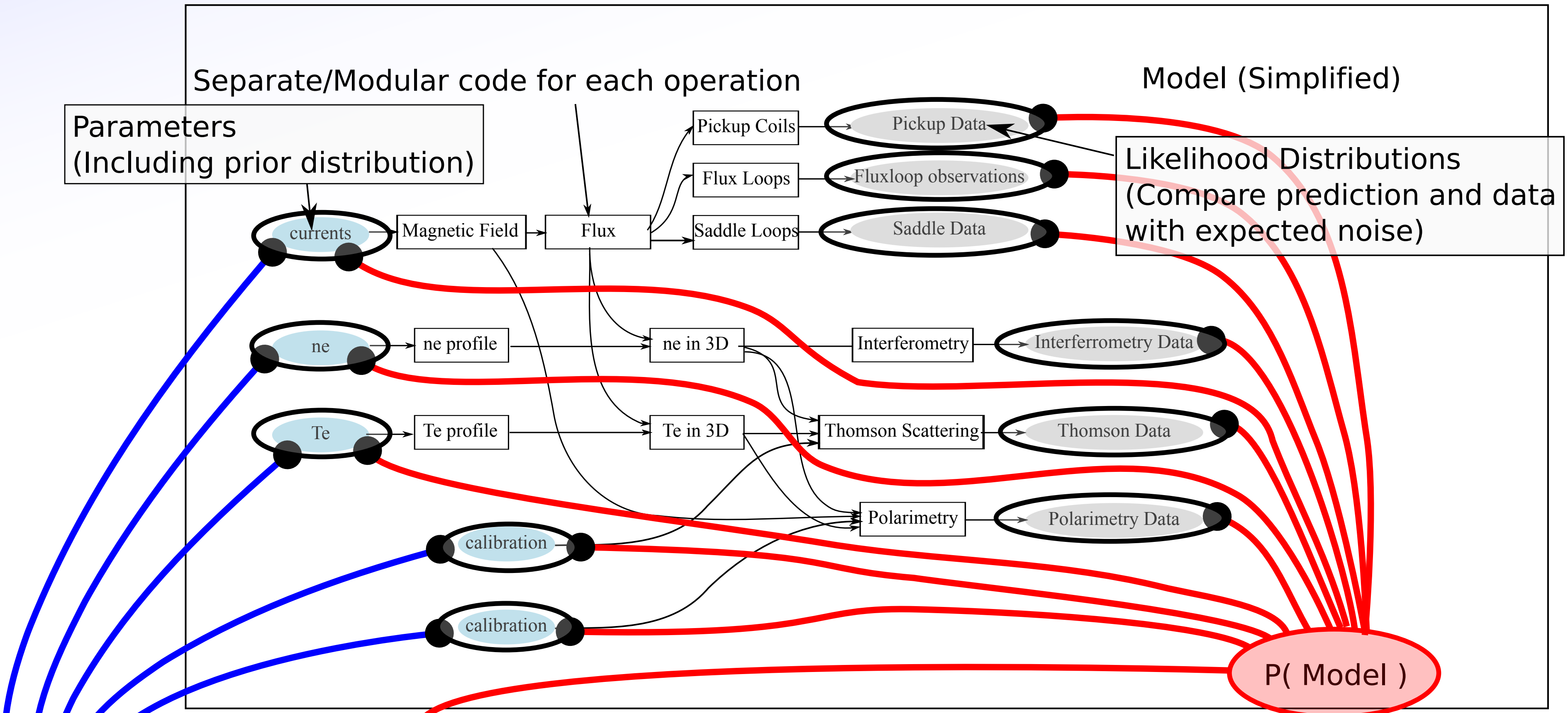


'Samples' = Profiles at points drawn from posterior distribution.
Fully represents **uncertainties and degeneracy**



Forward modelling and Bayesian Inference

Minerva framework for Bayesian combined modelling:



Bayes Theorem: P(Te, Ne, J | Data) ~ P(D | Ne, Te, J) P(Te, Ne, J)

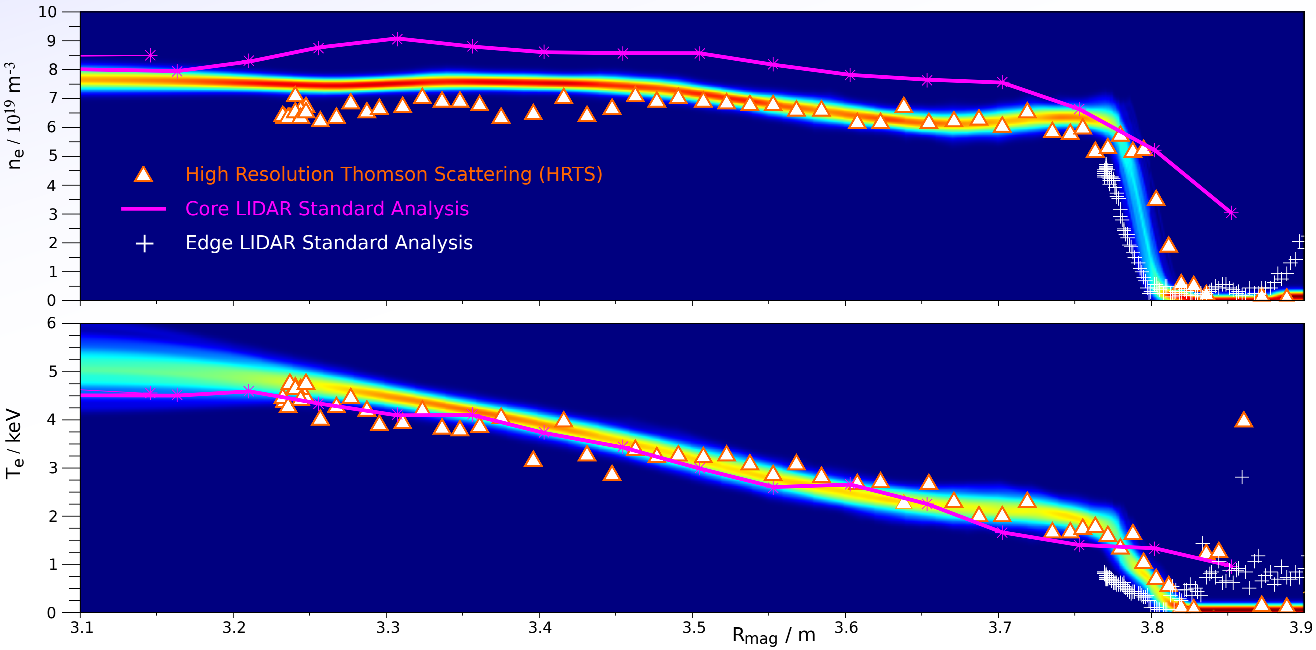
Practically: Solve and explore using external algorithms:

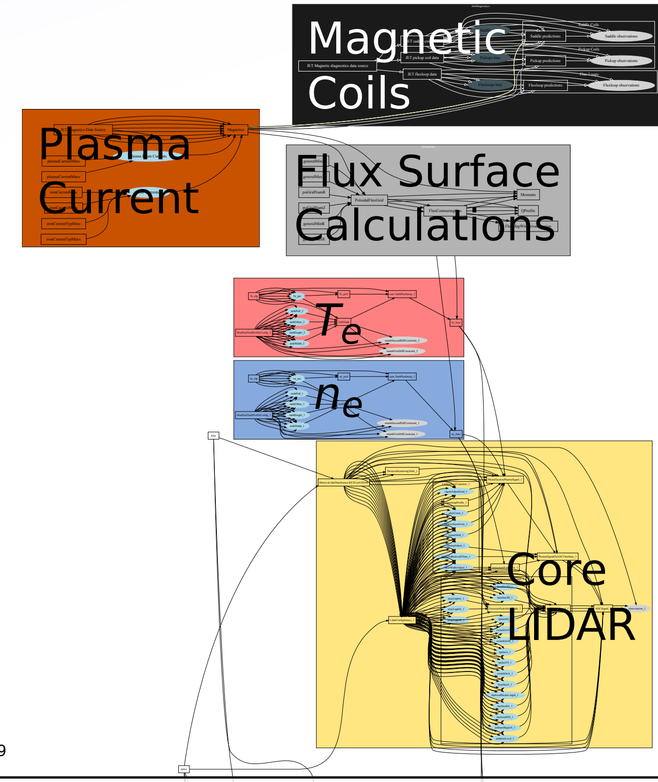
Linear Gaussian Solver
(Best fit and PDF
covariance)

