

Bayesian Analysis Results from JET: Thomson Scattering and Equilibrium

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D. C. McDonald³, A. Meakins³, E. Solano³, JET-EFDA Collaborators*

1: Blackett Laboratory, Imperial College, London SW7 2BZ, UK

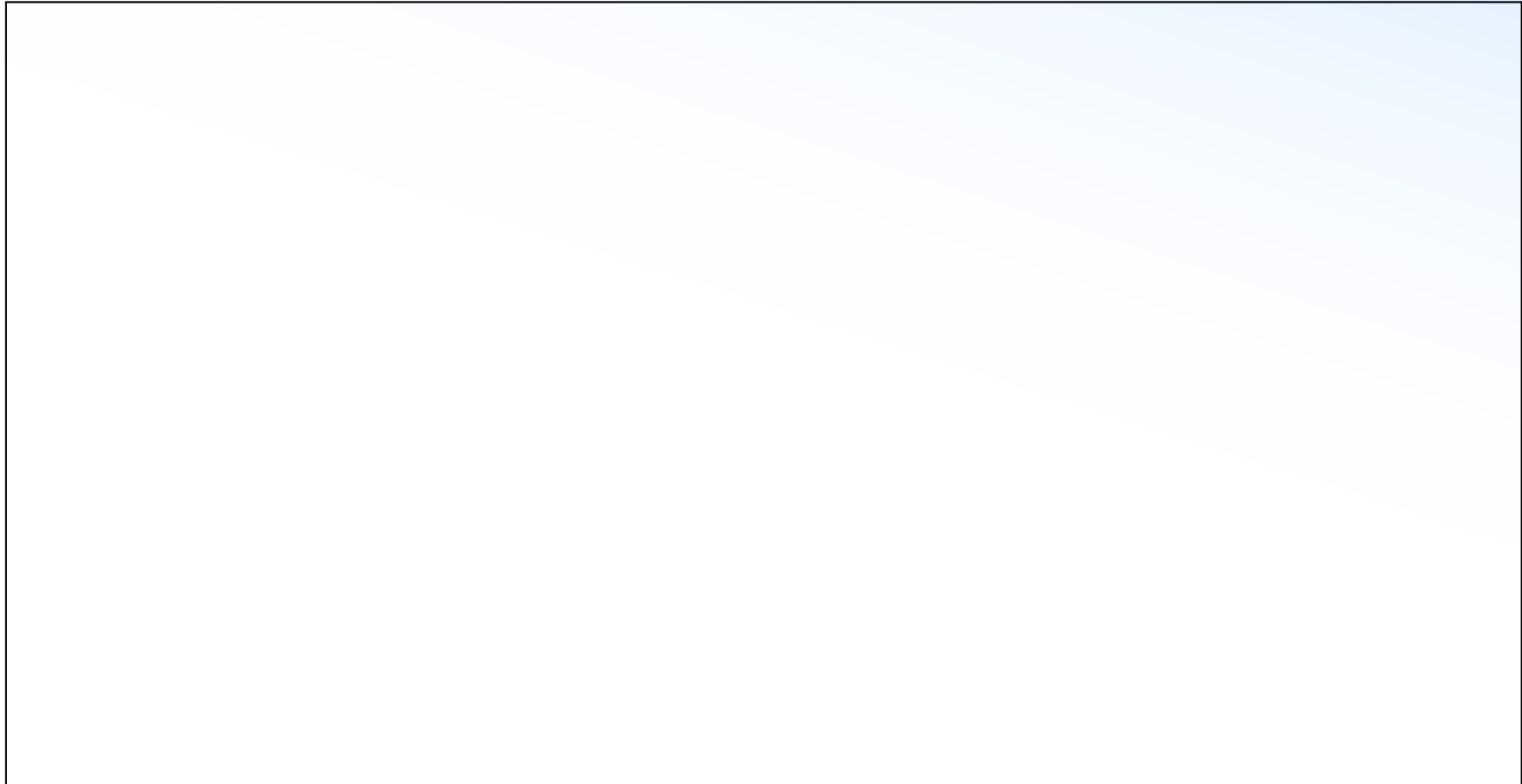
2: Max Planck Institute, Teilinstitut Greifswald, Germany

3: UKAEA Fusion Association, Culham Science Centre, OX14 3DB, UK

* See the Appendix of F. Romanelli et al., Fusion Energy Conference 2008 (Proc. 22nd Int. FEC Geneva) IAEA

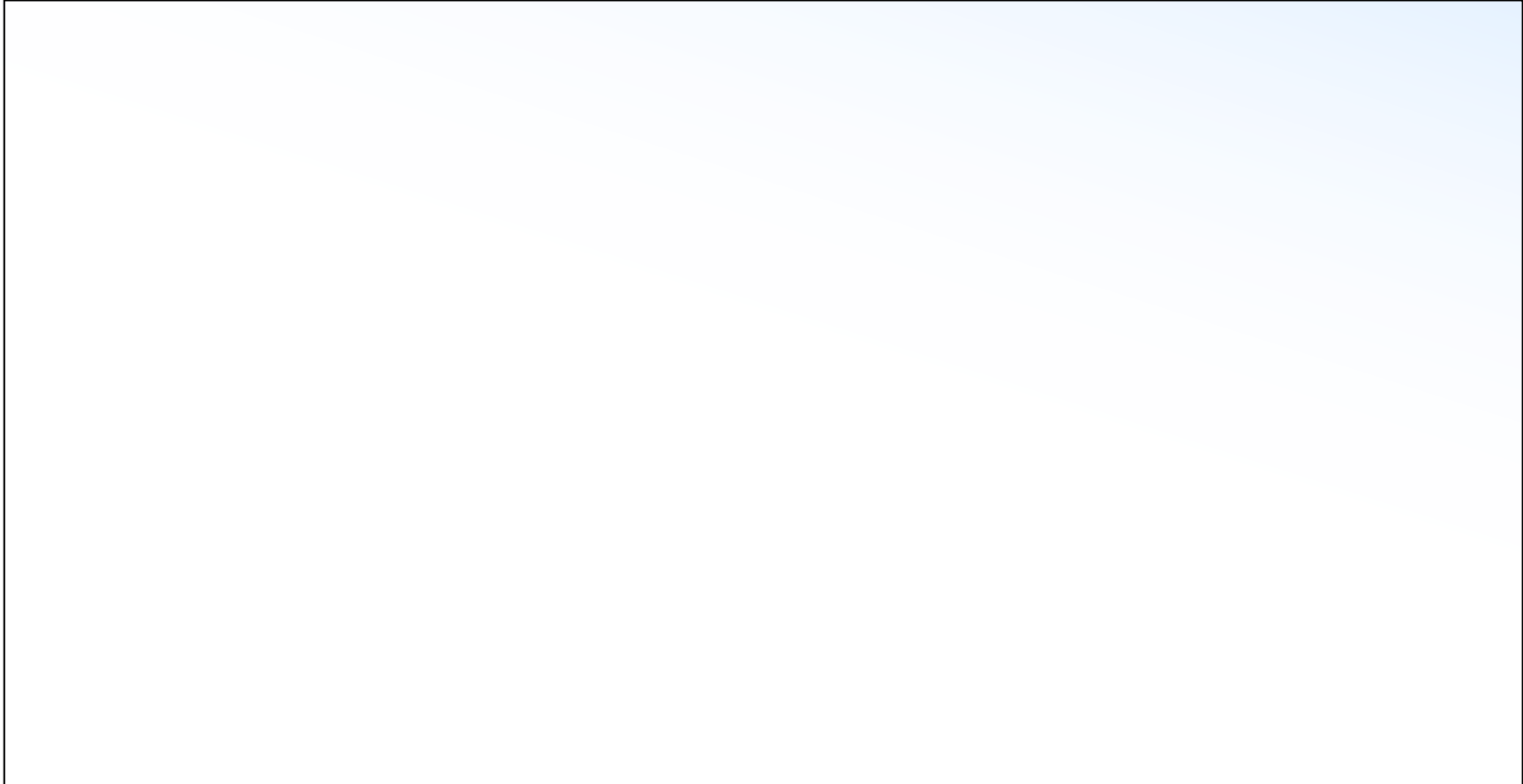
Forward Modelling and Bayesian Inference

The basic idea:



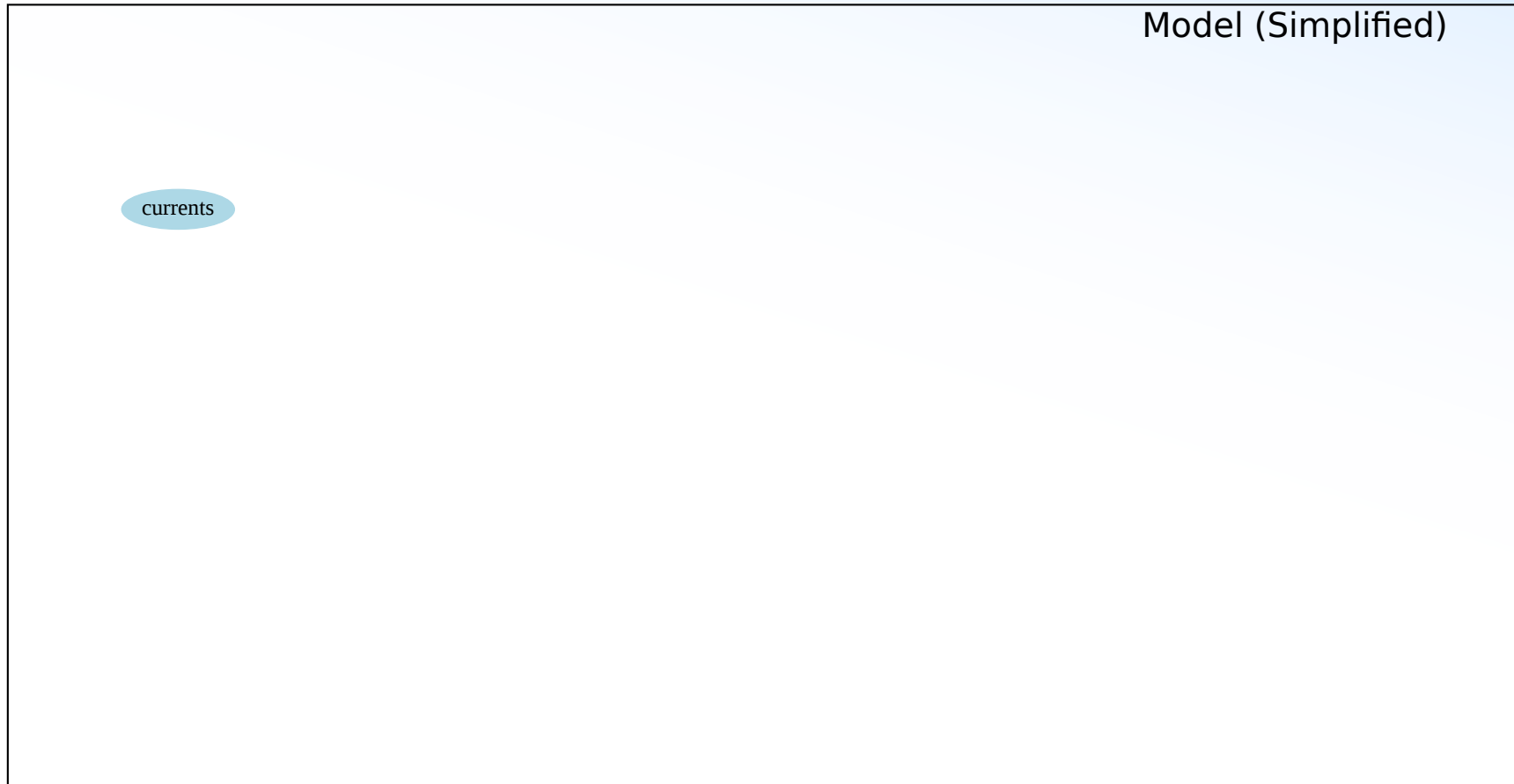
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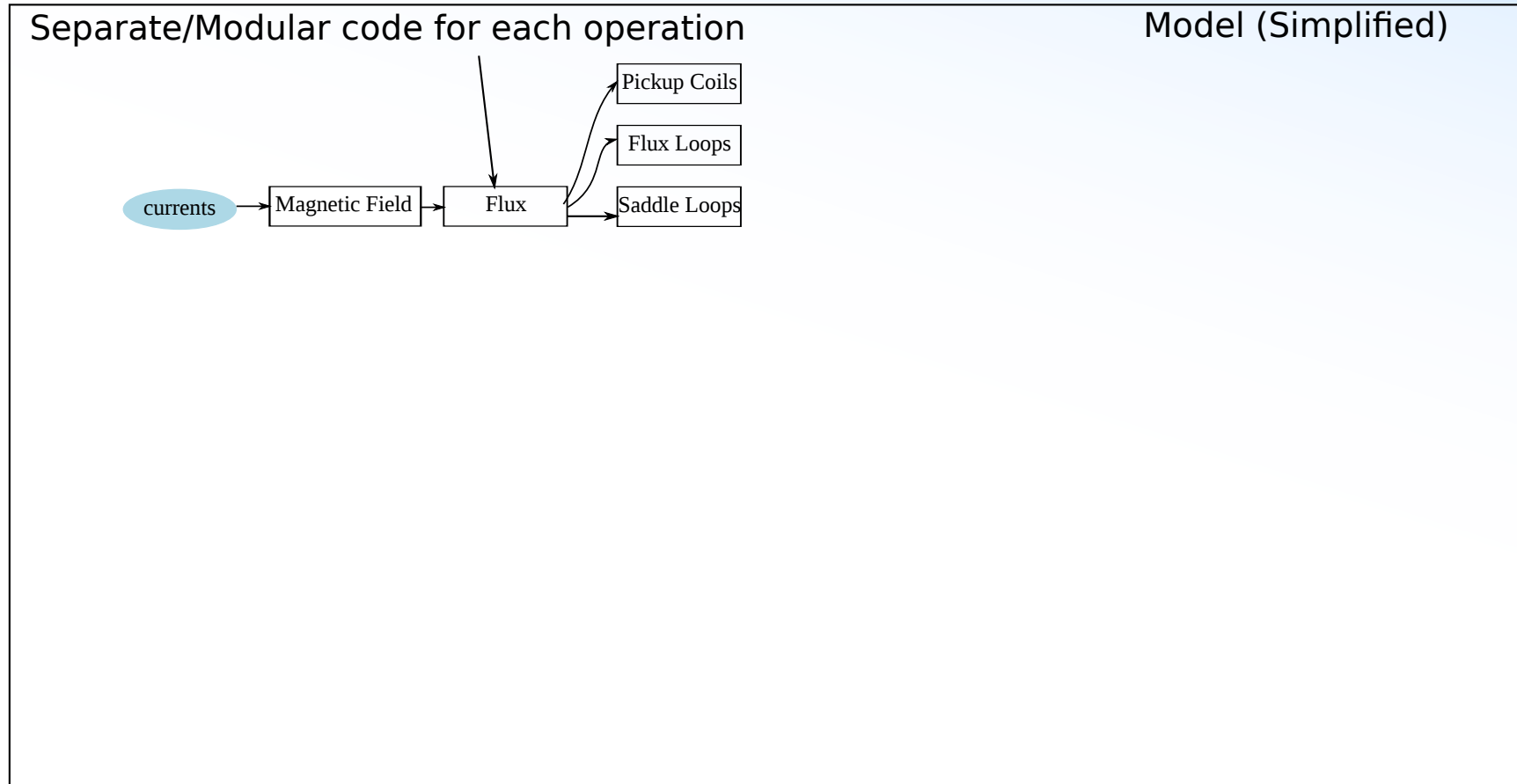
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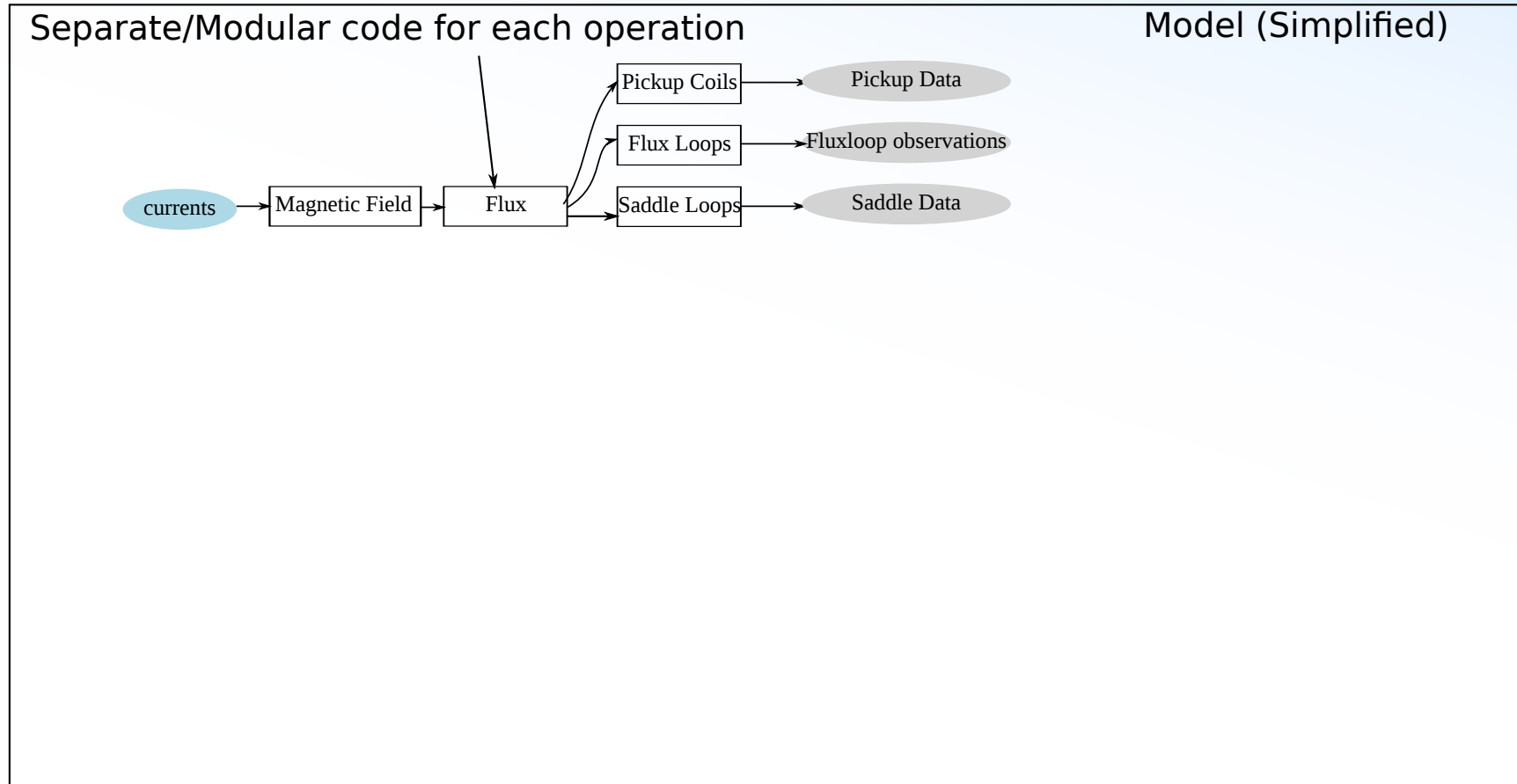
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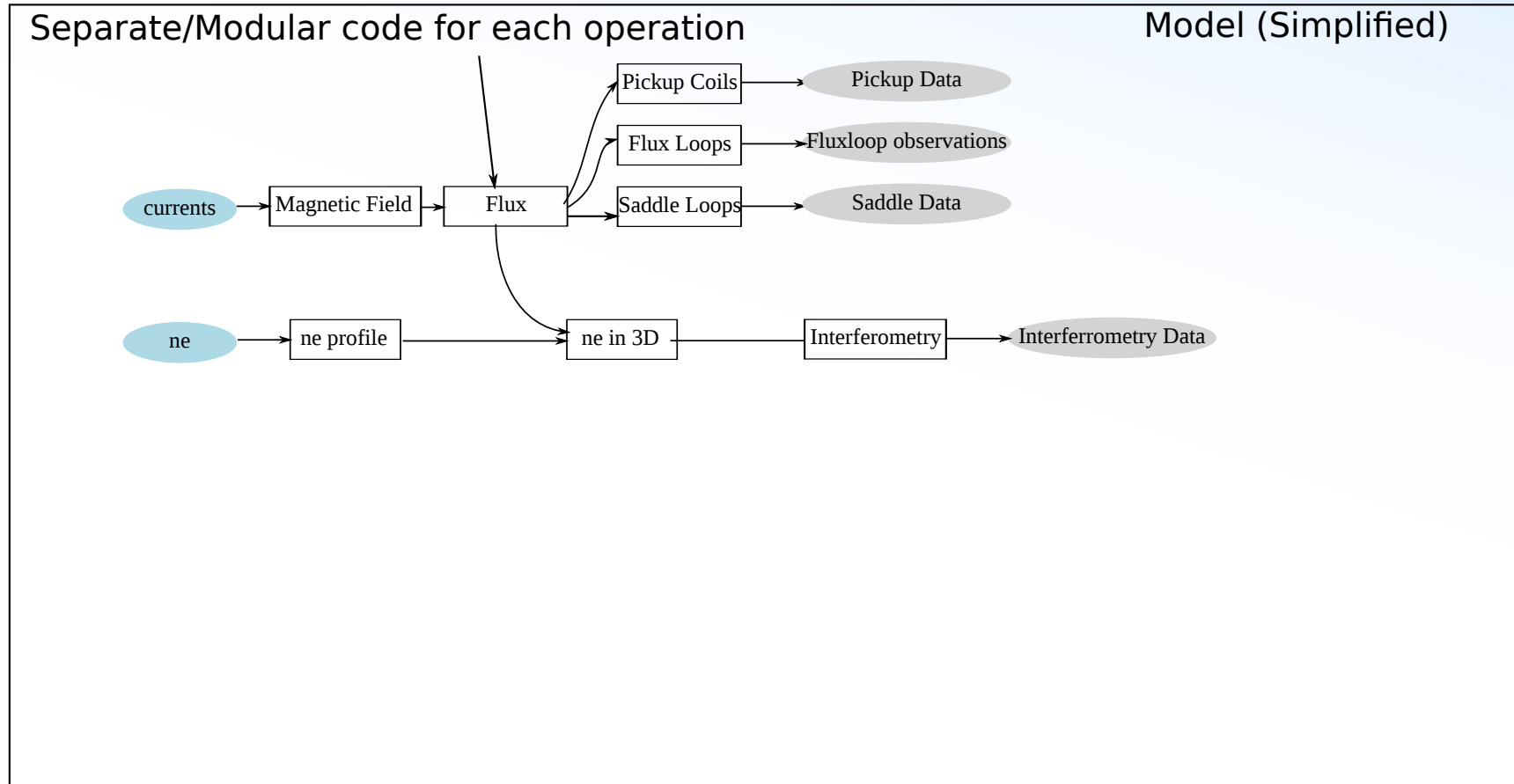
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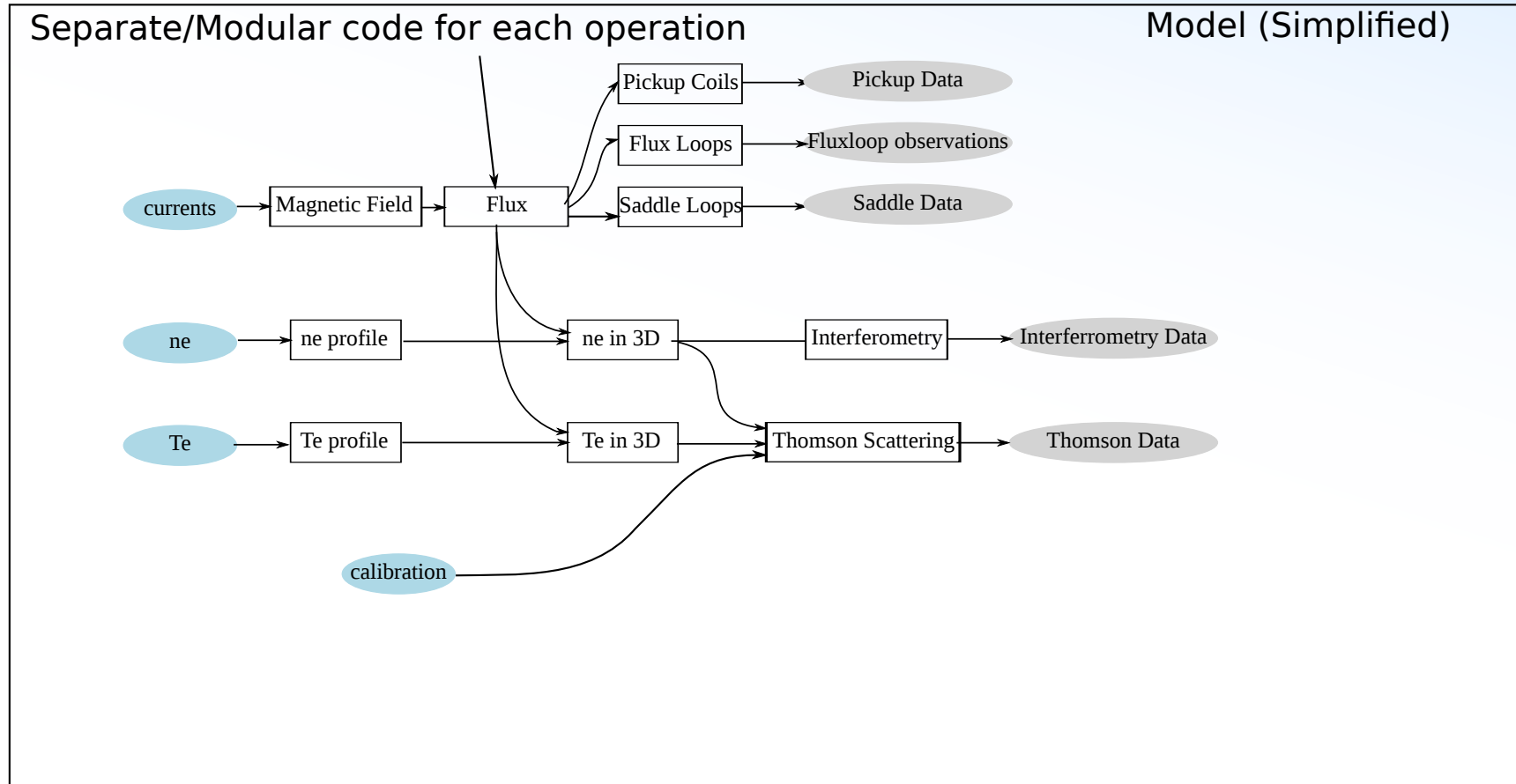
