Bayesian Analysis Results from JET: Thomson Scattering and Equilibrium

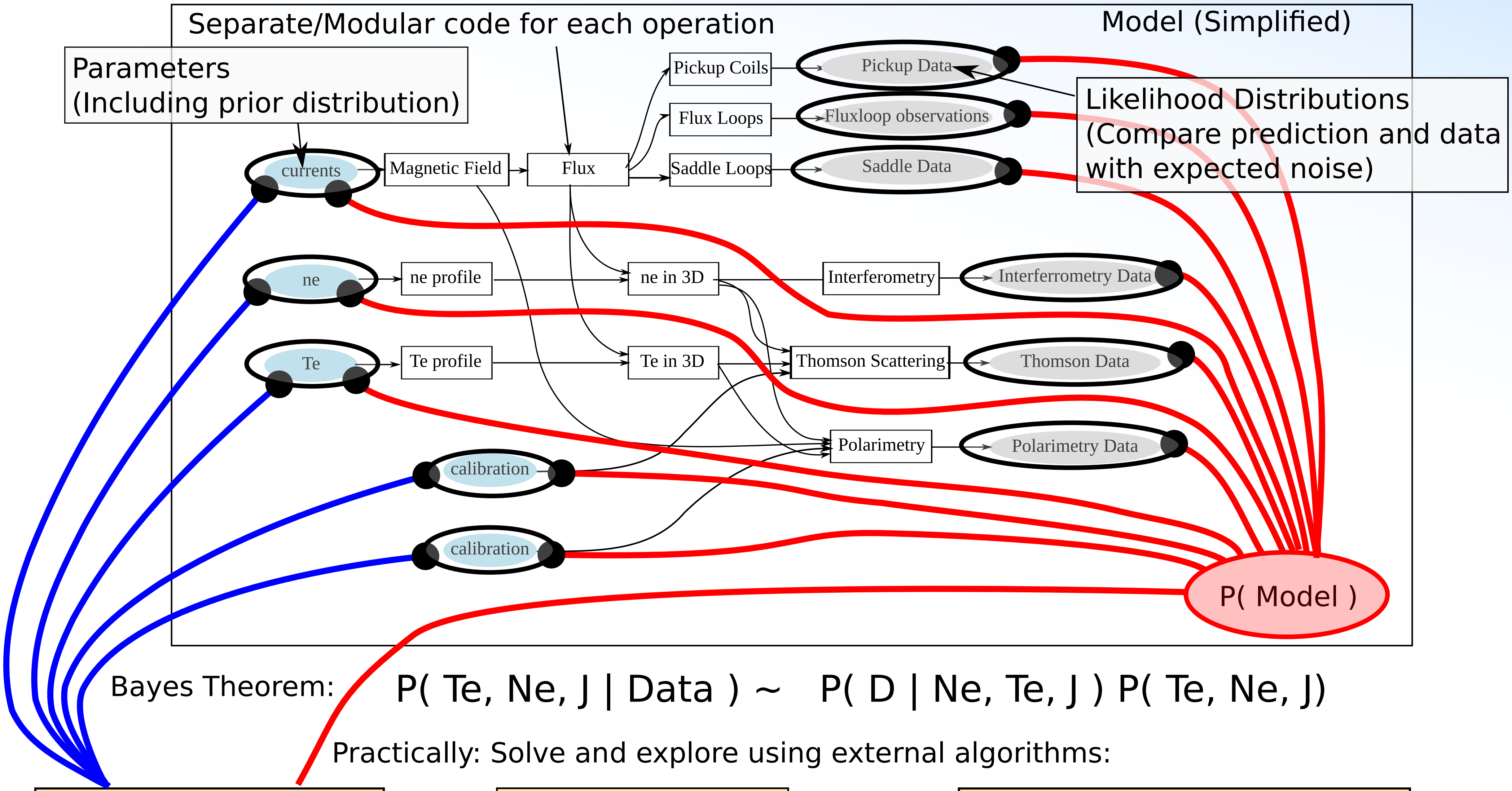
O. P. Ford,[1] J. Svensson,[2] M. Beurskens,[3] A. Boboc,[3] J. Flanagan,[3] M. Kempenaars[3]
D. C. McDonald,[3] A. Meakins,[3] E. Solano,[3] JET-EFDA Collaborators*

1: Blackett Laboratory, Imperial College, London SW7 2BZ, UK
2: Max Planck Institute, Teilinstitut Greifswald, Germany
3: UKAEA Fusion Association, Culham Science Centre, OX14 3DB, UK

* See the Appendix of F. Romanelli et al., Fusion Energy Conference 2008 (Proc. 22nd Int. FEC Geneva) IAEA

Forward Modelling and Bayesian Inference

The basic idea:



Separate/Modular code for each operation

Model (Simplified)

Parameters
(Including prior distribution)

Likelihood Distributions
(Compare prediction and data
with expected noise)

Bayes Theorem: $P(\, Te, Ne, J \mid Data \,) \sim \; P(\, D \mid Ne, Te, J \,)\; P(\, Te, Ne, J)$

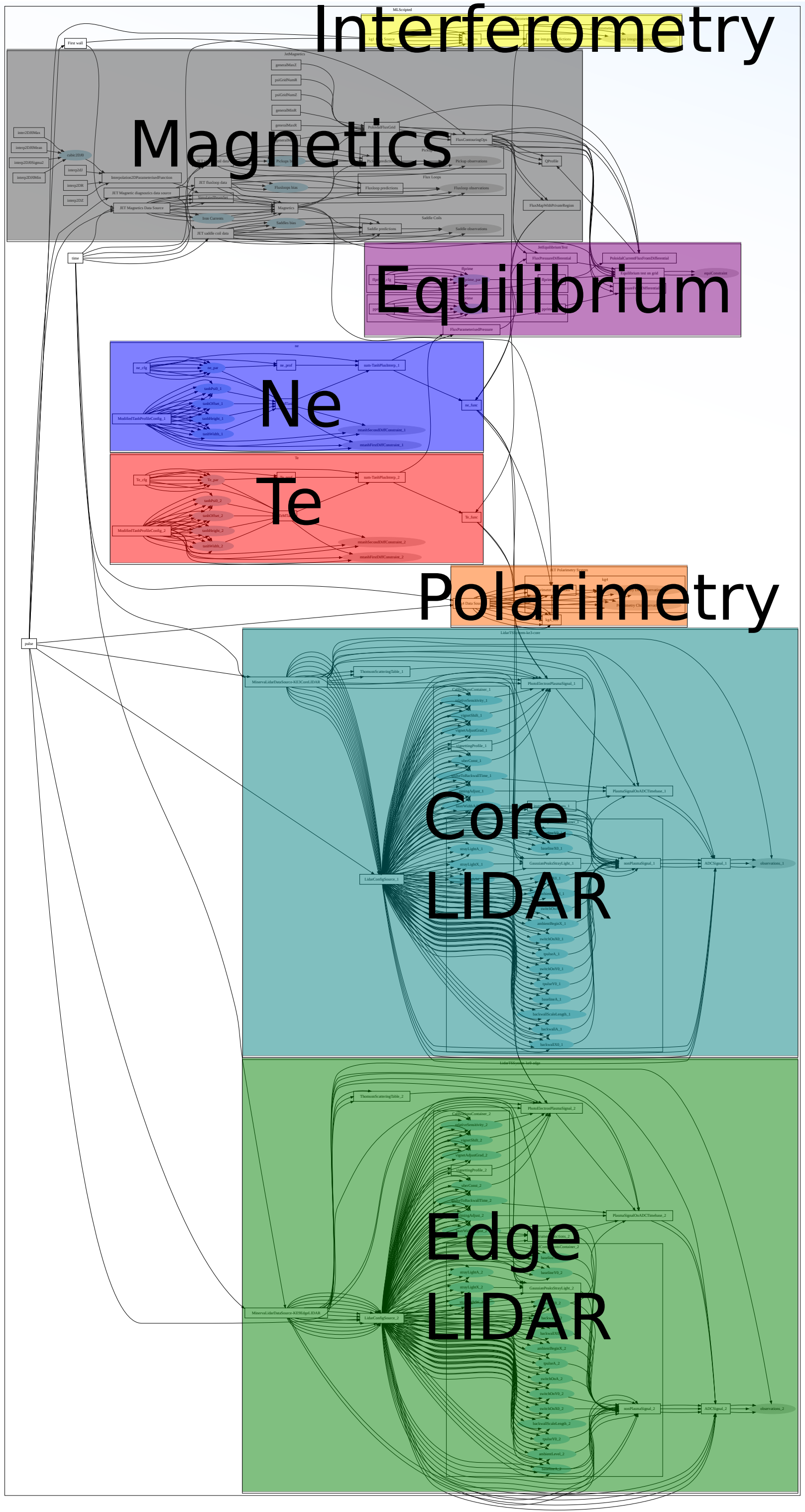
Practically: Solve and explore using external algorithms:

Linear Gaussian Solver
(Best fit and PDF
covariance)

Genetic Algorithms
(Non-linear best fit)

Metropolis Hastings
MCMC Non-linear Exploration:
--> Uncertainty

Software and Models



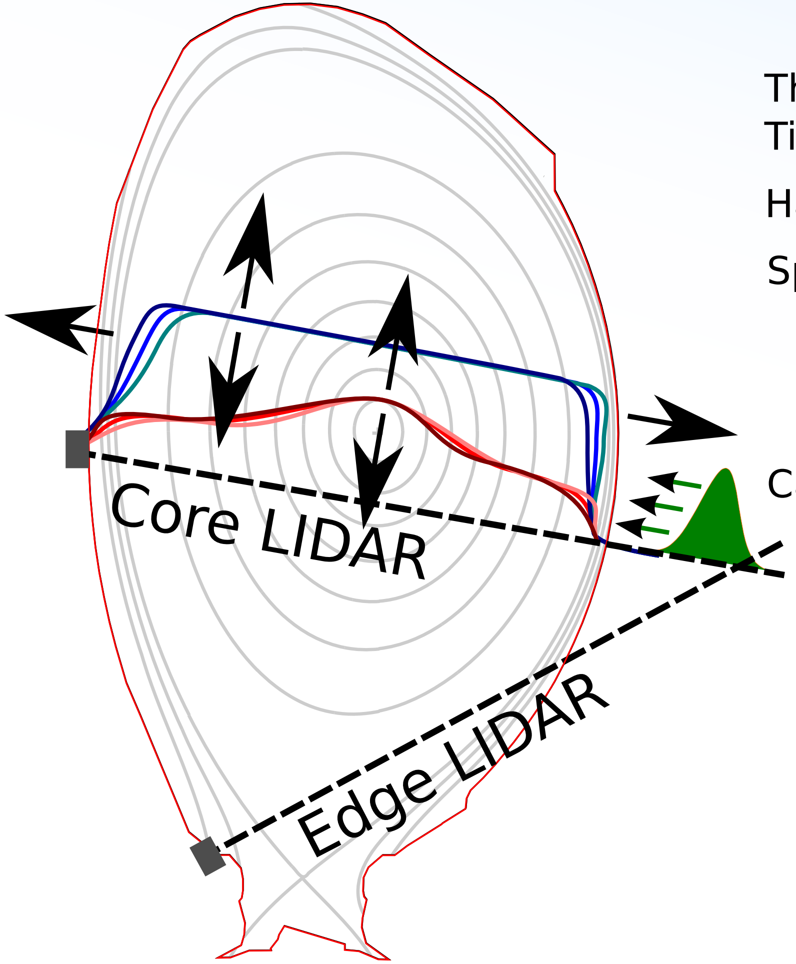
Write nodes and wire them together.
Software framework handles the rest.
Even automatically generates the graphical representation.
We can re-wire the graph and redefine/modify the problem at will, even during a run.

Parts previously written:
- Magnetics (field/flux calculations and JET magnetics)
- Interferometry.

Parts I've written as part of my PhD:
- Polarimetry
- Core LIDAR
- Edge LIDAR
- Equilibrium (Grad-Shafranov Test)
- Various Ne/Te profile models.
- +(Parallelised and developed outer algorithms)

Other parts written during the past 3 years:
- JET MSE
- JET Reflectometry
- JET Infrared strikepoint camera
- MAST Magnetics
- MAST MSE
- MAST Thomson Scattering
- ... and a few others ...

Core + Edge LIDAR: The systems and the problem



Thomson Scattering diagnostics with single spectrometer.
Time of flight for positioning.

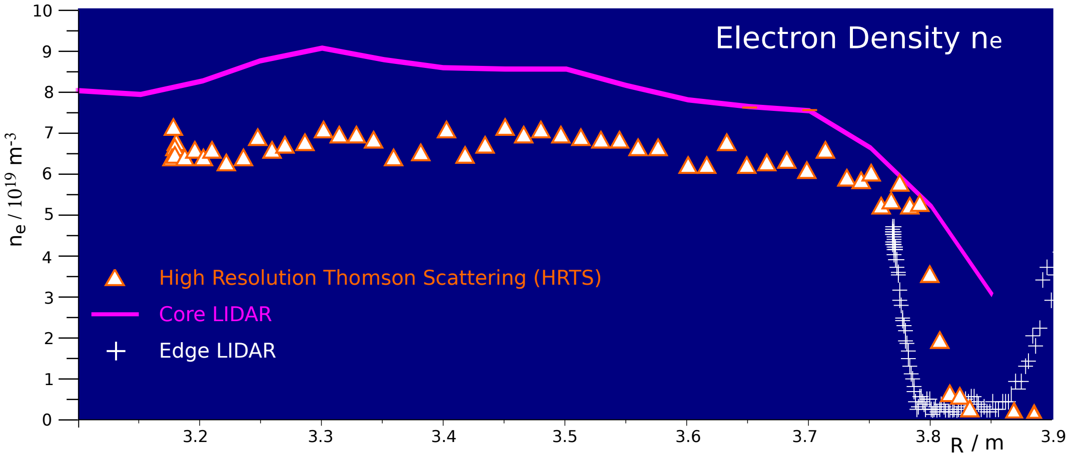
Hardware system very complex.

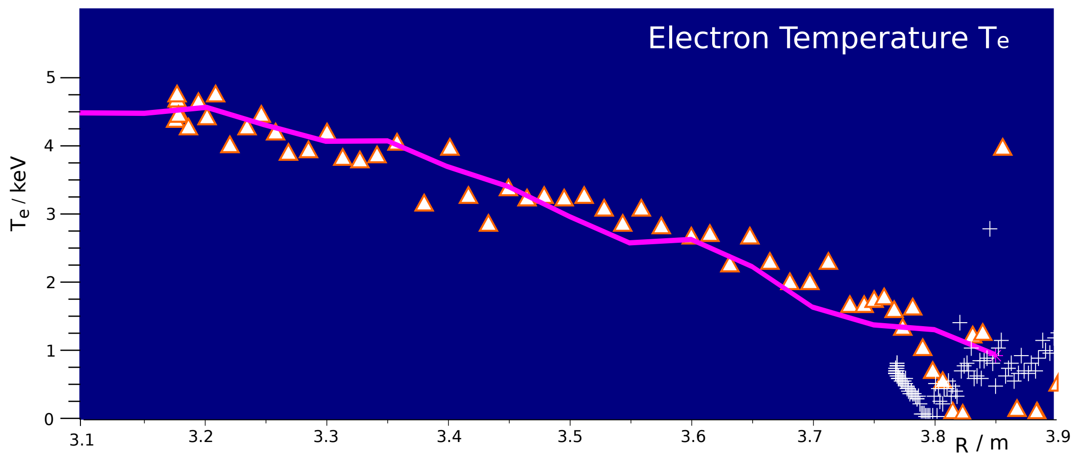
Spatial Resolution:

- Effective convolution of light signal.
- If ignored: Convolves n_e but complex effect on T_e.
- No problem for forward modelling: we just convolve the signal.

Calibrations:

- Beam dump position + timing --> Uncertain position.
- Optical transmission + laser energy --> n_e magnitude.
- Spectrometer Relative Sensitivities --> T_e magnitude.
- Relative Channel timing --> T_e + n_e shape!