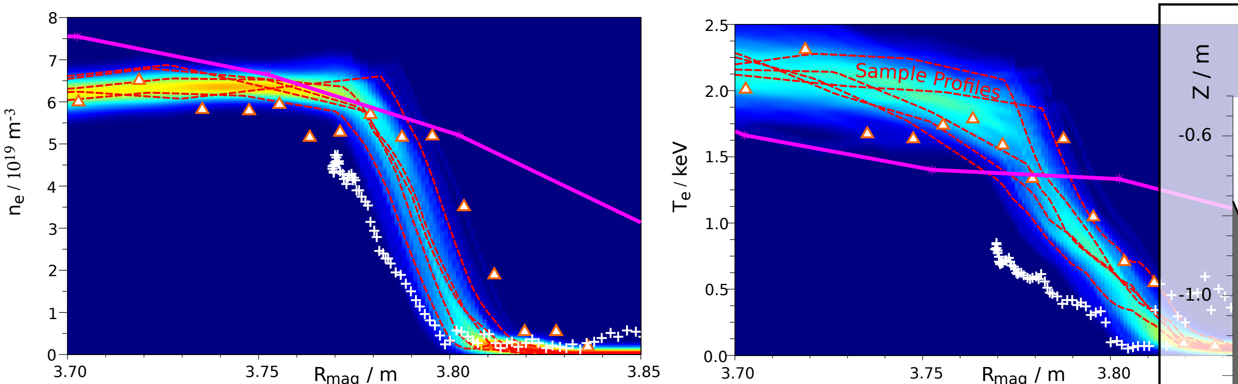
Genetic Algorithms
(Non-linear best fit)

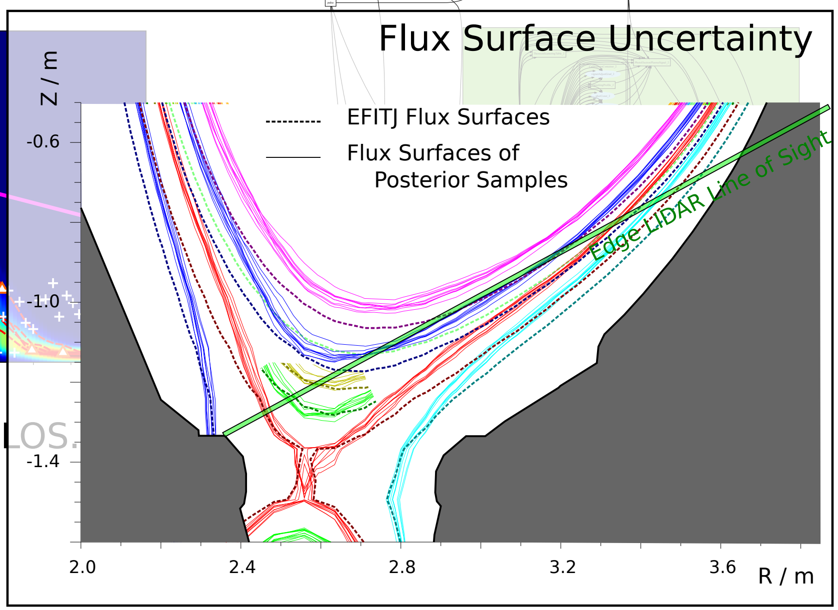
Metropolis Hastings
MCMC Non-linear Exploration:
--> Uncertainty

Connect magnetics model and run inversion.





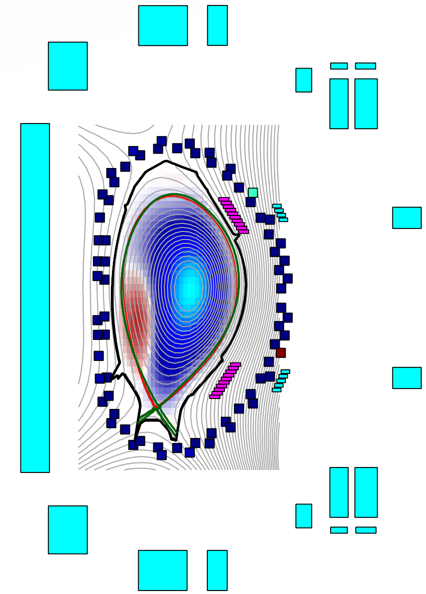




The TS diagnostics provide information on plasma current near LOS.

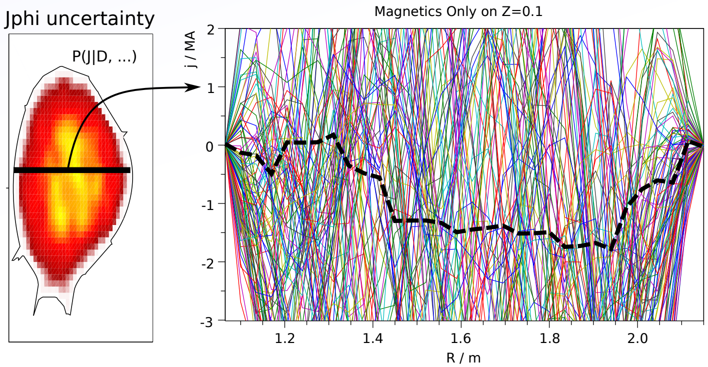
Plasma current one of the most important and least diagnosed parameters in Tokamaks.

IMSE + Current Tomography



Put description of AUG coils and some pickups into Minerva so we can now do Current Tomorgraphy and Bayesian Equilibrium for AUG.

For magnetics only, we have the usual tomography situation:



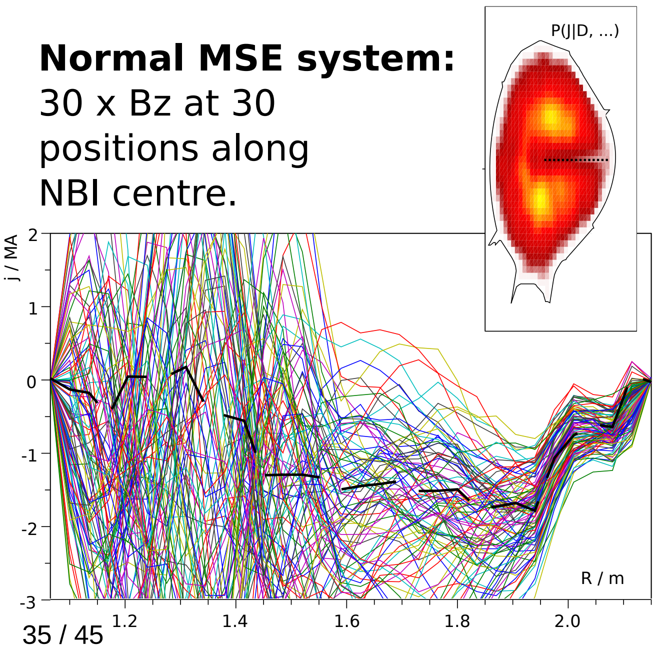
(Almost) no prior/regularisation

↓

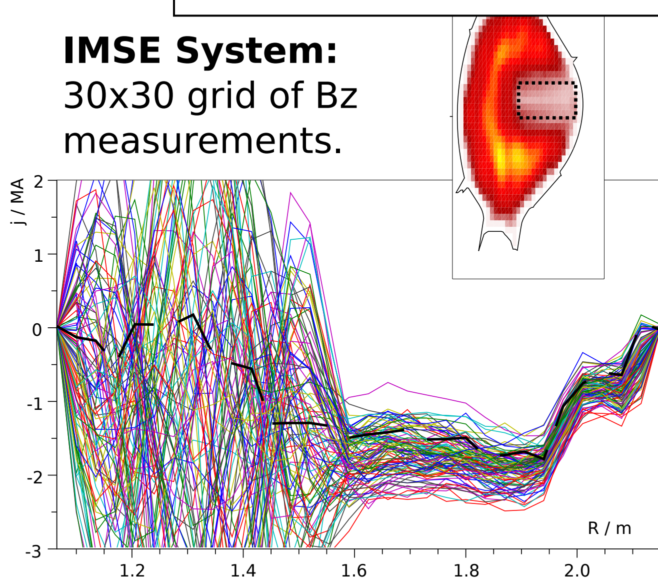
(Almost) infinite uncertainty
(but B/psi still good)

Each case has 900 measurements at sigma = 10mT.
So difference is only in the ***type*** of information.

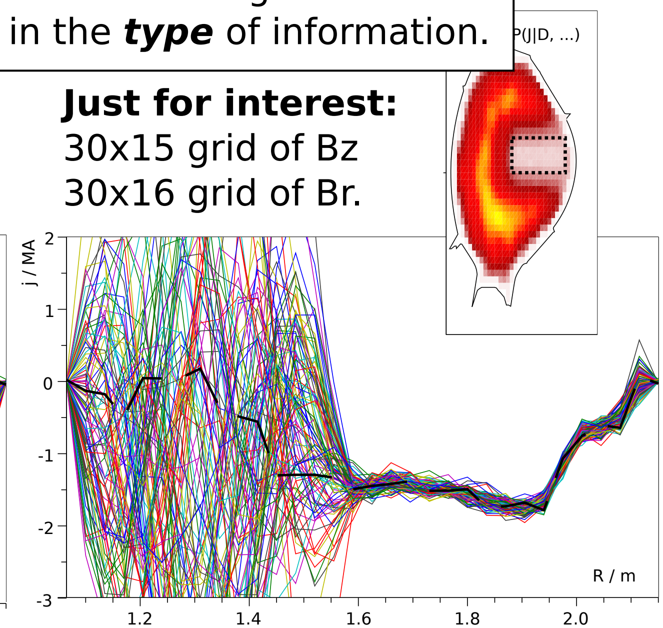
Normal MSE system:
30 x Bz at 30 positions along NBI centre.