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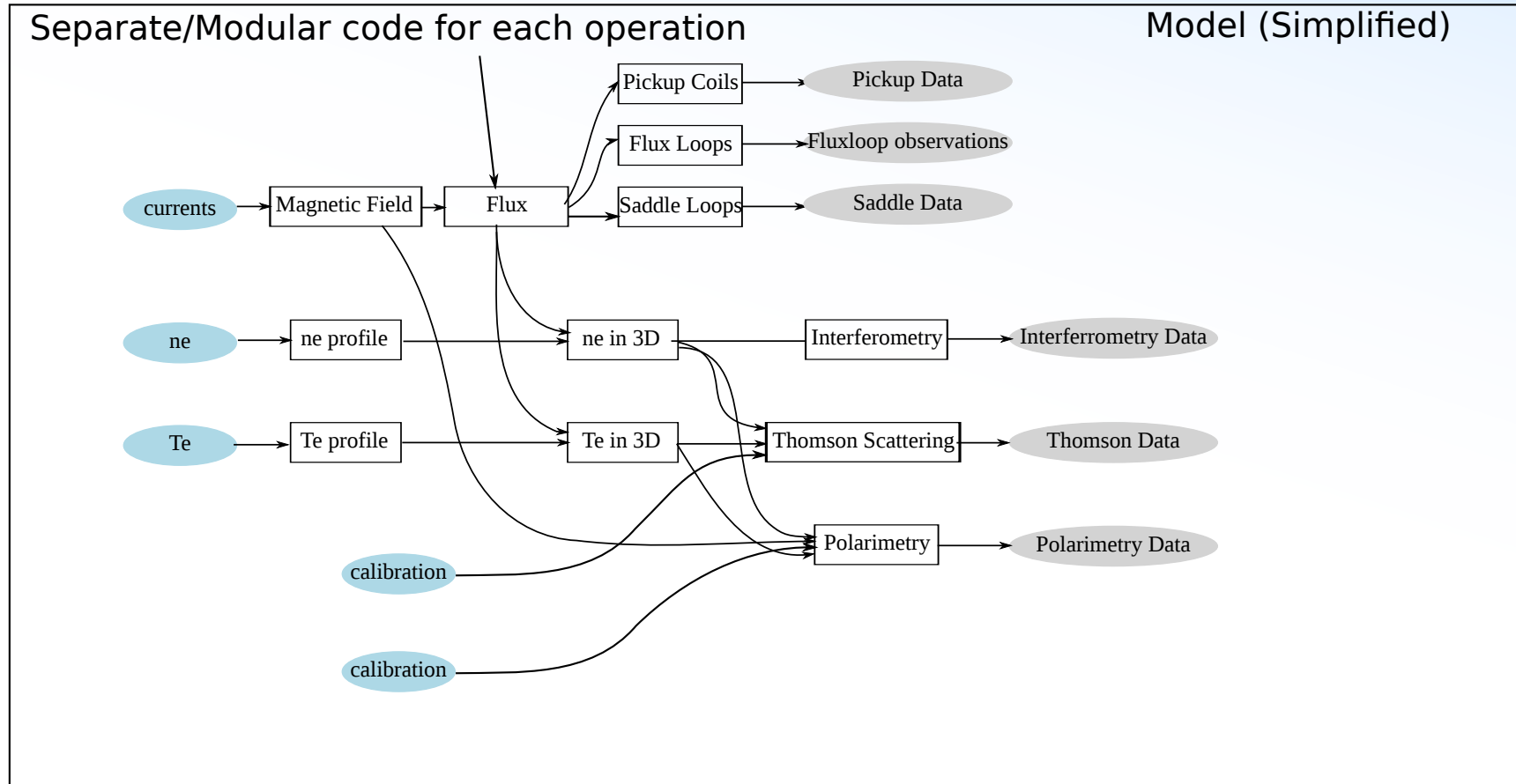
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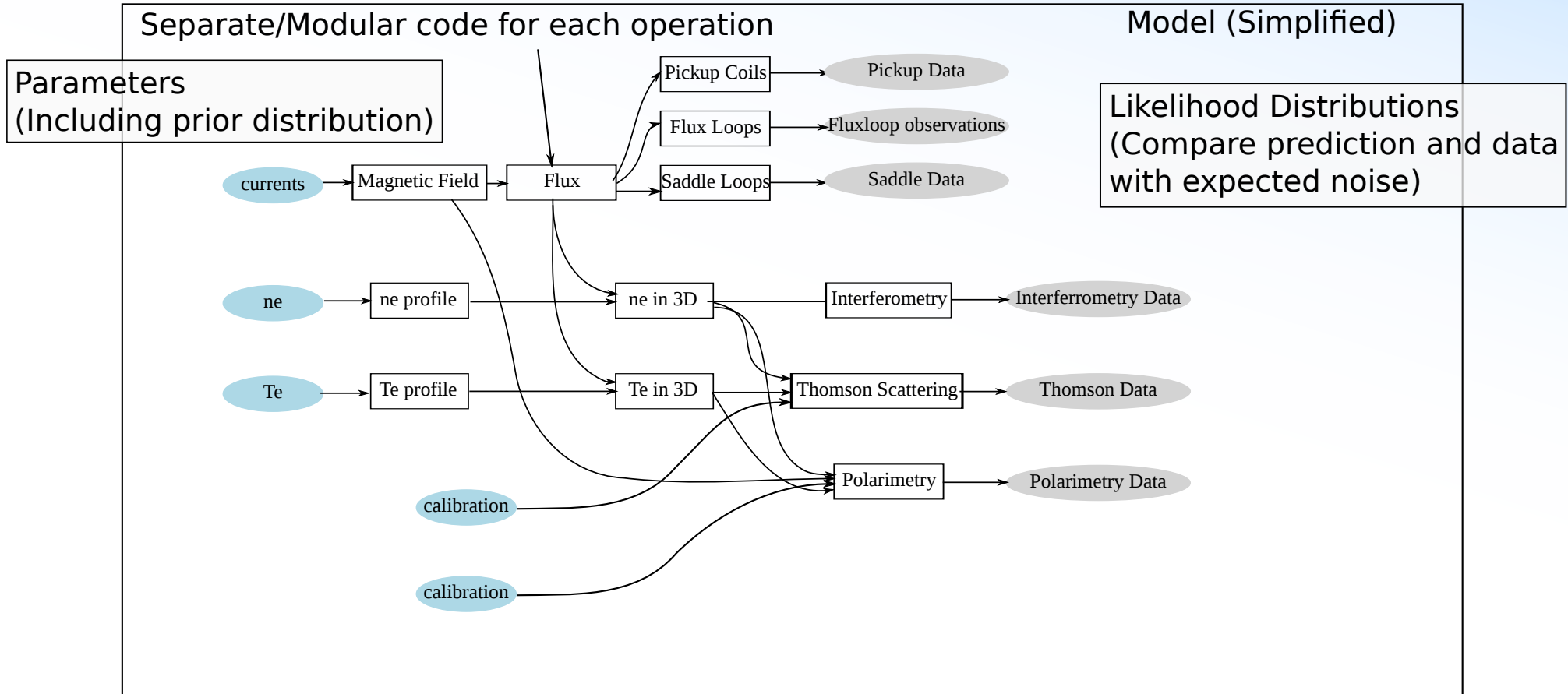
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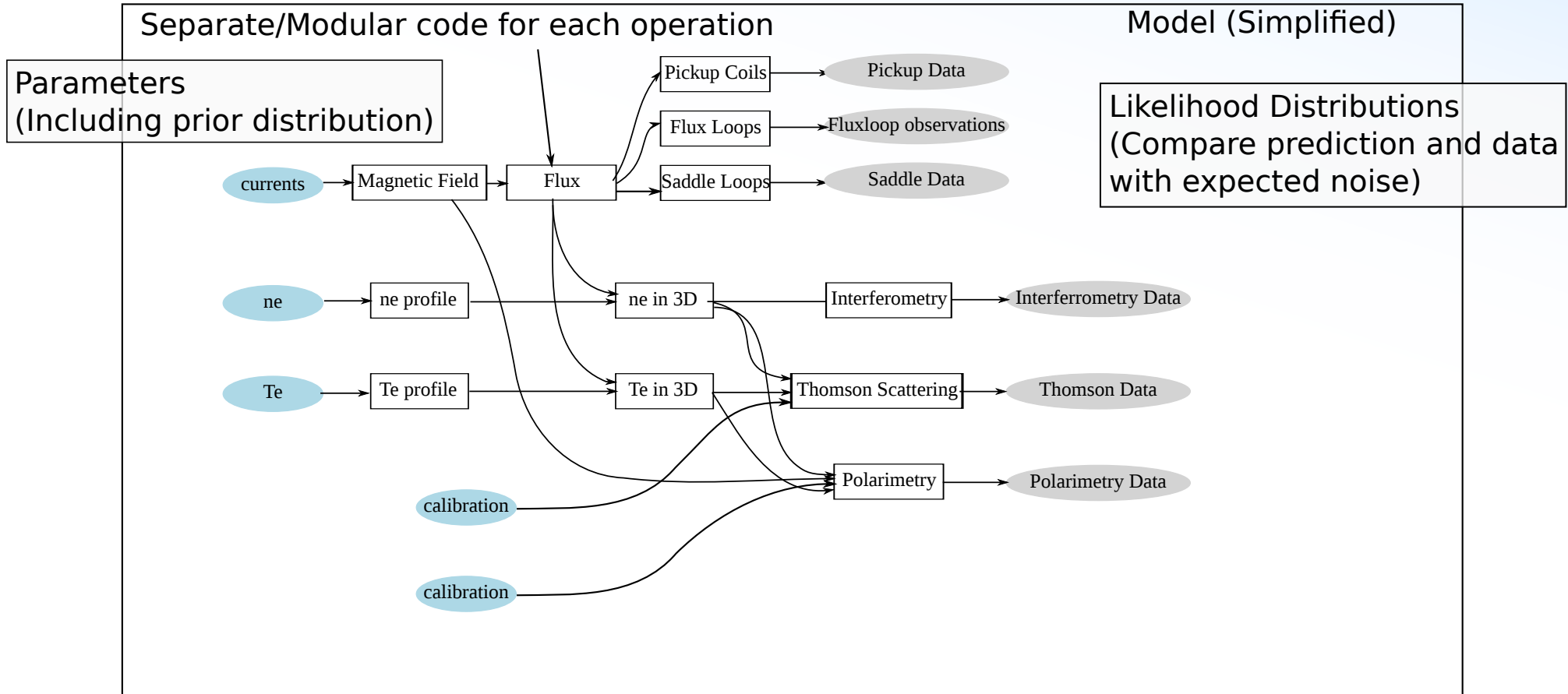
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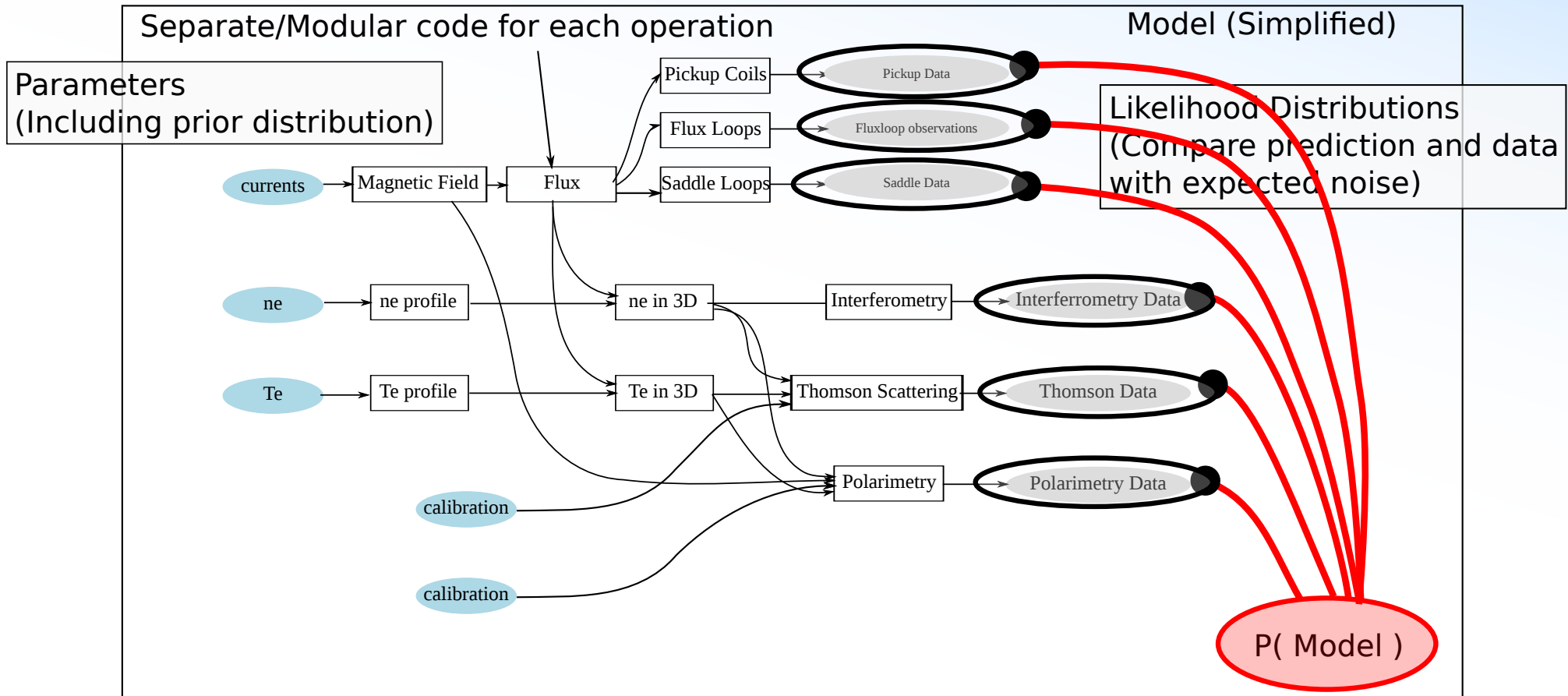