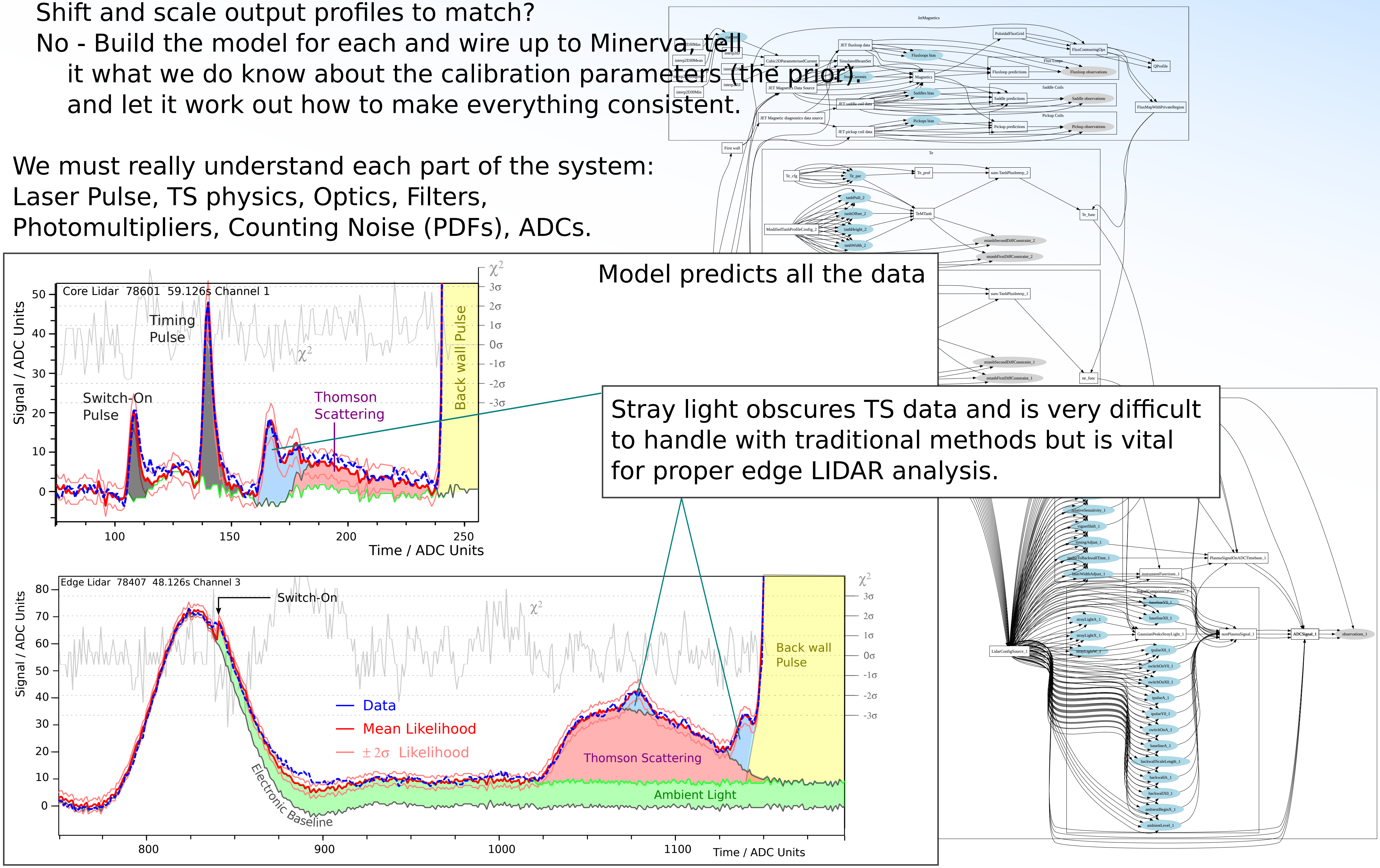




Core + Edge LIDAR: The model

So how do we deal with disagreement with other diagnostics?
  Shift and scale output profiles to match?
  No - Build the model for each and wire up to Minerva, tell
    it what we do know about the calibration parameters (the prior)
    and let it work out how to make everything consistent.

We must really understand each part of the system:
Laser Pulse, TS physics, Optics, Filters,
Photomultipliers, Counting Noise (PDFs), ADCs.



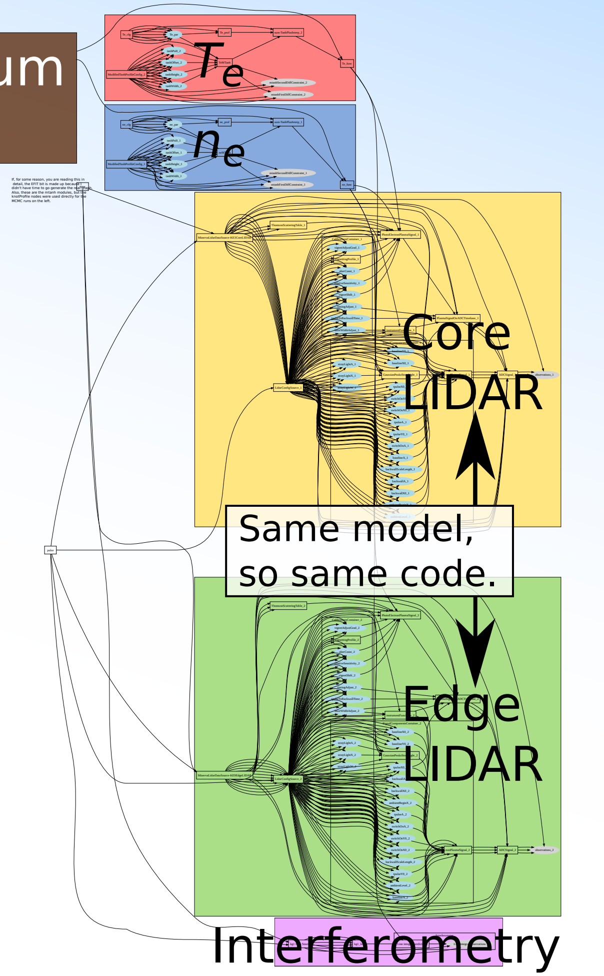
Model predicts all the data

Stray light obscures TS data and is very difficult to handle with traditional methods but is vital for proper edge LIDAR analysis.

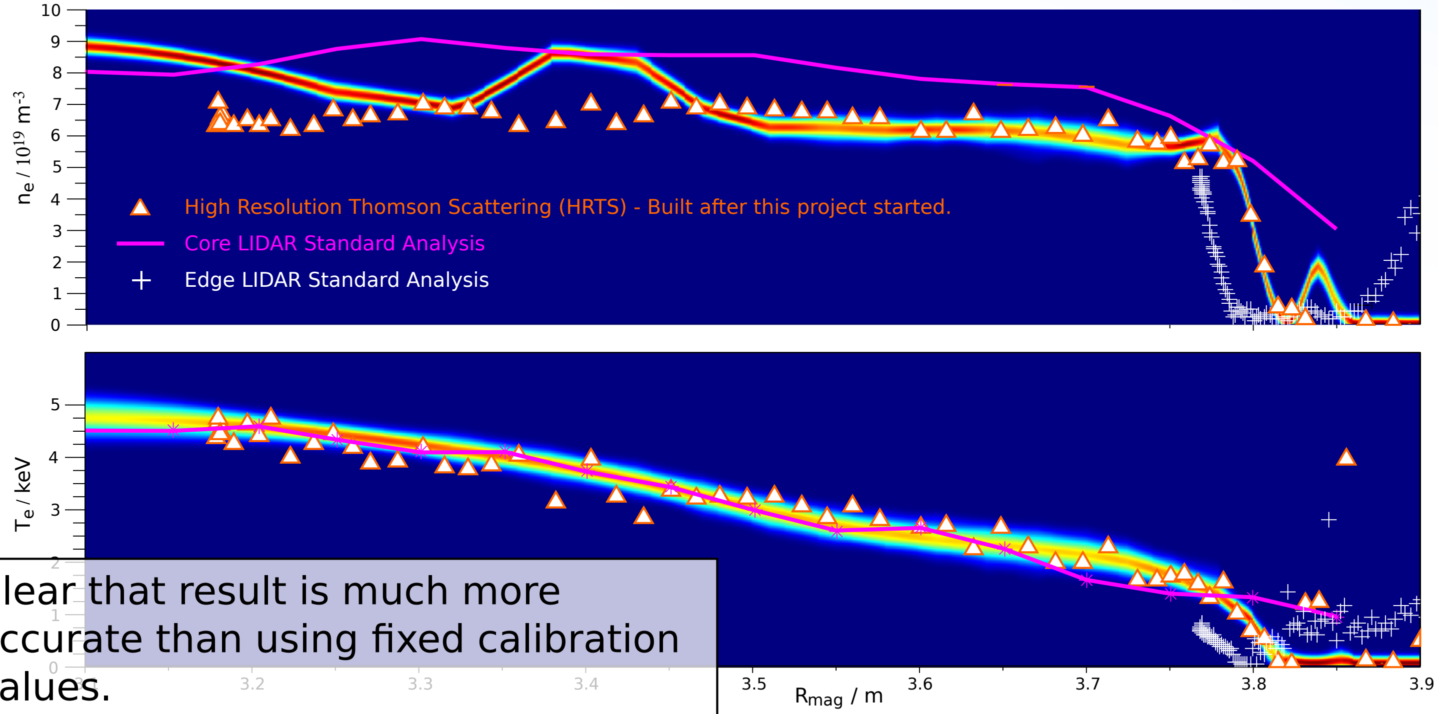
Core LIDAR + Edge LIDAR + Interferometry