IMSE System:
30x30 grid of Bz measurements.

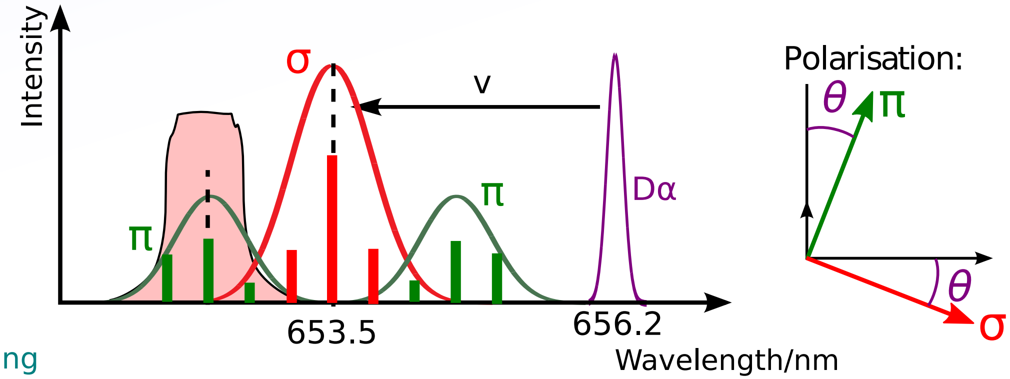


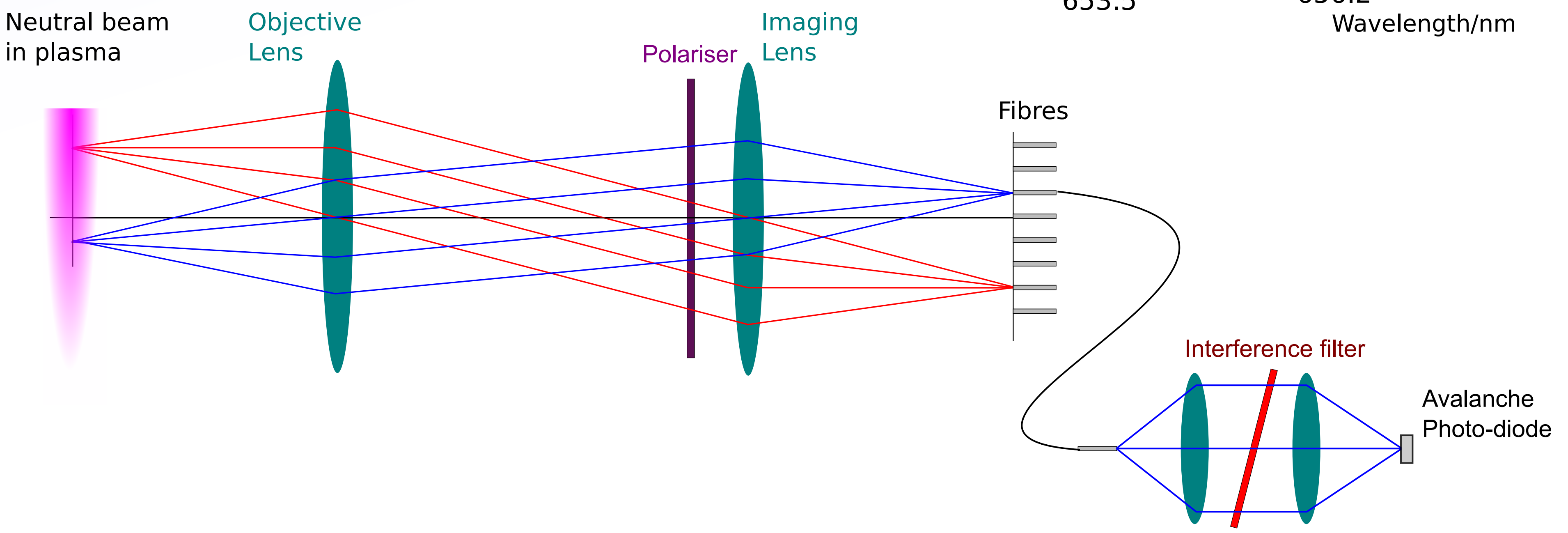
Just for interest:
30x15 grid of Bz
30x16 grid of Br.



Motional Stark Effect Polarimetry

Typical hardware:
- Temperature/tile tuned interference filter





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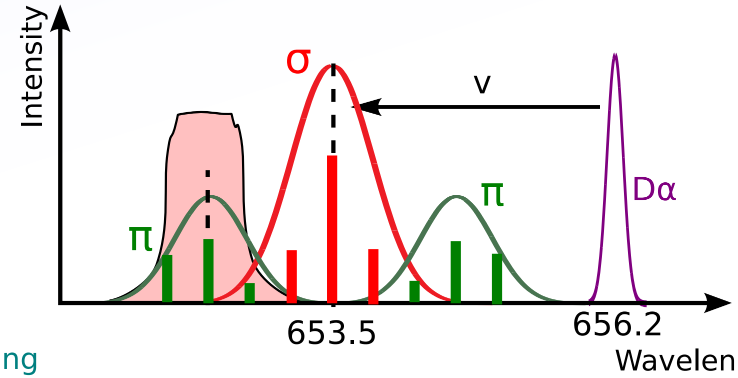

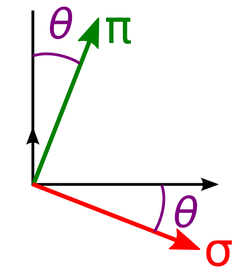

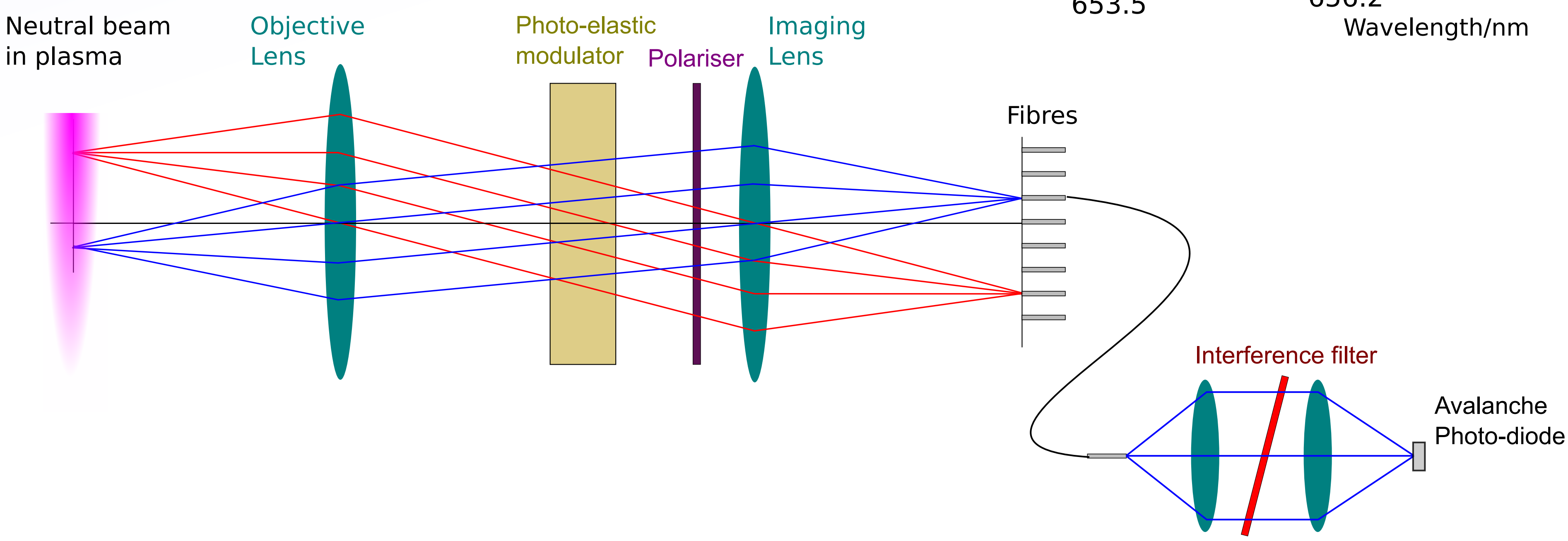

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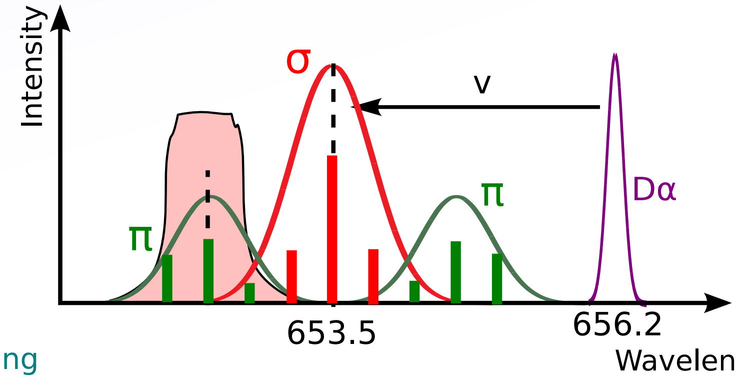