Bayes Theorem:

$$P(Te, Ne, J | Data) \sim P(D | Ne, Te, J) P(Te, Ne, J)$$

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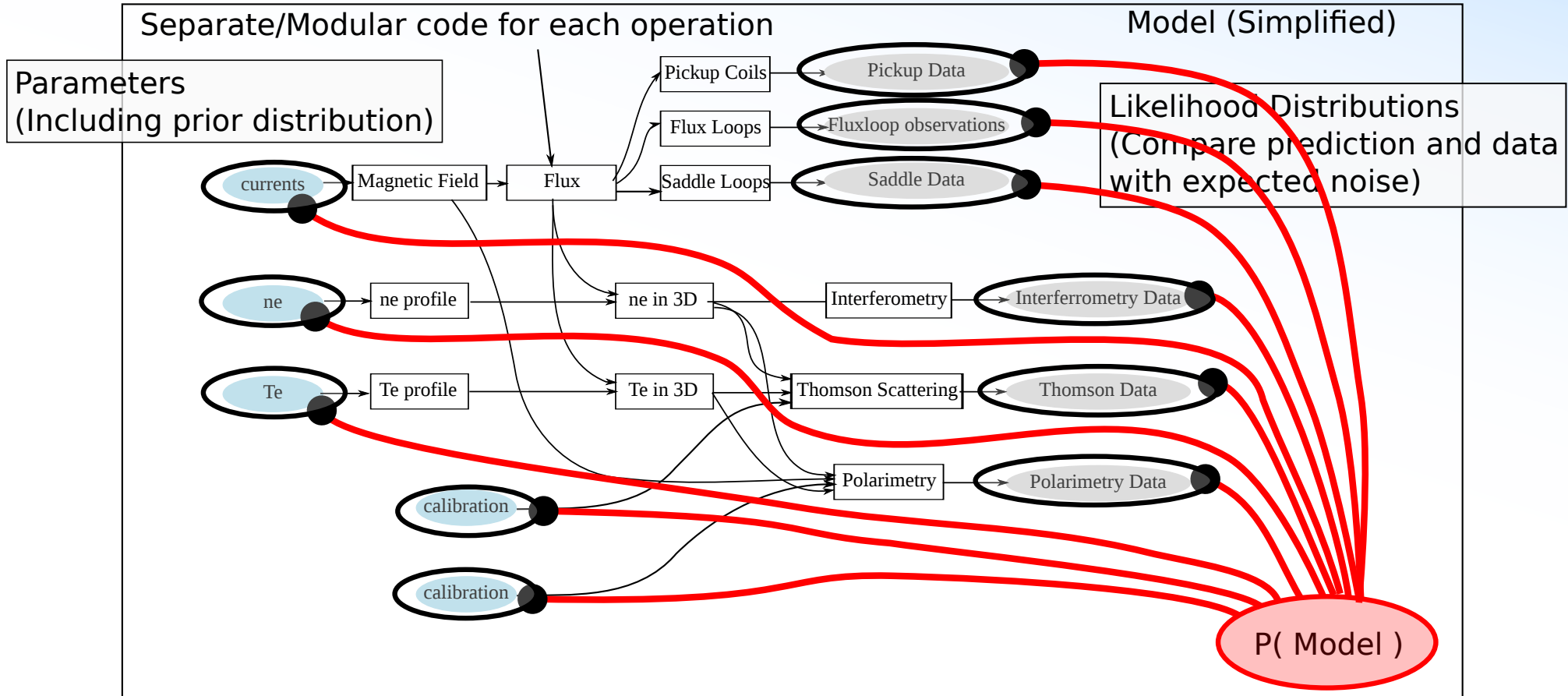