A typical high density H-mode pulse:
- Connect up the model.
- Give all calibrations some uncertainty (what we believe).
- Give some less trusted calibrations almost complete freedom (uniform prior).
- Throw the complete problem at the distributed GA for MAP (best fit) and then at the distributed MCMC for the PDF (uncertaint...

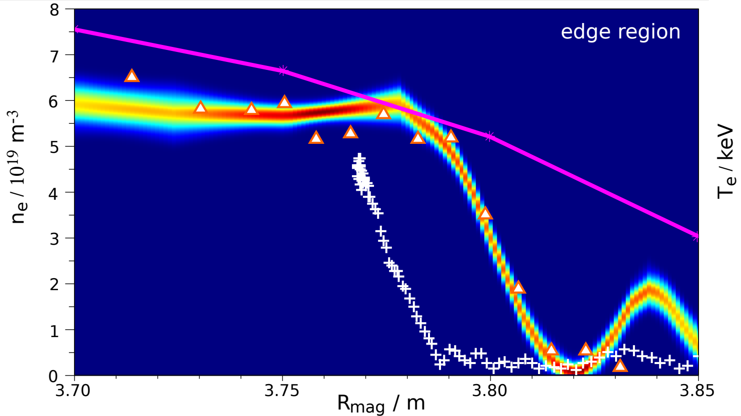
Equilibrium Code

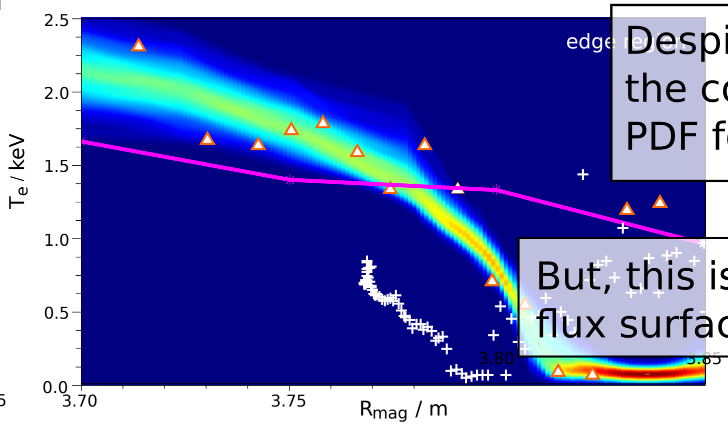


Same model, so same code.



Clear that result is much more accurate than using fixed calibration values.



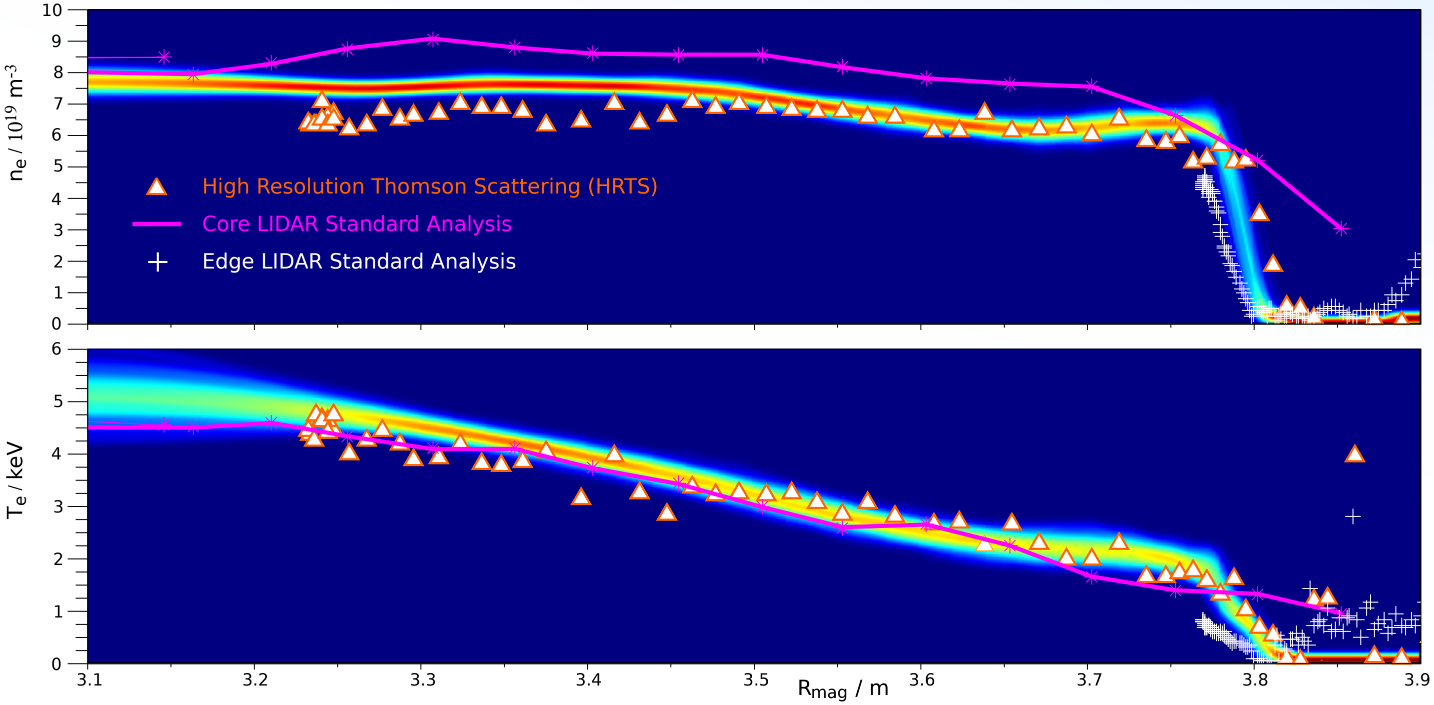


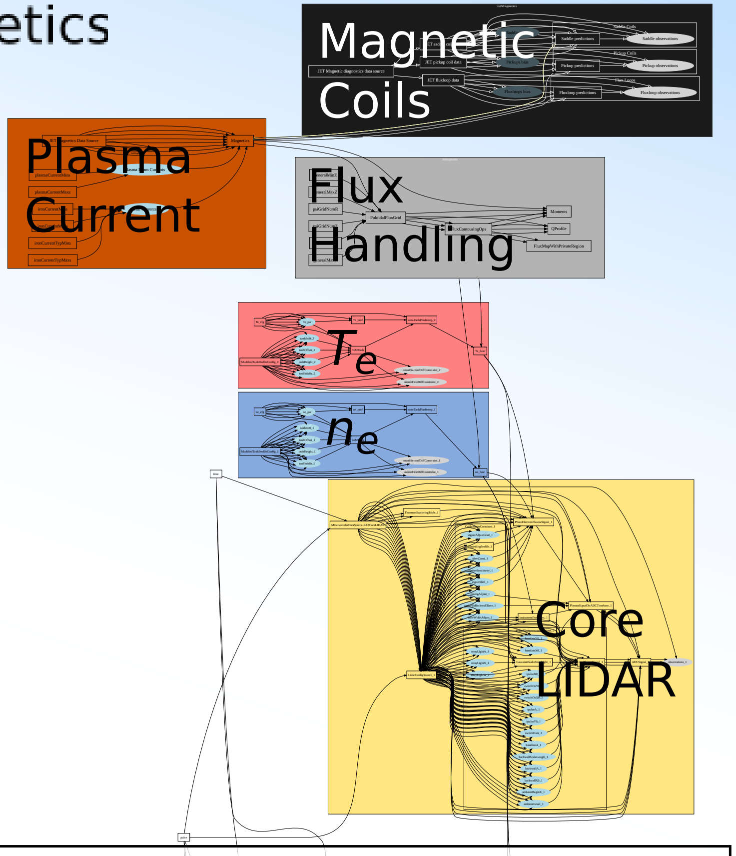
Despite completely free *Te* calibration, the combination can fix Te and gives a PDF for the calibration values.