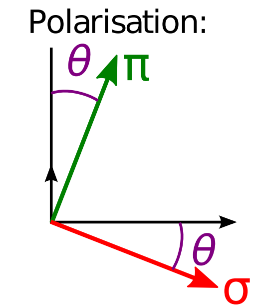

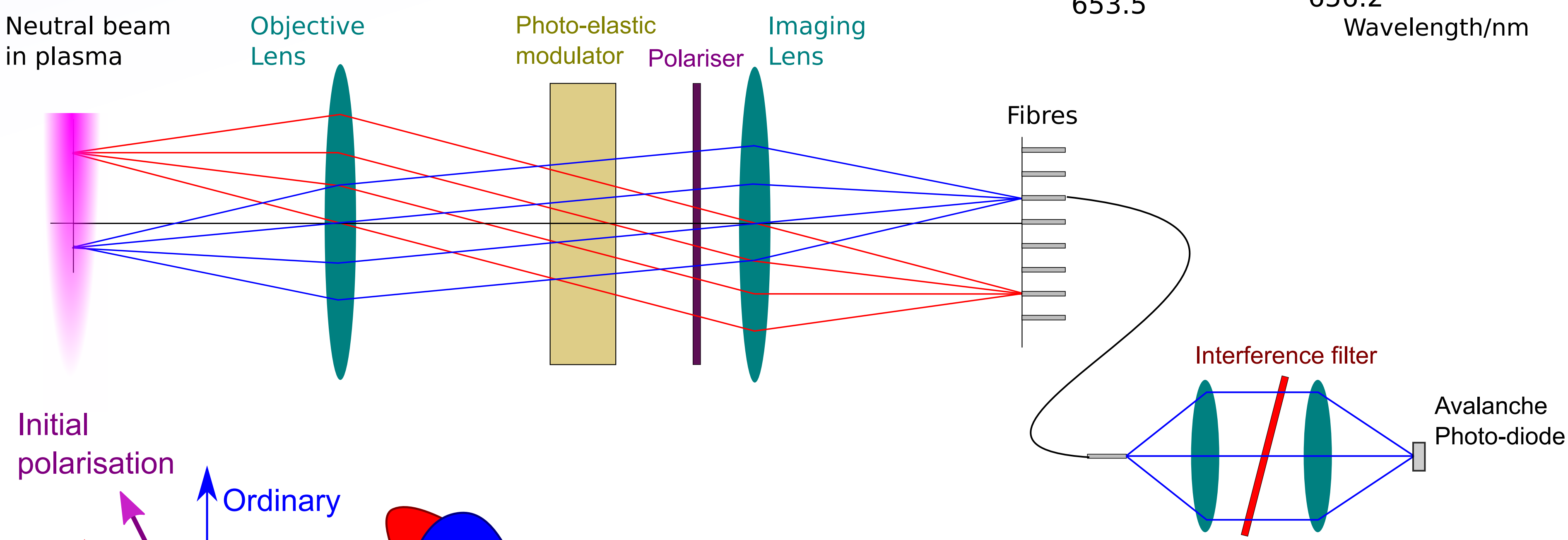

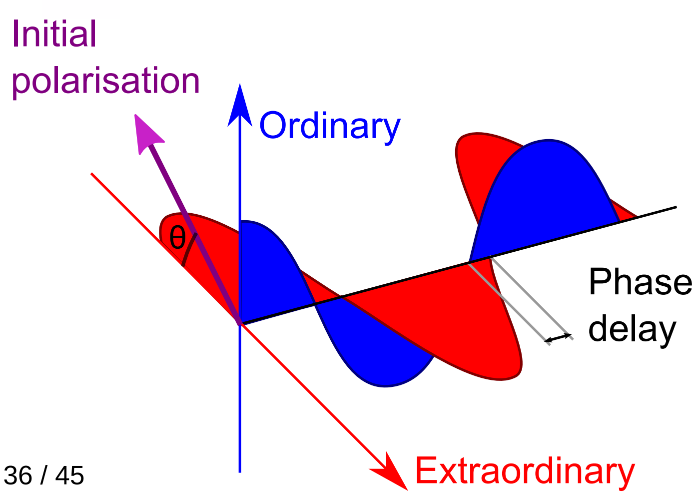

- PEMS at ~45° to initial polarisation
- Splits incoming polarisation into E and O waves.
- Introduces phase shift φ between E and O

Motional Stark Effect Polarimetry

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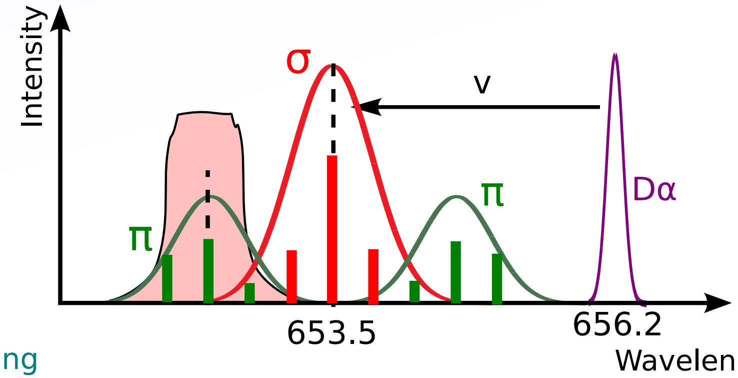
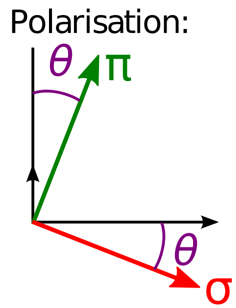
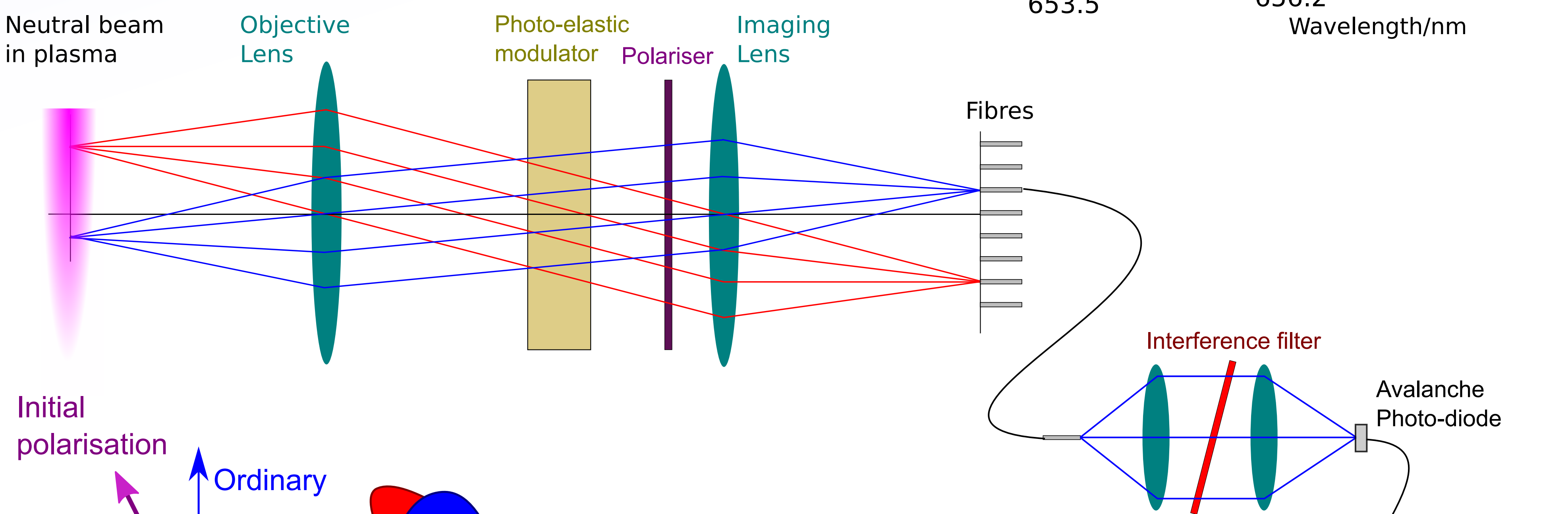
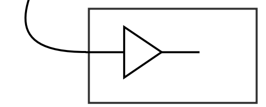
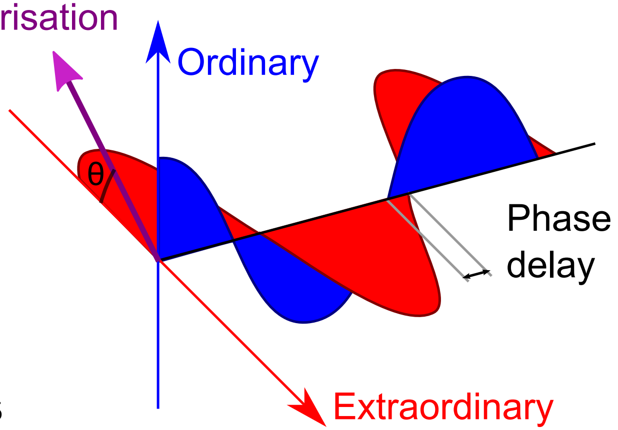
- Temperature/tile tuned interference filter
- Photo-elastic modulator (PEM)