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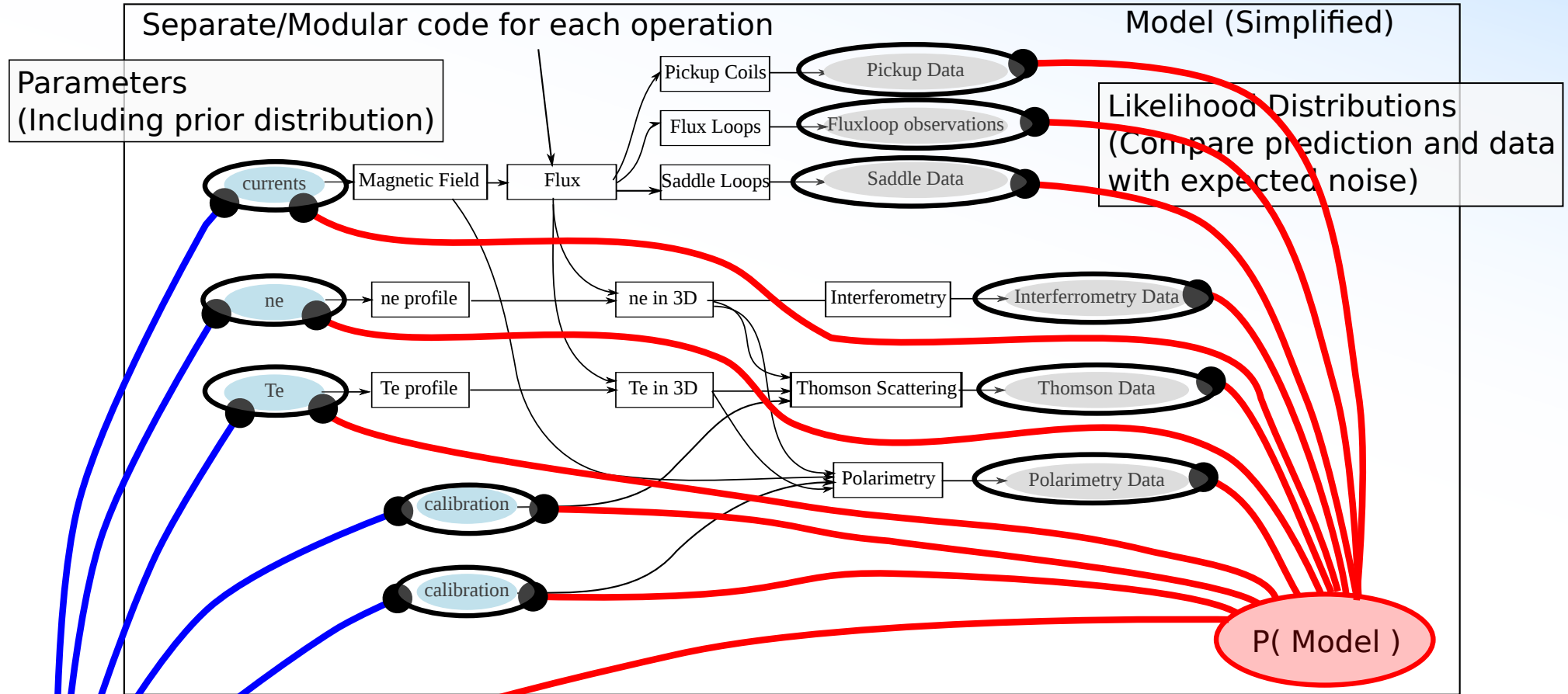


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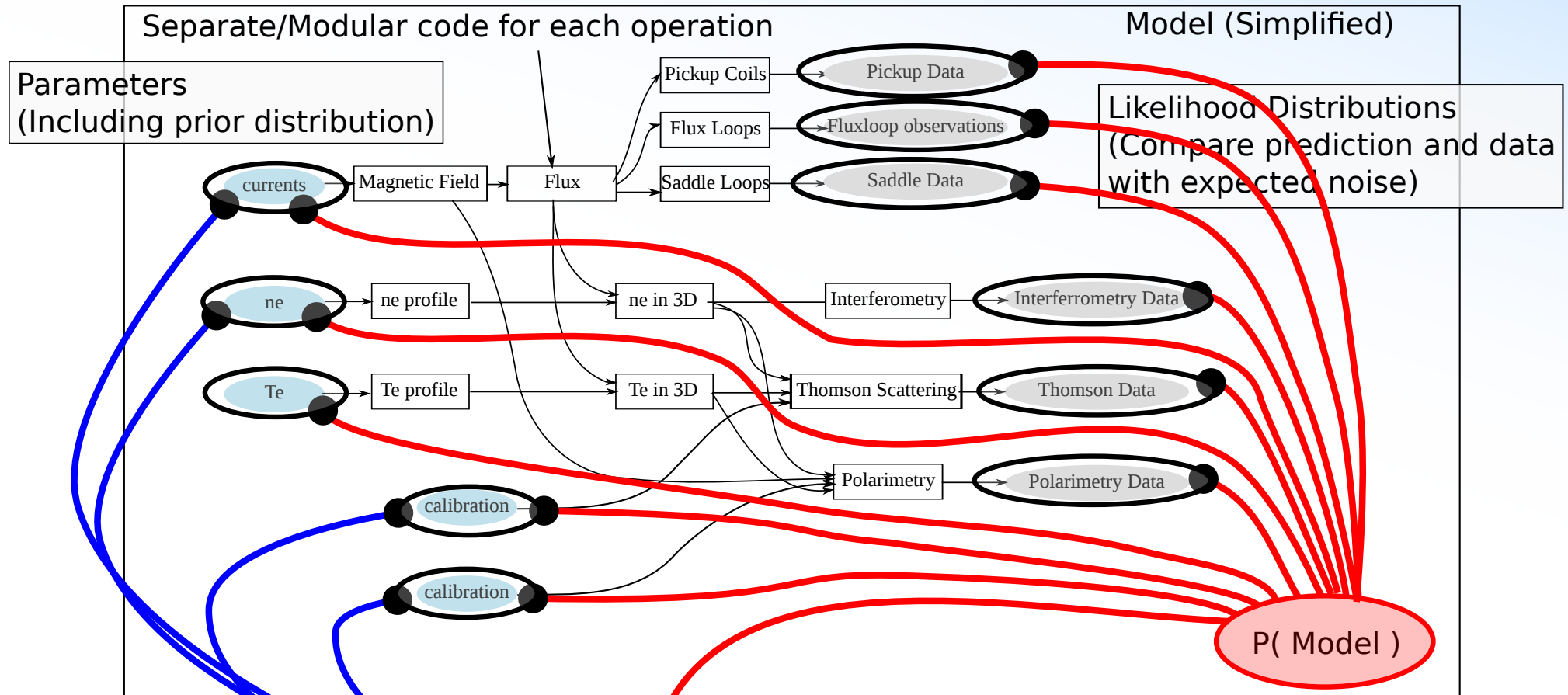
$$P(\text{Te, Ne, J} | \text{Data}) \sim \frac{P(\text{D} | \text{Ne, Te, J}) P(\text{Te, Ne, J})}{P(\text{Data})}$$

Practically: Solve and explore using external algorithms:

Linear Gaussian Solver
(Best fit and PDF covariance)

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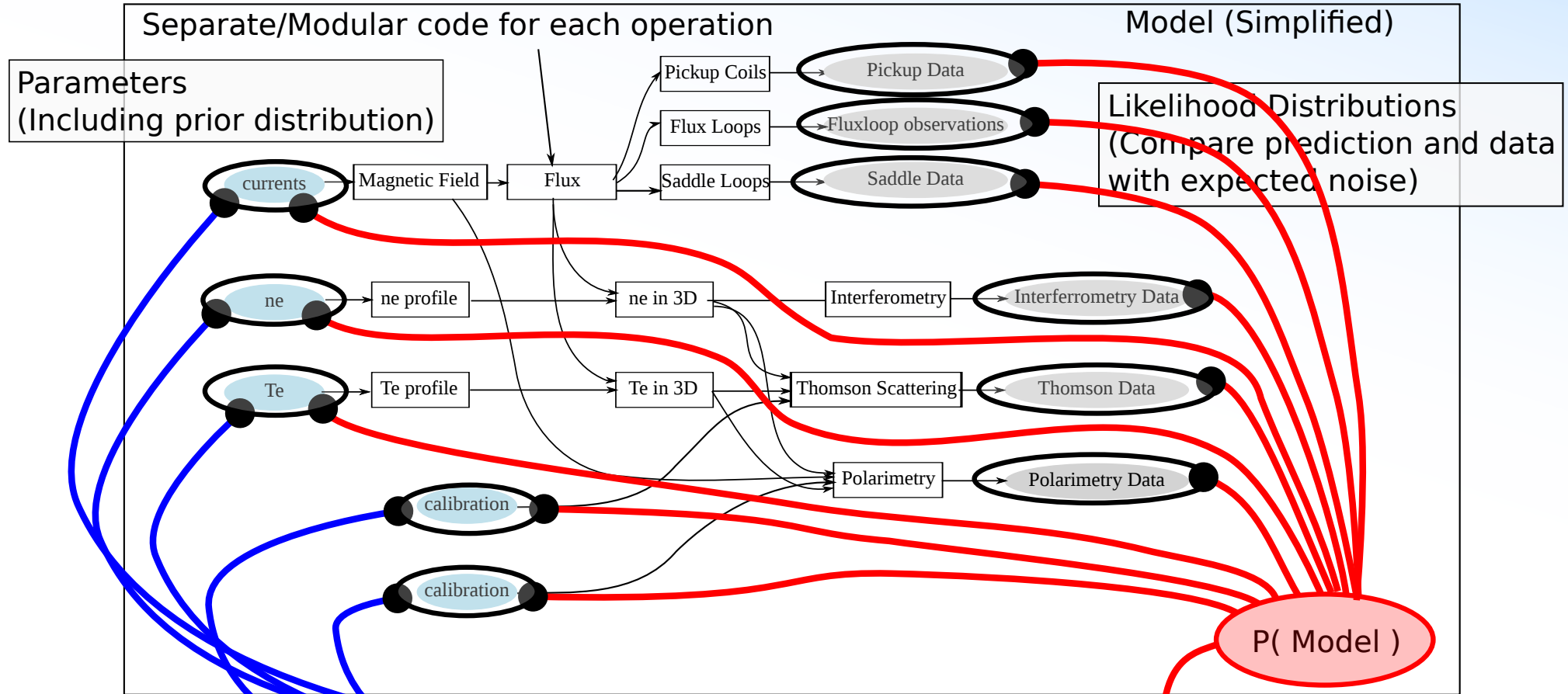
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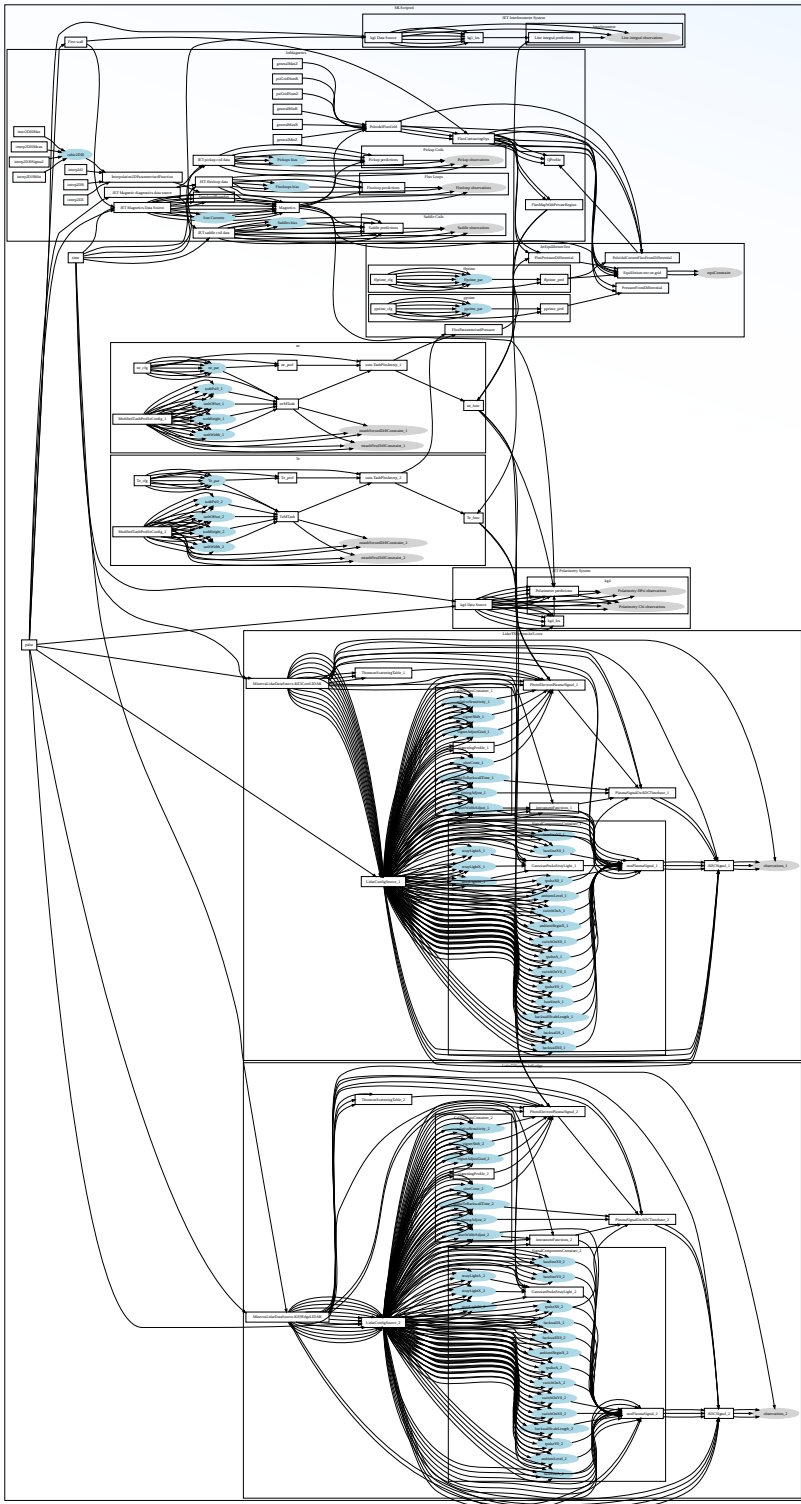
Metropolis Hastings
MCMC Non-linear Exploration:
--> Uncertainty