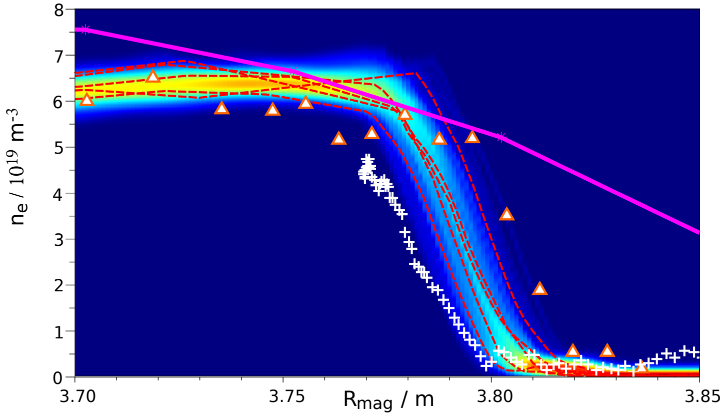
But, this isn't complete - we are still using flux surfaces fixed to the equilibrium code.

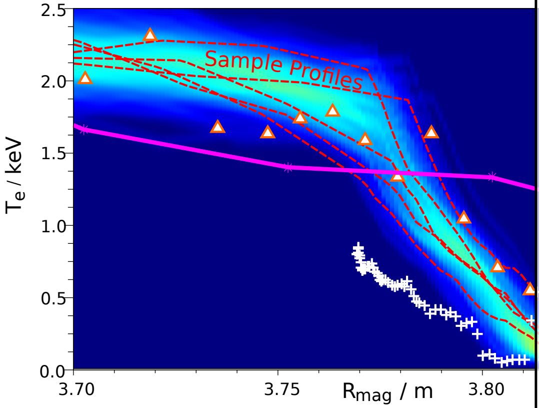
Core LIDAR + Edge LIDAR + Interferometry + Magnetics

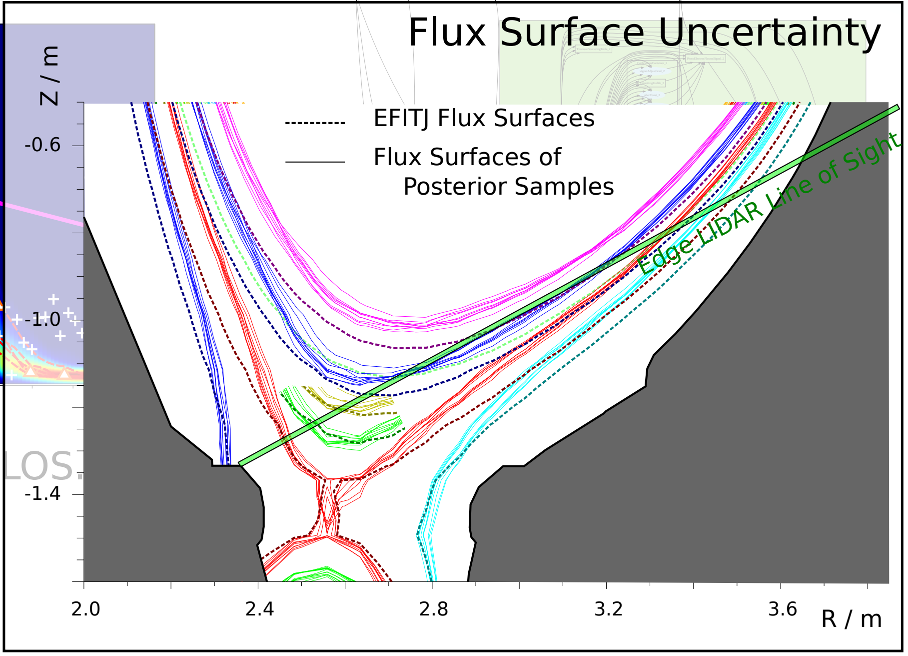
Connect magnetics model and run inversion.









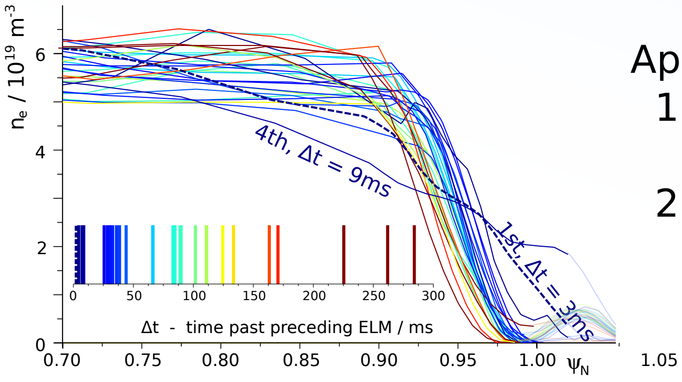


The TS diagnostics provide information on plasma current near LOS.

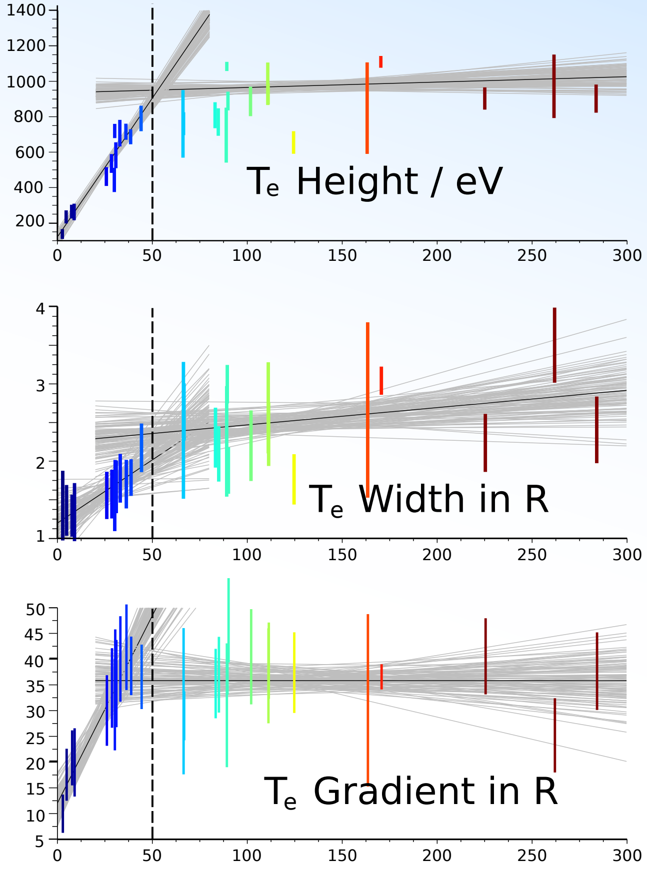
Plasma current one of the most important and least diagnosed parameters in Tokamaks.