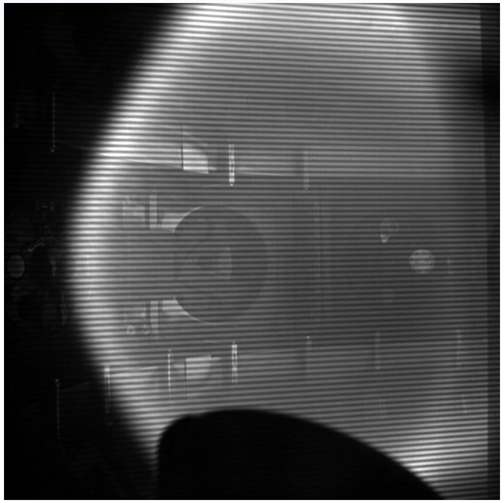


- PEMS at ~45° to initial polarisation
- Splits incoming polarisation into E and O waves.
- Introduces phase shift φ between E and O
- Modulates *Δφ* in time
- E and O waves interfere due to final polariser.
- Modulation of *Δφ* in time from which *θ* can be recovered.

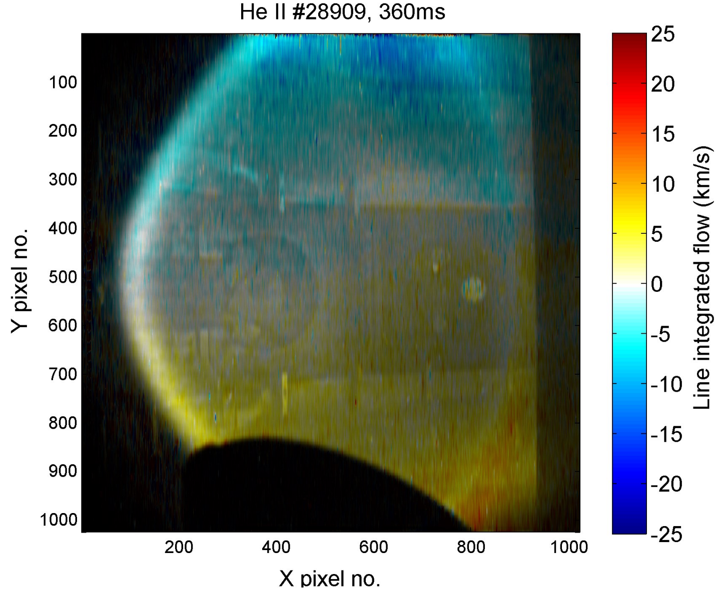
Coherence Imaging

Some results of neutral Helium flow in the (relatively) cold edge of MAST:

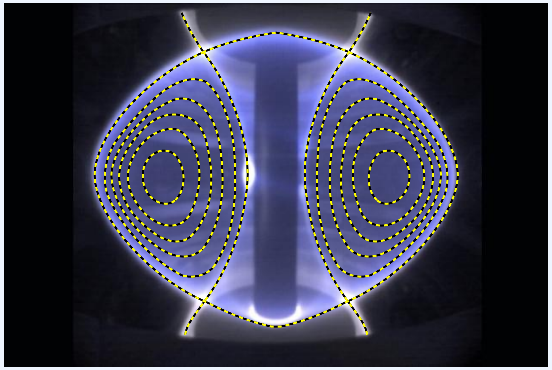
Raw Image:



Helium Flow Velocity:

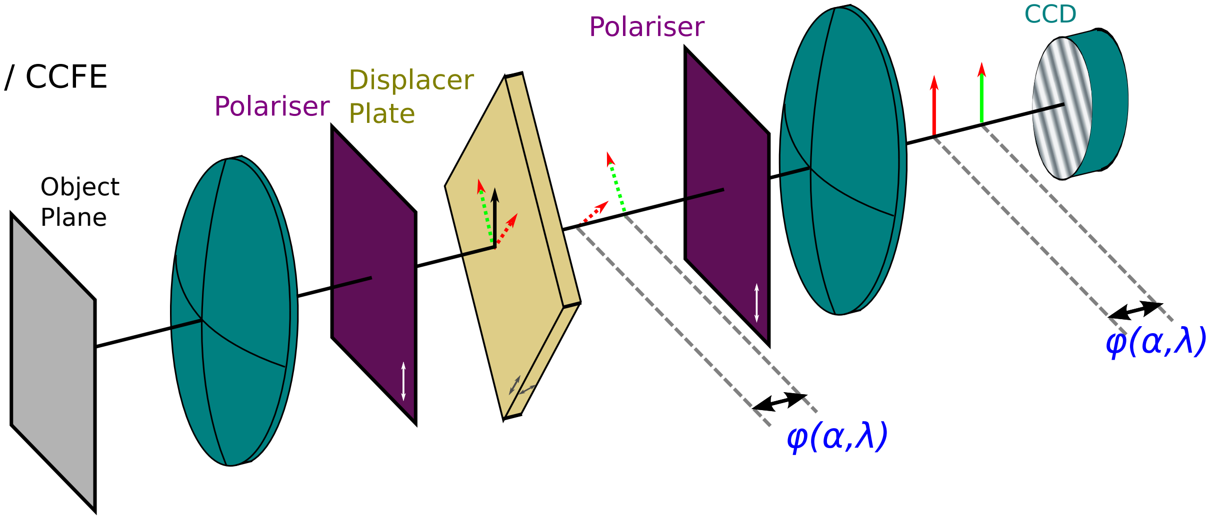


MAST
Mega Amp Spherical Tokamak,
CCFE, Culham, UK



MAST is a 'spherical' Tokamak. The torus has a very small major radius compared to it's minor radius, but is still a Tokamak.

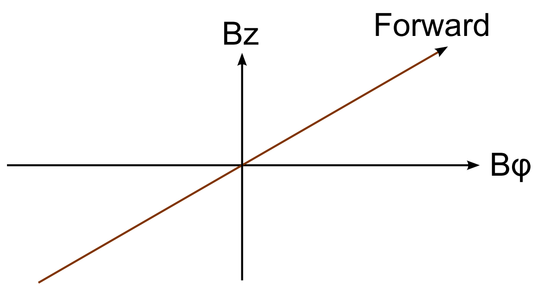
*With thanks to Scott Silburn, Durham University / CCFE
[S. Silburn et. al. 40th EPS Conf. on plasma phys. 2013]



IMSE - Calibration

Absolute q_0 requires absolute $d\theta/dR$
How can we calibrate θ, (or $d\theta/dR$)?

- Run the same plasma with reversed field --> Reversed pitch angle



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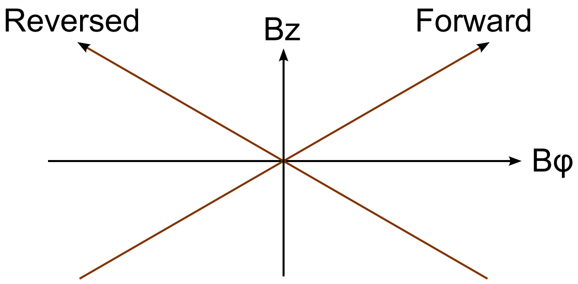

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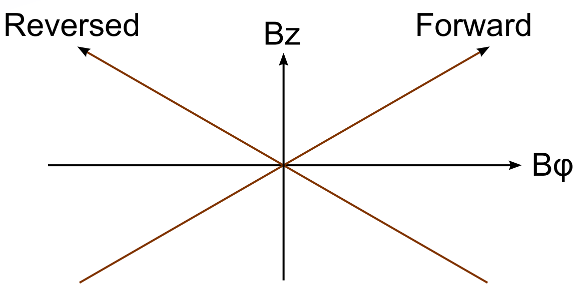