Software and Models

Write nodes and wire them together.
Software framework handles the rest.

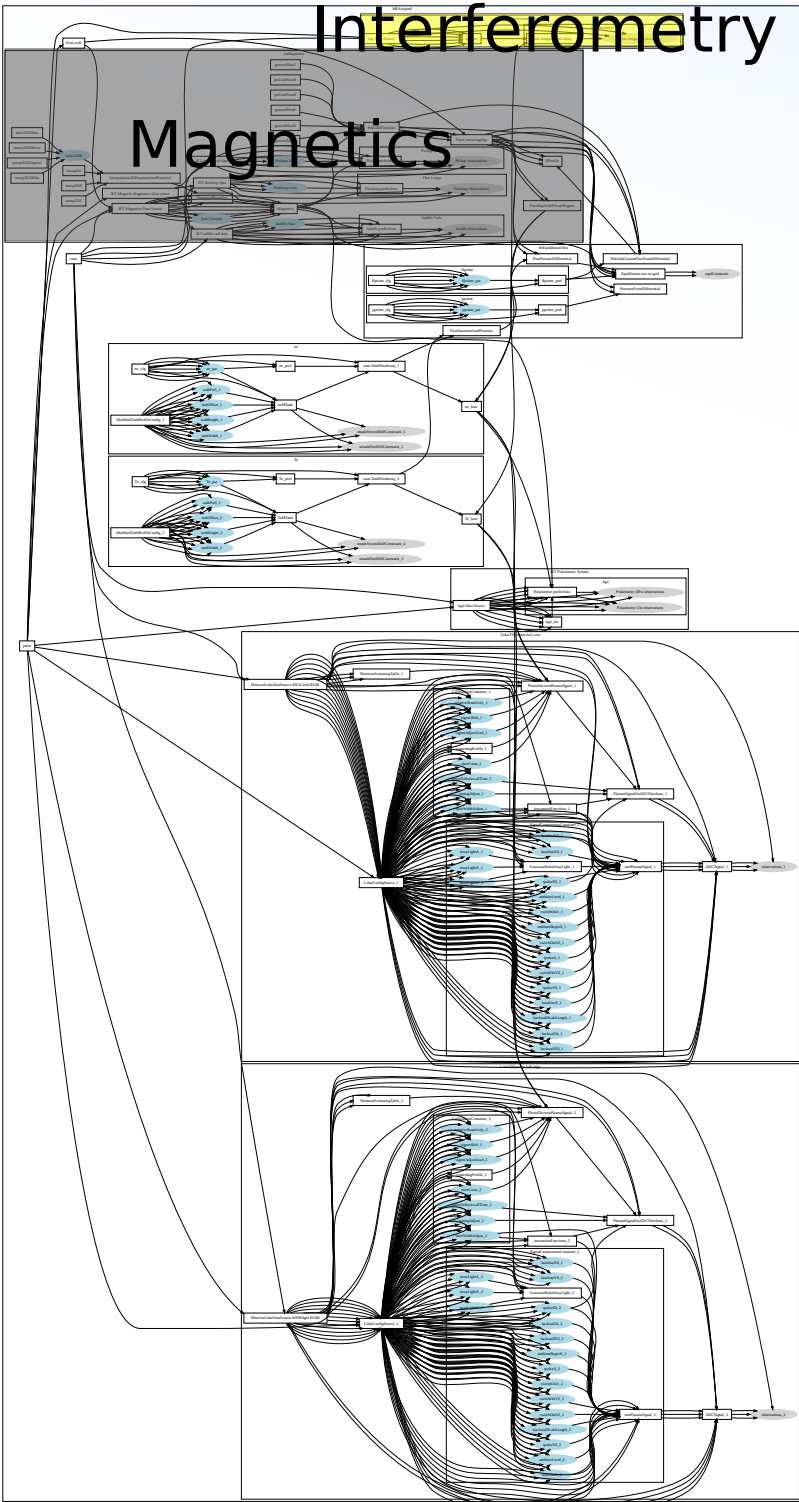
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Interferometry

Magnetics



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We can re-wire the graph and redefine/modify the problem
at will, even during a run.

Parts previously written:

- Magnetics (field/flux calculations and JET magnetics)
- Interferometry.

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Equilibrium

Ne

Te

Polarimetry

Core LIDAR

Edge LIDAR

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- Core LIDAR
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- Equilibrium (Grad-Shafranov Test)
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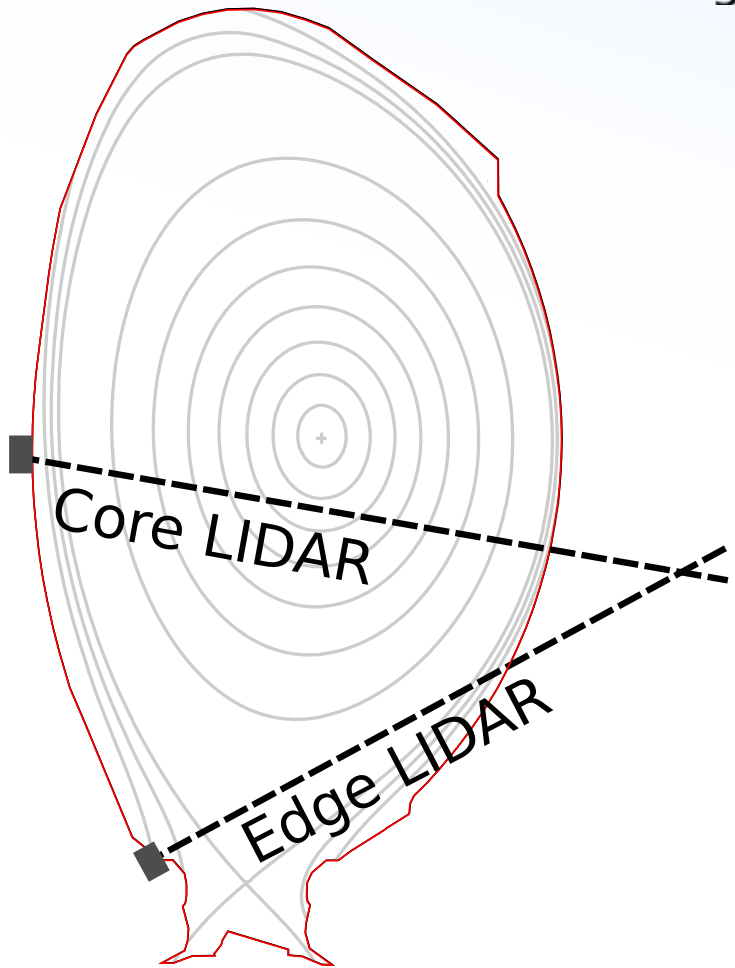
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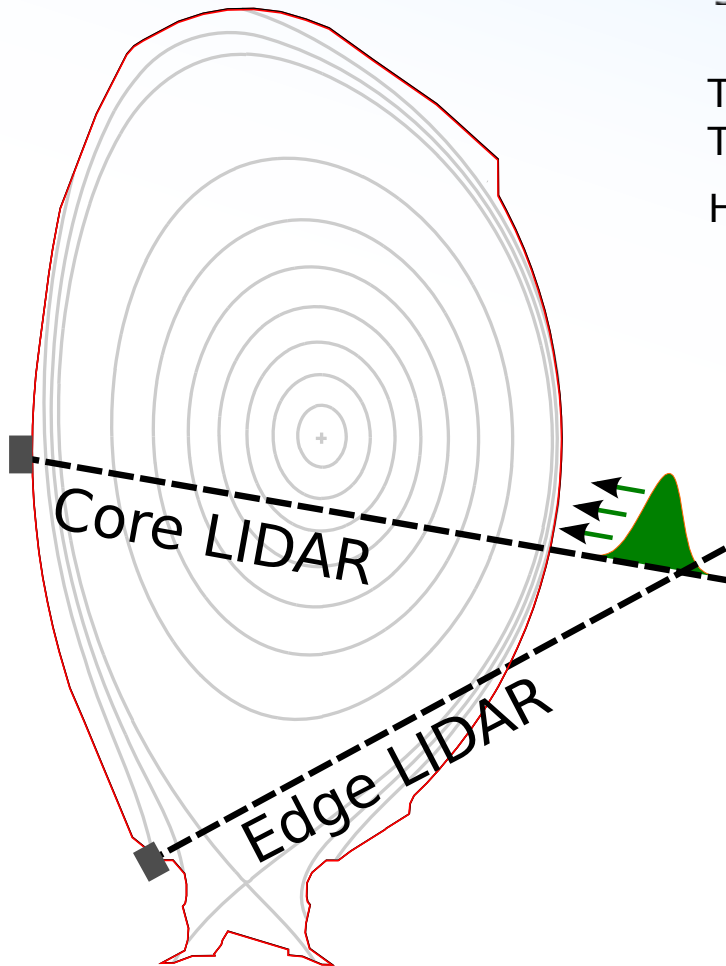
- JET MSE
- JET Reflectometry
- JET Infrared strikepoint camera
- MAST Magnetics
- MAST MSE
- MAST Thomson Scattering
- ... and a few others ...