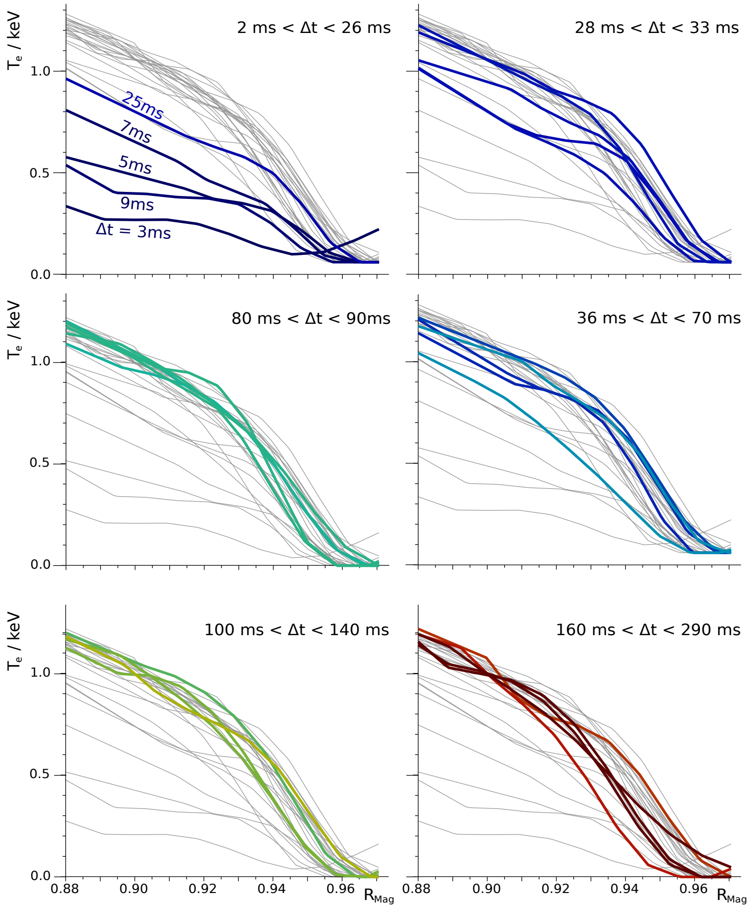
Core LIDAR + Edge LIDAR + Interferometry: Pedestal Evolution Study

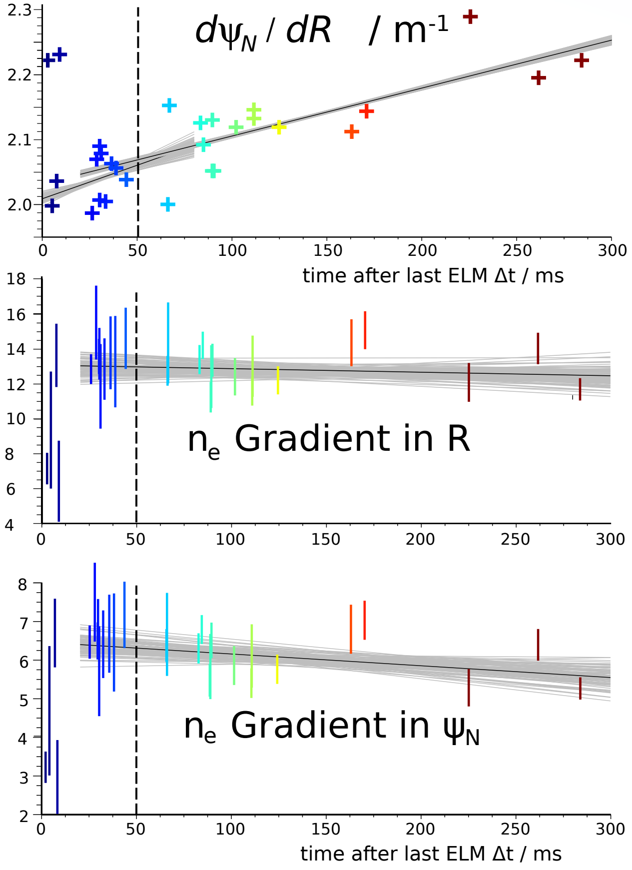
Looked in detail at evolution of ne/Te pedestals through the ELM cycle. 28 time points over 6 almost identicle pulses.



Appears to be two distinct phases for *Te*:
1) Rapid rise in height and gradient during first 50ms.
2) Slow rise in height and width at fixed gradient until next ELM.







Some indication that *ne* pedestal has a fixed gradient in real space despite compression of flux surfaces.

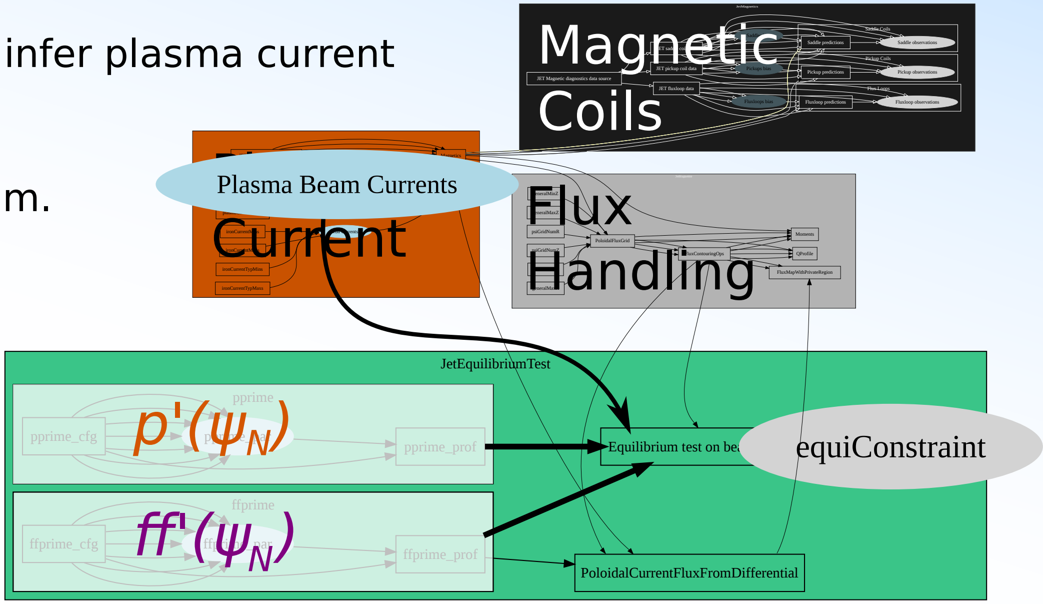
Equilbrium I

Inference of plasma current and flux surfaces P(ψ_N | ...) is the big problem.

With enough extra diagnostics, it might be possible to infer plasma current accurately, entirely from data.

For now, we can add the prior assumption of Equilibrium.
(Isotropic and no flow)

$$J_\phi = Rp' + \frac{\mu_0}{R} f f'$$



NB: It's not immediately clear how restrictive force balance (GS equation) actually is, since it is almost always used with strong prior constraints on *p*' (or *p* - the equilibirum pressure) and *ff*' (or *f* - the poloidal current flux). With weak contraints on *p*' and *ff*', the space of possible solutions is still very large.

Assume GS equality is almost correct: assign a PDF on difference:
P(*J*, *p*', *ff*') = G(*J* - *Rp*' - *ff*'/*R*;   0, σ_{GS}) with small σ_{GS}.

The posterior P(*J*, *p*', *ff*' | D_{diags} + ~Equilibrium) will include all possible combinations of *J*, *p*' and *ff*' that are consistent with the diagnostics, the priors and describe a plasma very close to equilbrium.

Adding to model (and the code) is fairly trivial, but, the problem is now very hard for the external algorithms to handle due to non-linear 1000D+ posterior.