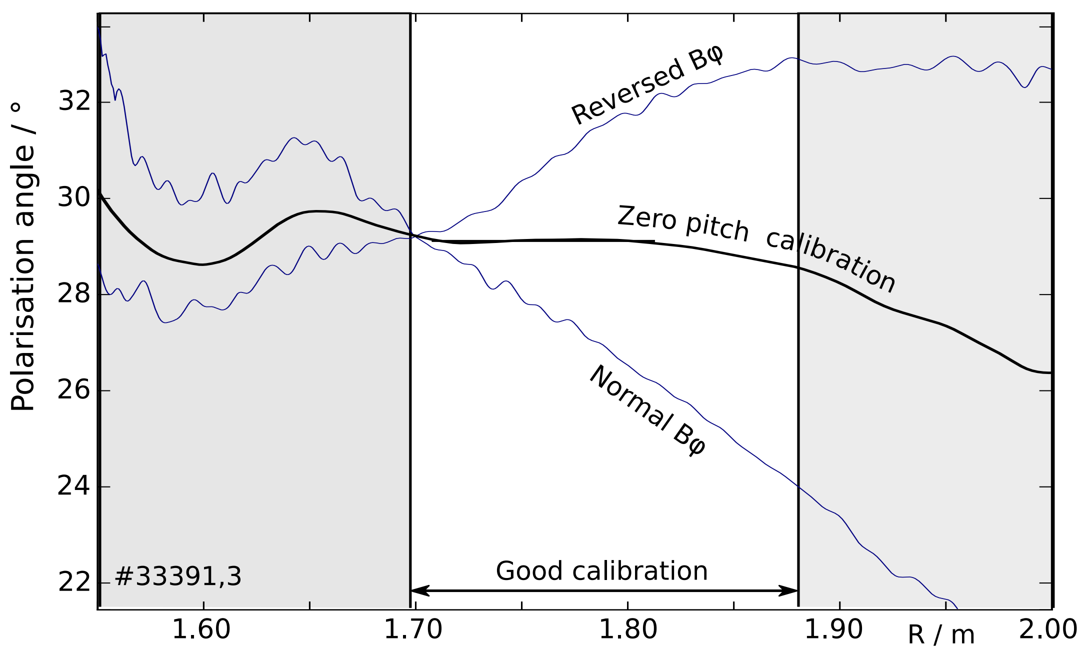

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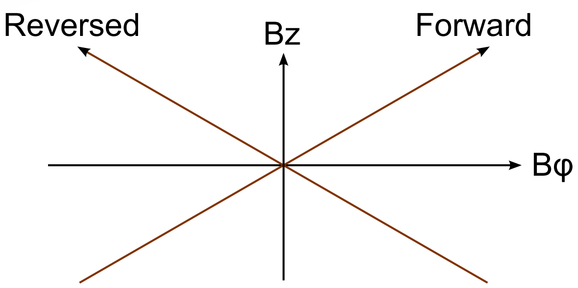
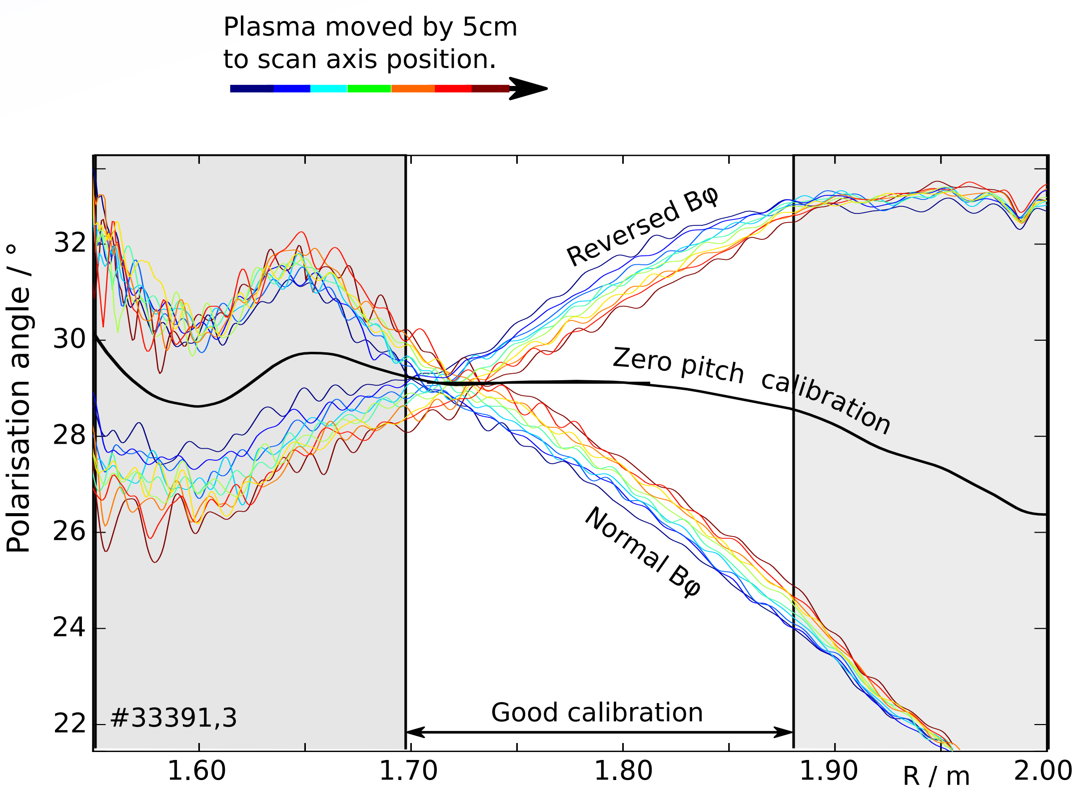
- Run the same plasma with reversed field --> Reversed pitch angle
- Also scan axis position to confirm meeting point (magnetic axis) agrees with 0 pitch angle.





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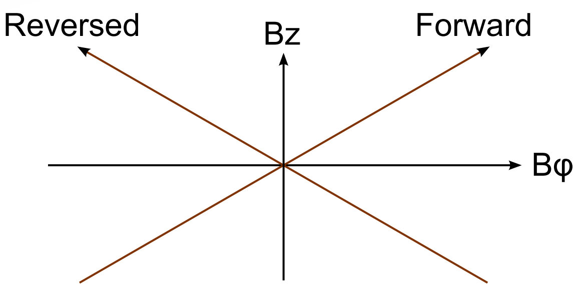
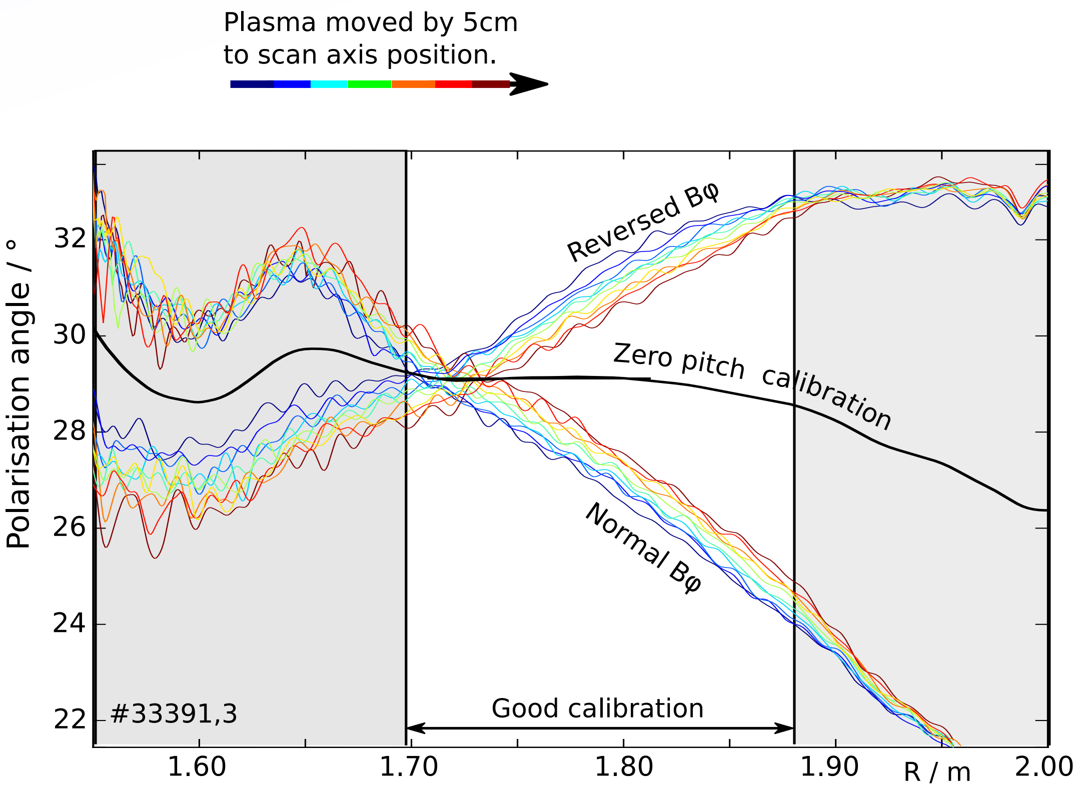
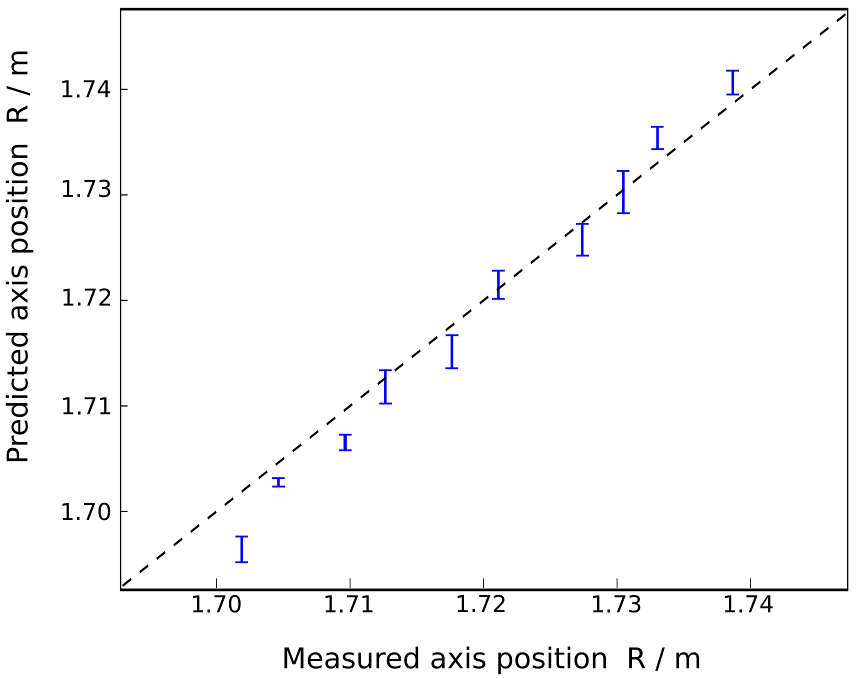