Core + Edge LIDAR: The systems and the problem

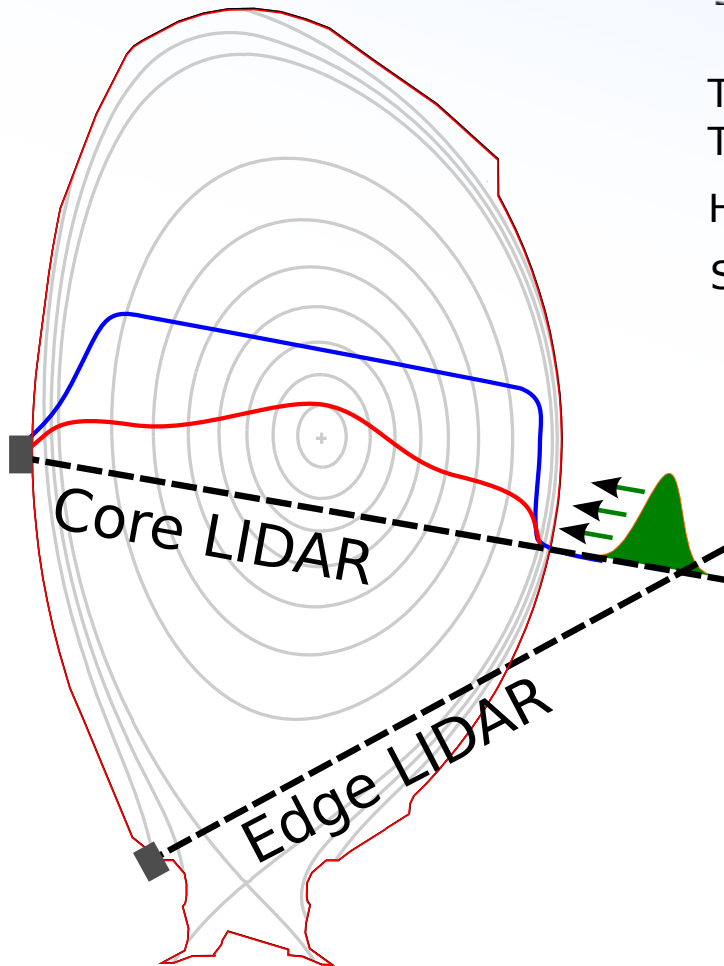


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Thomson Scattering diagnostics with single spectrometer.
Time of flight for positioning.
Hardware system very complex.



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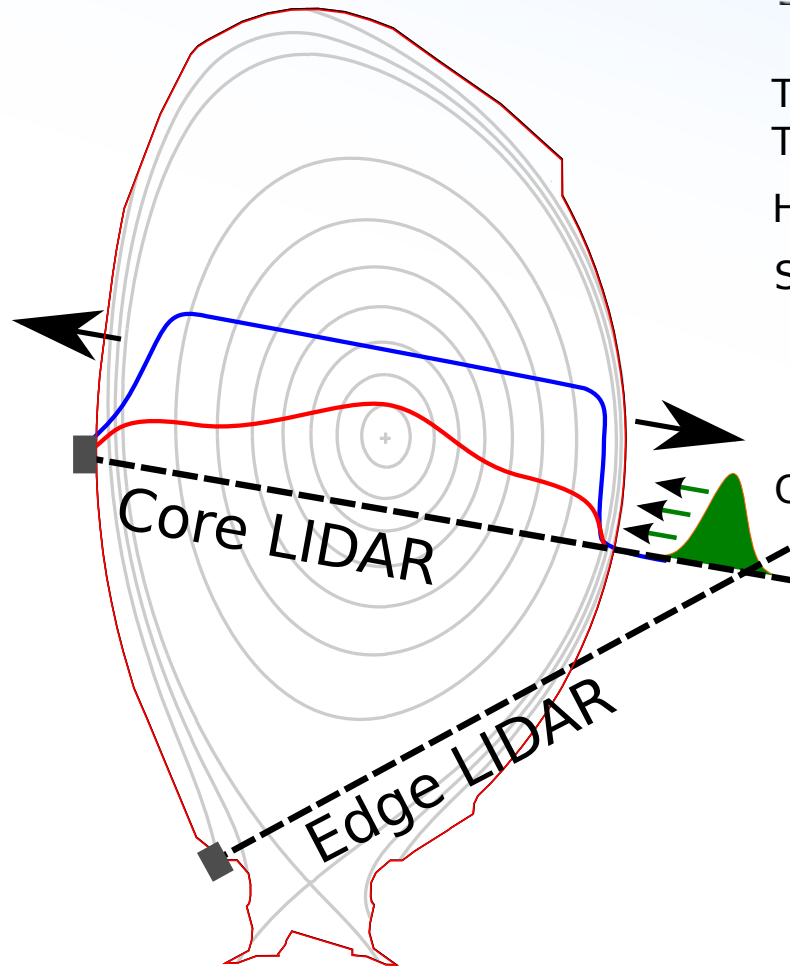
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Effective convolution of light signal.

If ignored: Convolves n_e but complex effect on T_e .

No problem for forward modelling: we just convolve the signal.