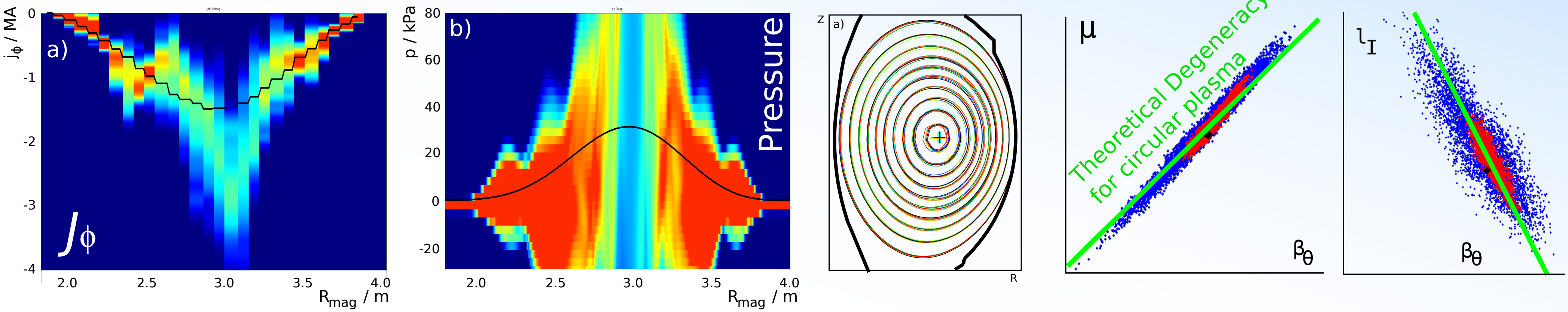
1) Parallelise the linear solver and iterate to find MAP
(slower but more stable than EFIT).

2) Exploring the PDF only just possible for simpler current profile shapes.

Equilbrium II: Posterior Exploratiand MAP estimates.

78601 High ne
H-Mode (pellets)

For simpler L-mode plasmas, we can explore the PDF, and recover the theoretically predicted degeneracy.

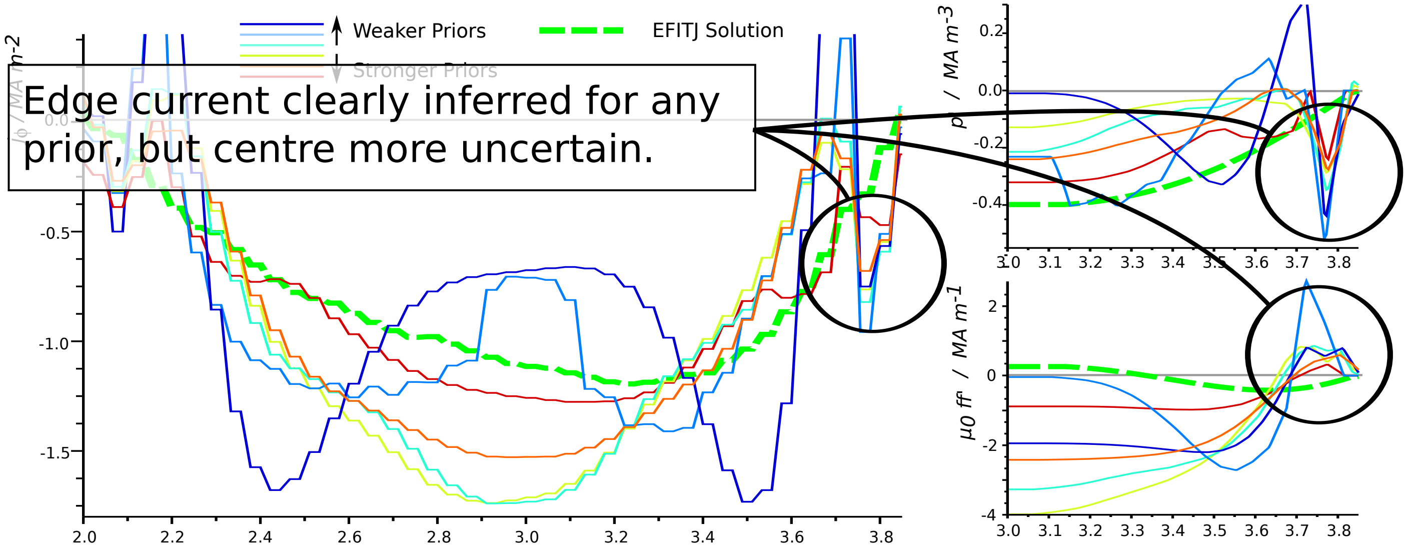


Because of modularity, we can switch parametrisation and priors of *J*, *p'* and *ff'* at will and on-the-fly.
For H-Mode:

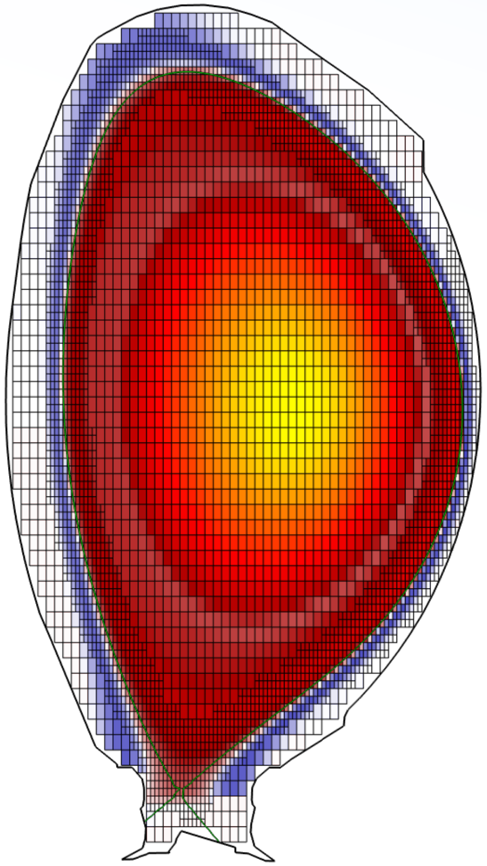
J_ϕ: Current beams with higher resolution near edge

$p'(\psi_N)$, $ff'(\psi_N)$: 20 knots, weak smoothing priors.

Too non-linear with too high-dimensionality (4732D) for current MCMC algorithms.
Study MAP with different priors:



Edge current clearly inferred for any prior, but centre more uncertain.

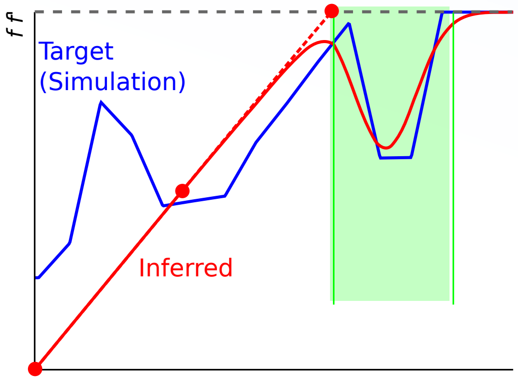


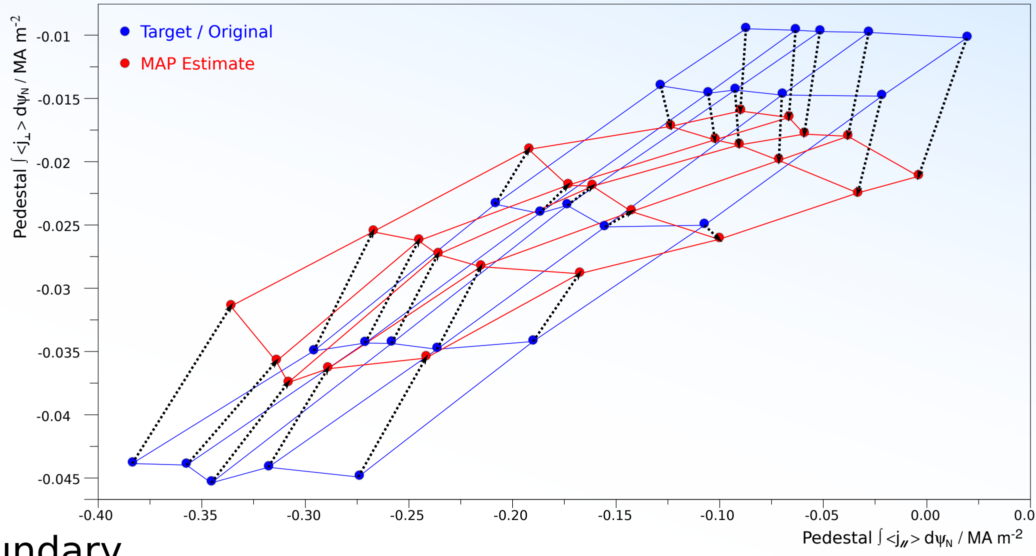
Equilbrium III: Pedestal current evolution