Meeting point well predicted by equilibrium:



IMSE - Calibration

Absolute q_0 requires absolute $d\theta/dR$
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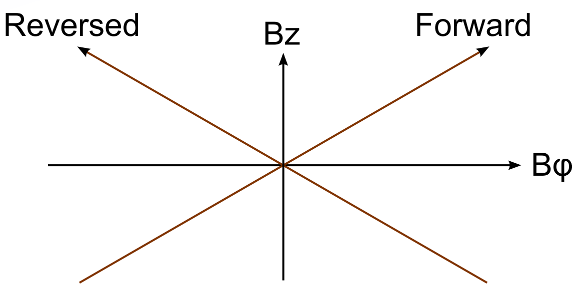
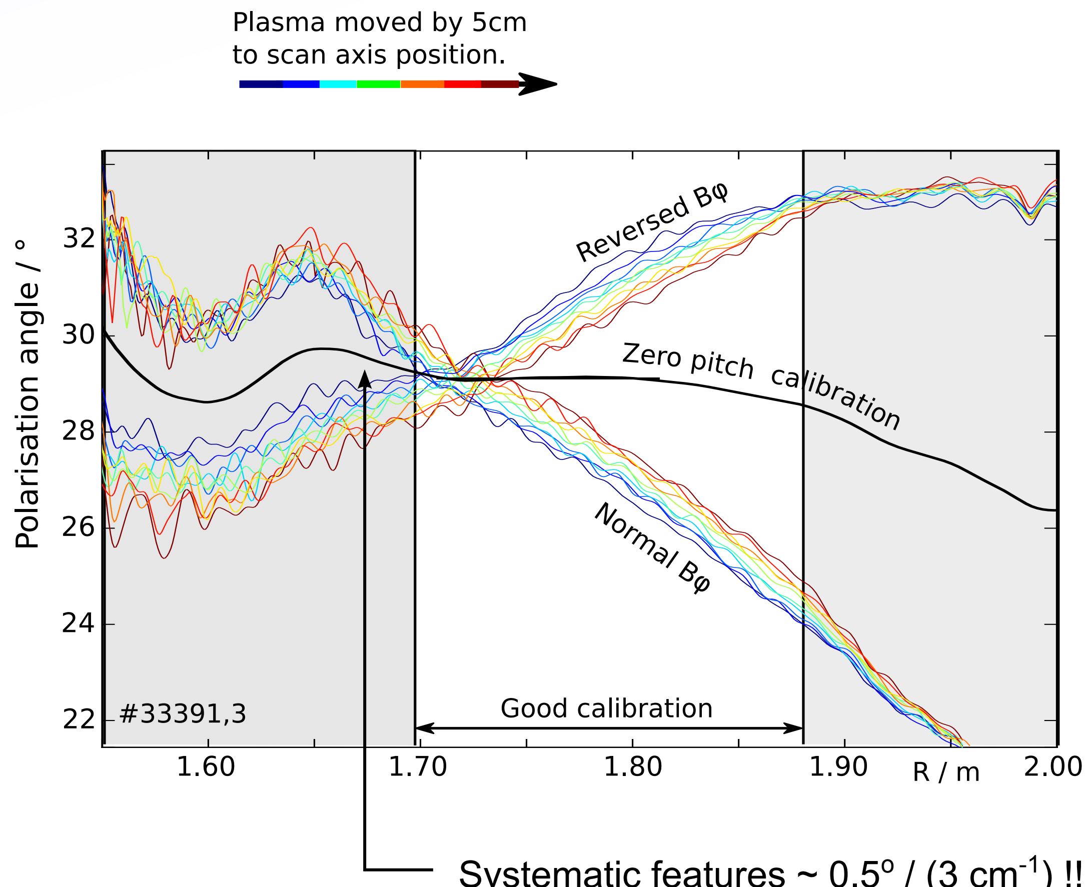
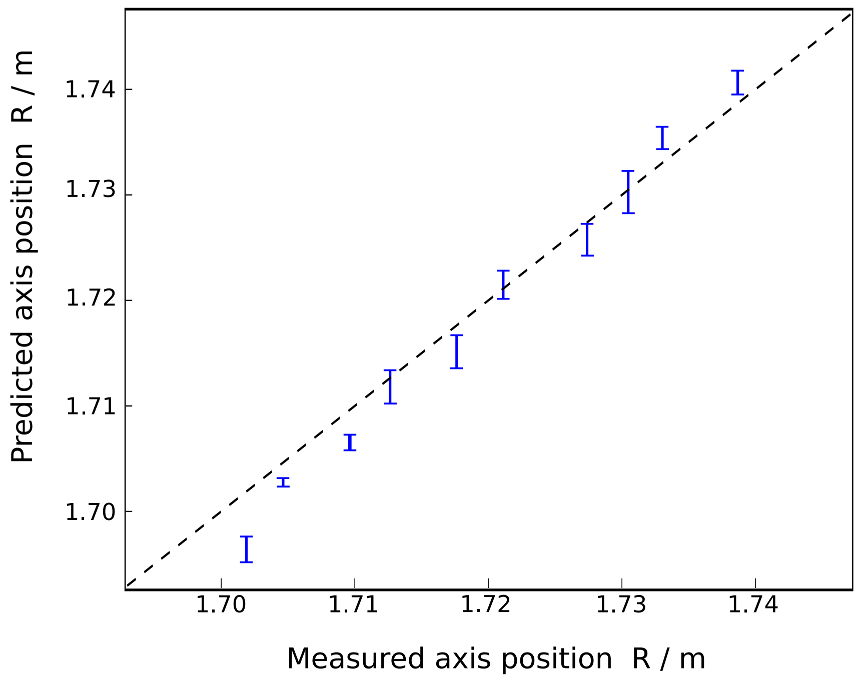




Meeting point well predicted by equilibrium:



Uncertainties in j/q without internal measurements

#32232 3.0s

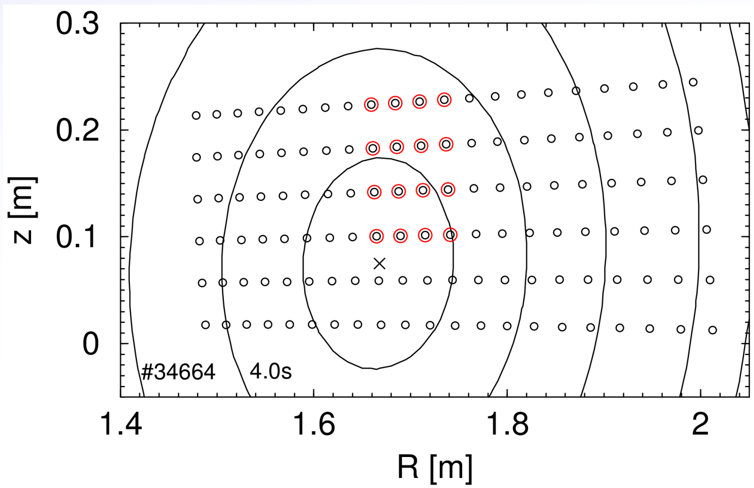
Uncertainties in j/q 1 MSE LOS

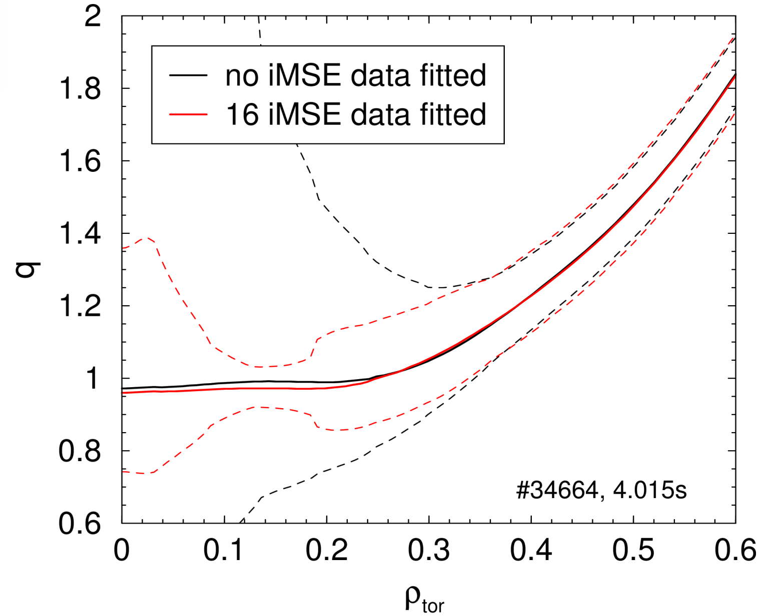
#32232 3.0s

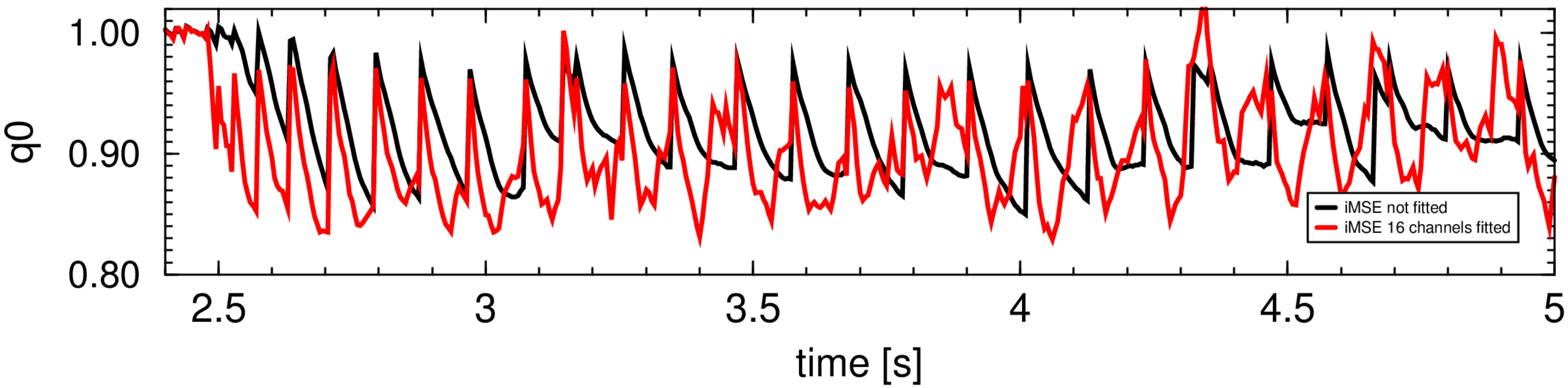
Uncertainties in j/q 12 IMSE LOS

#32232 3.0s

Status: iMSE fit (2)

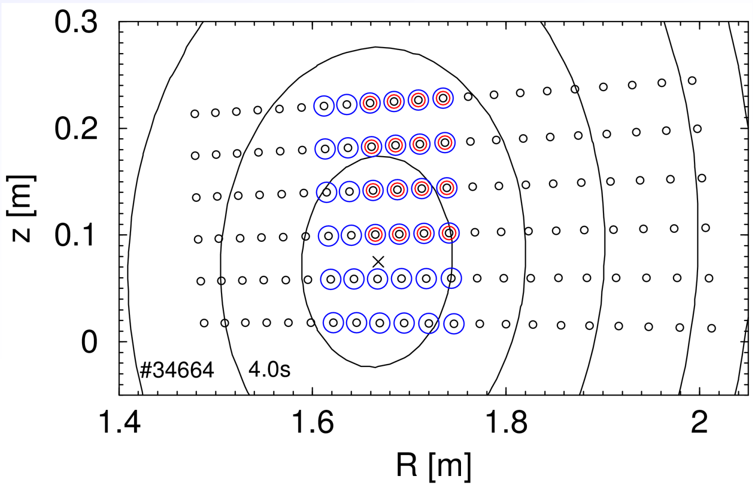


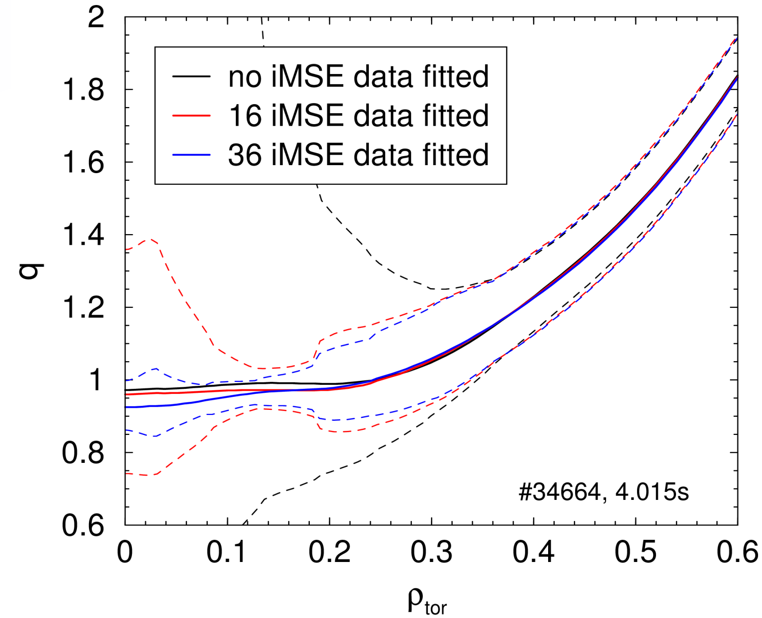


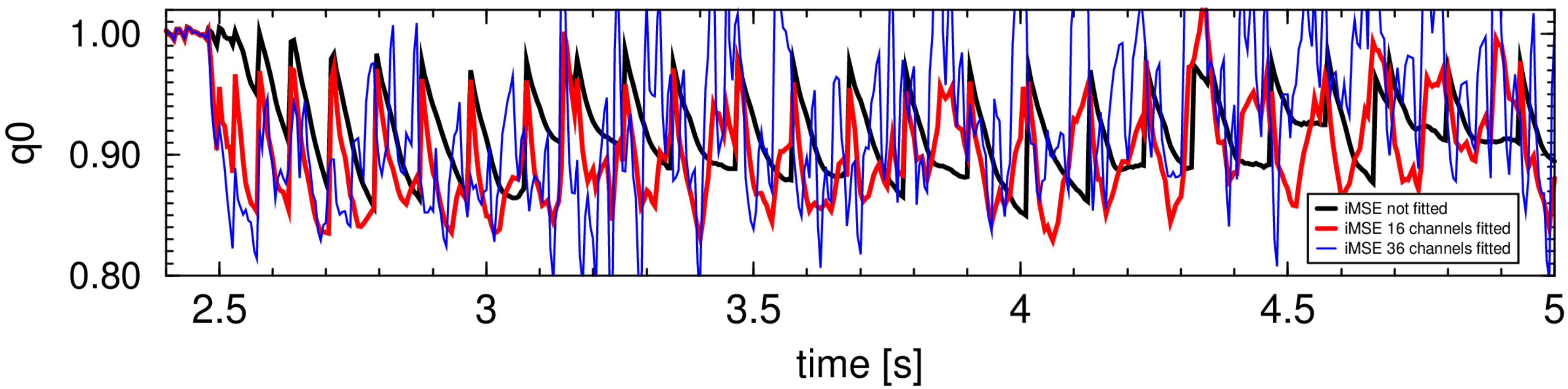


- 16 iMSE channels appear reasonable, but large q0 uncertainty

Status: iMSE fit (3)







- 16 iMSE channels appear reasonable, but large q0 uncertainty
- 36 iMSE channels not conclusive

Note: q0 estimation is the most challenging problem in current profile reconstruction!