78601 High ne
H-Mode (pellets)

Choose a good prior (e.g. Monotonic pressure), or use stronger parameterisation (e.g. Gaussian at edge):

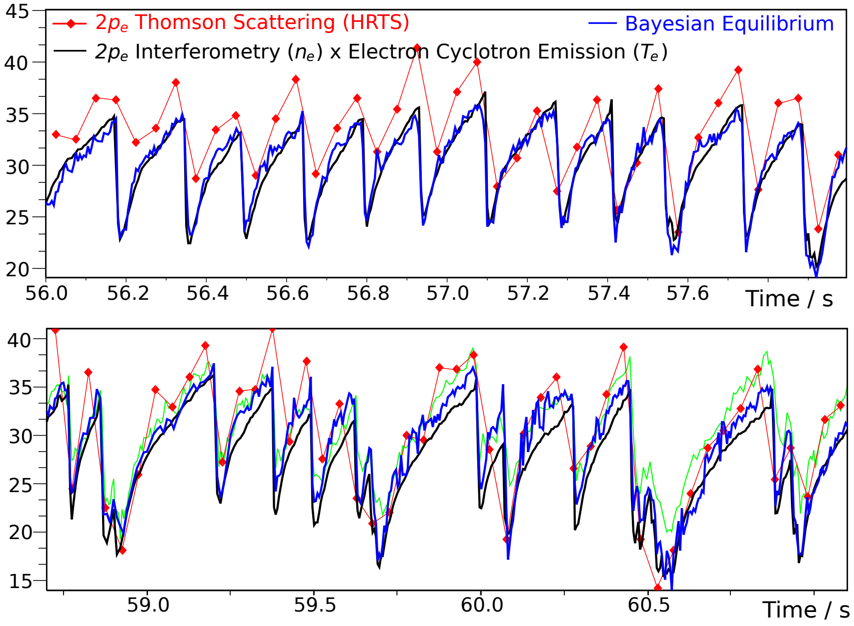
Easy to simulate data and invert to see what can be recovered:

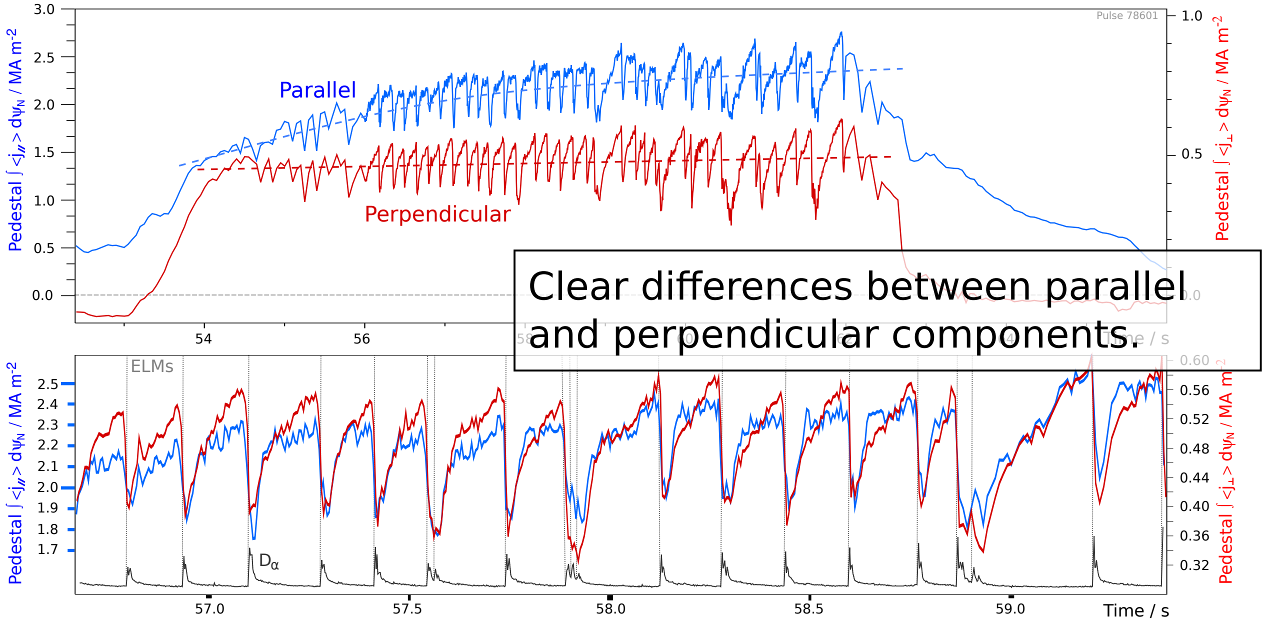




Surprising results:

- We CAN reconstruct information inside boundary.
- Can recover some information about pedestal current both the parallel AND perpendicular (i.e pressure) to the magnetic field.
- Evolution over ELM cycle follows pressure from kinetic measurements incredibly well:

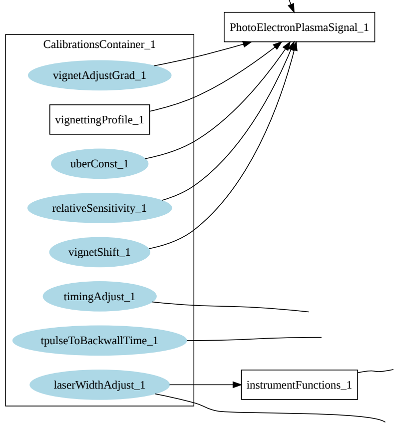




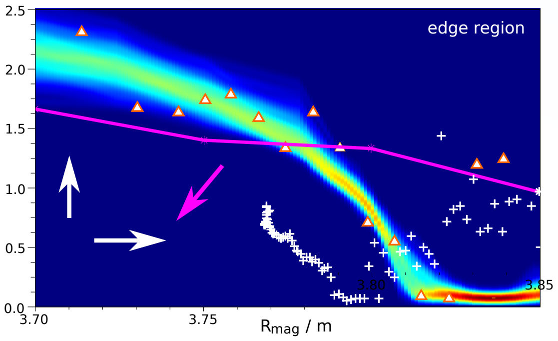
Clear differences between parallel and perpendicular components.

Summary

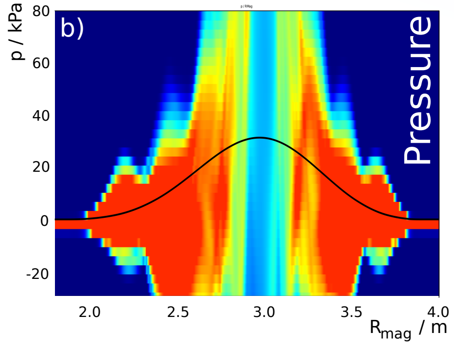
- Develop modular forward models for physics calculations and diagnostics.
- Build up a full description of each problem by connecting modular models.
- Use Bayesian Probability theory to invert data to a distribution over free parameters.



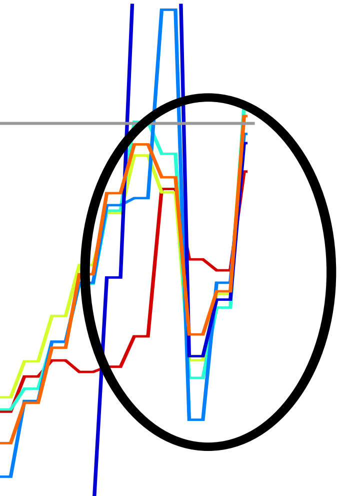
- Forward modelling allows easy handling of many calibration parameters and the complex uncertainties, they result in.



- Combining multiple diagnostics helps infer those calibration parameters from the data:
- Used to examine H-mode pedestal ne/Te evolution at very high spatial resolution.



- Use Bayesian 'posterior PDF' description to examine complex uncertainty in Tokamak equillibria without other strong prior assumptions.



- Surprising amount of detail recoverable from magnetics alone (no internal measurement) when these strong assumptions are not included.