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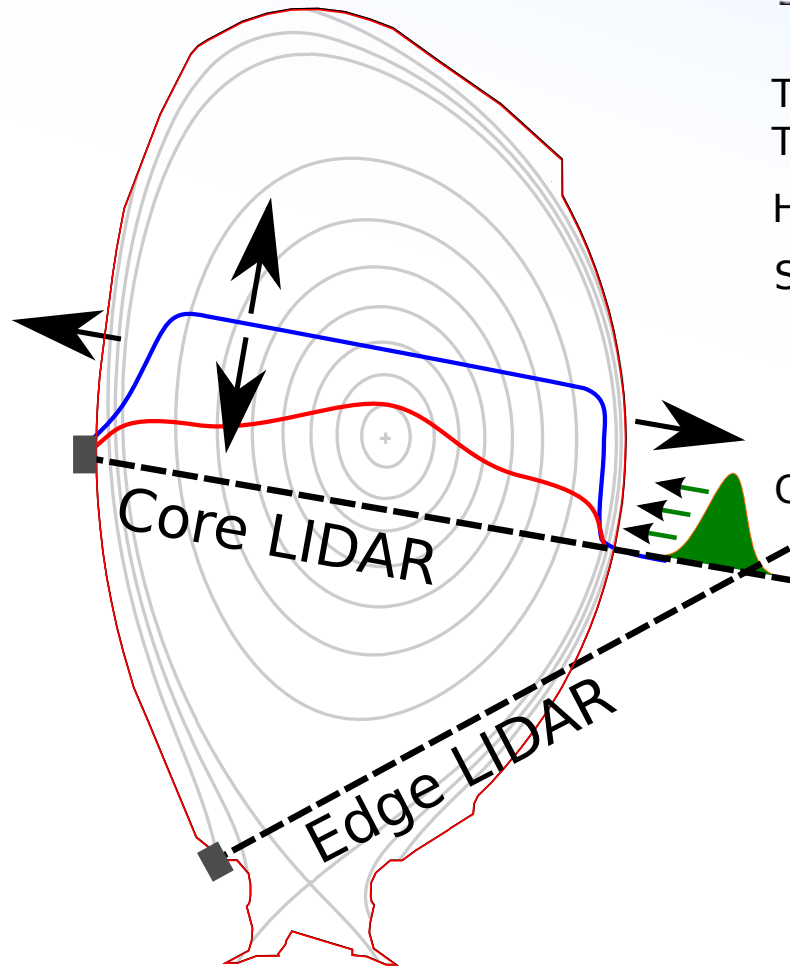
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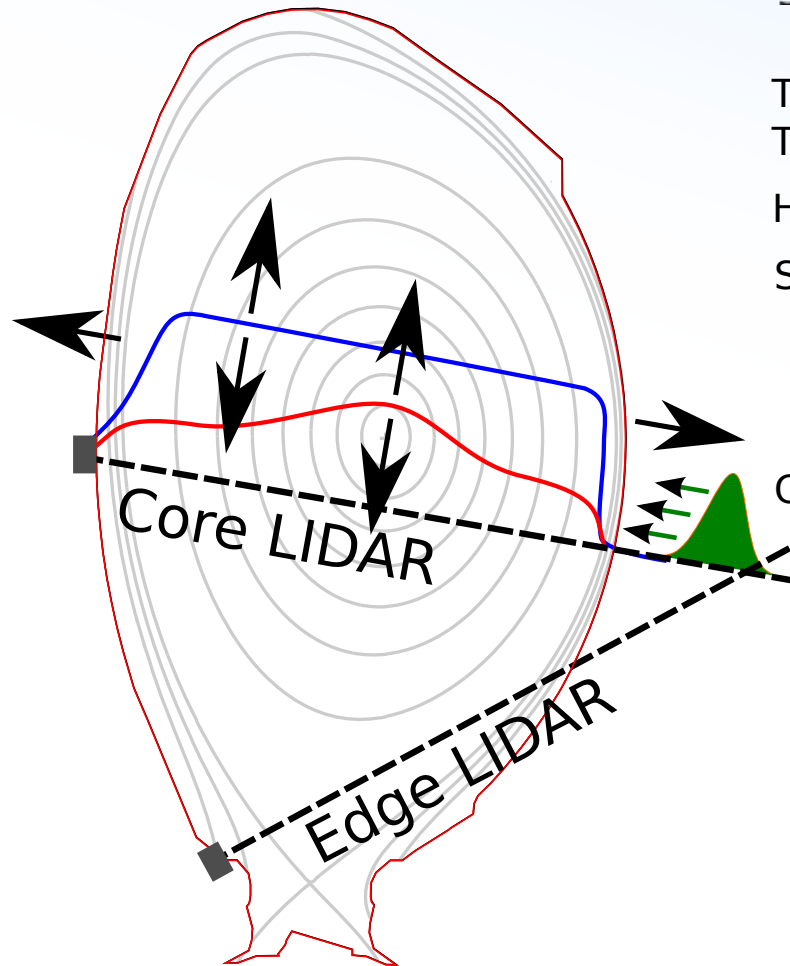
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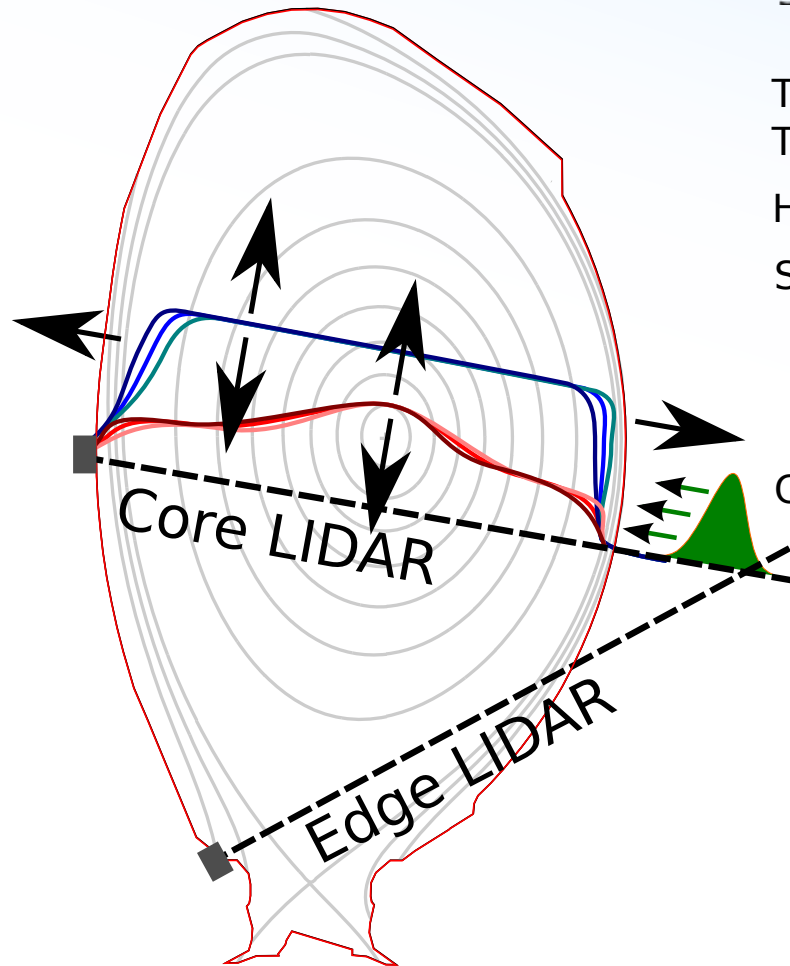
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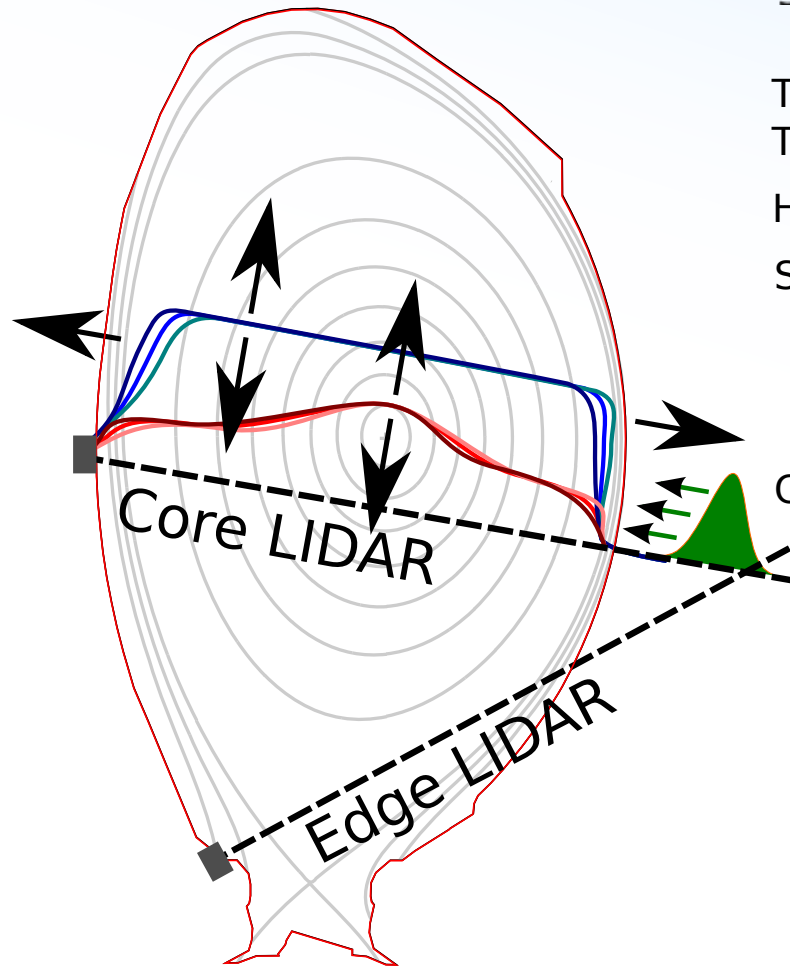
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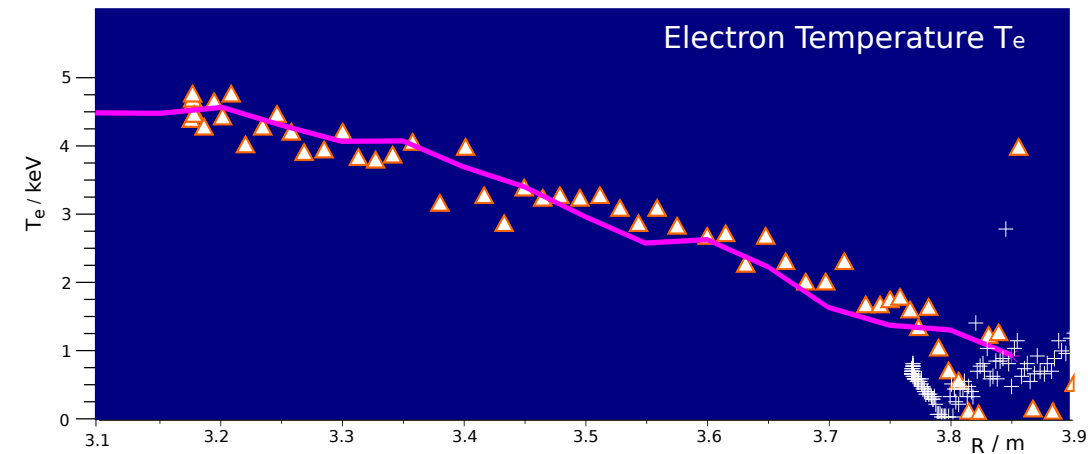
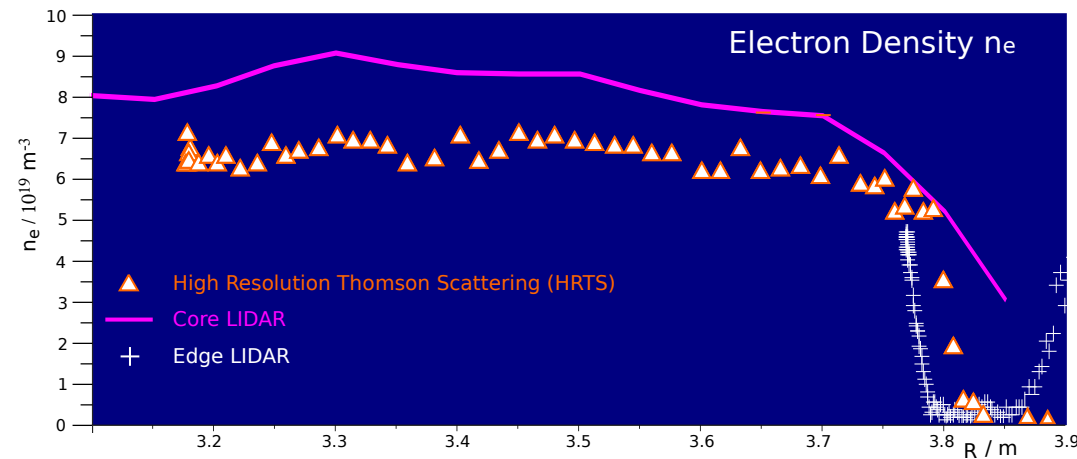
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Shift and scale output profiles to match?

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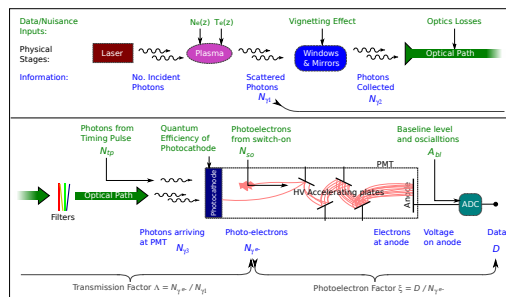
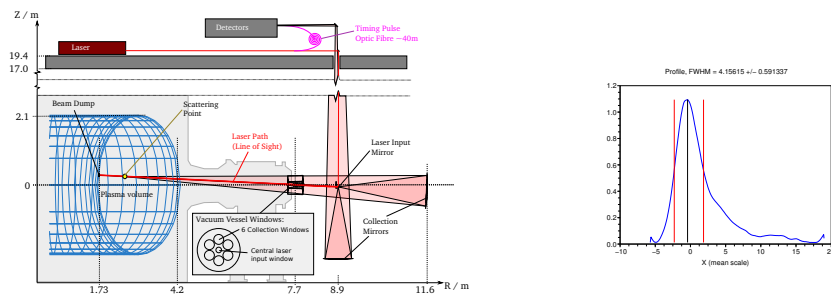
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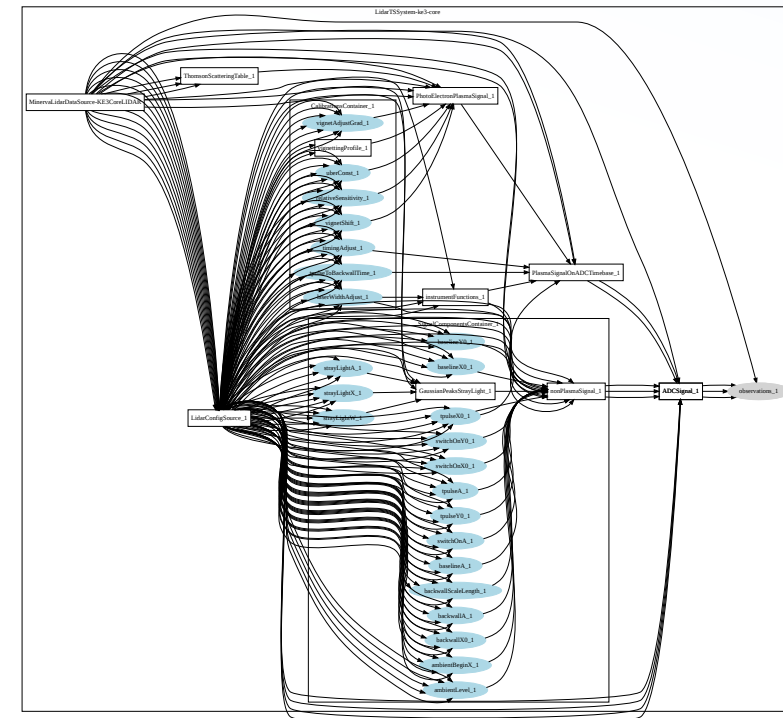
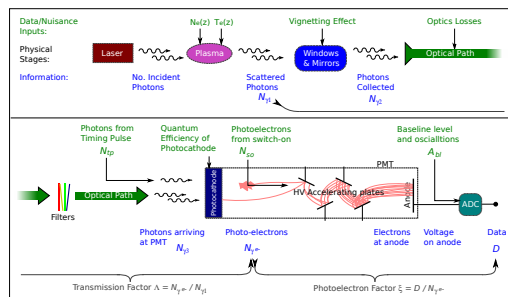
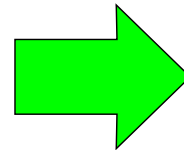
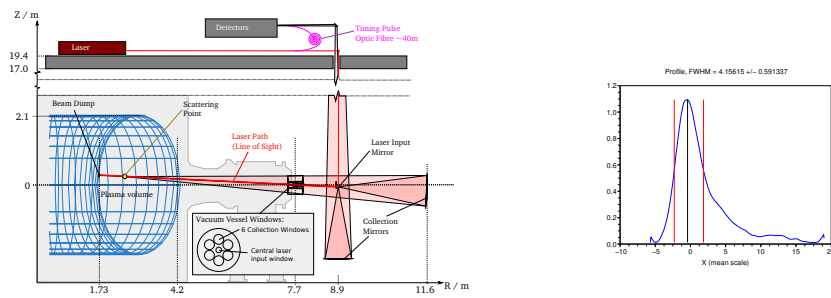
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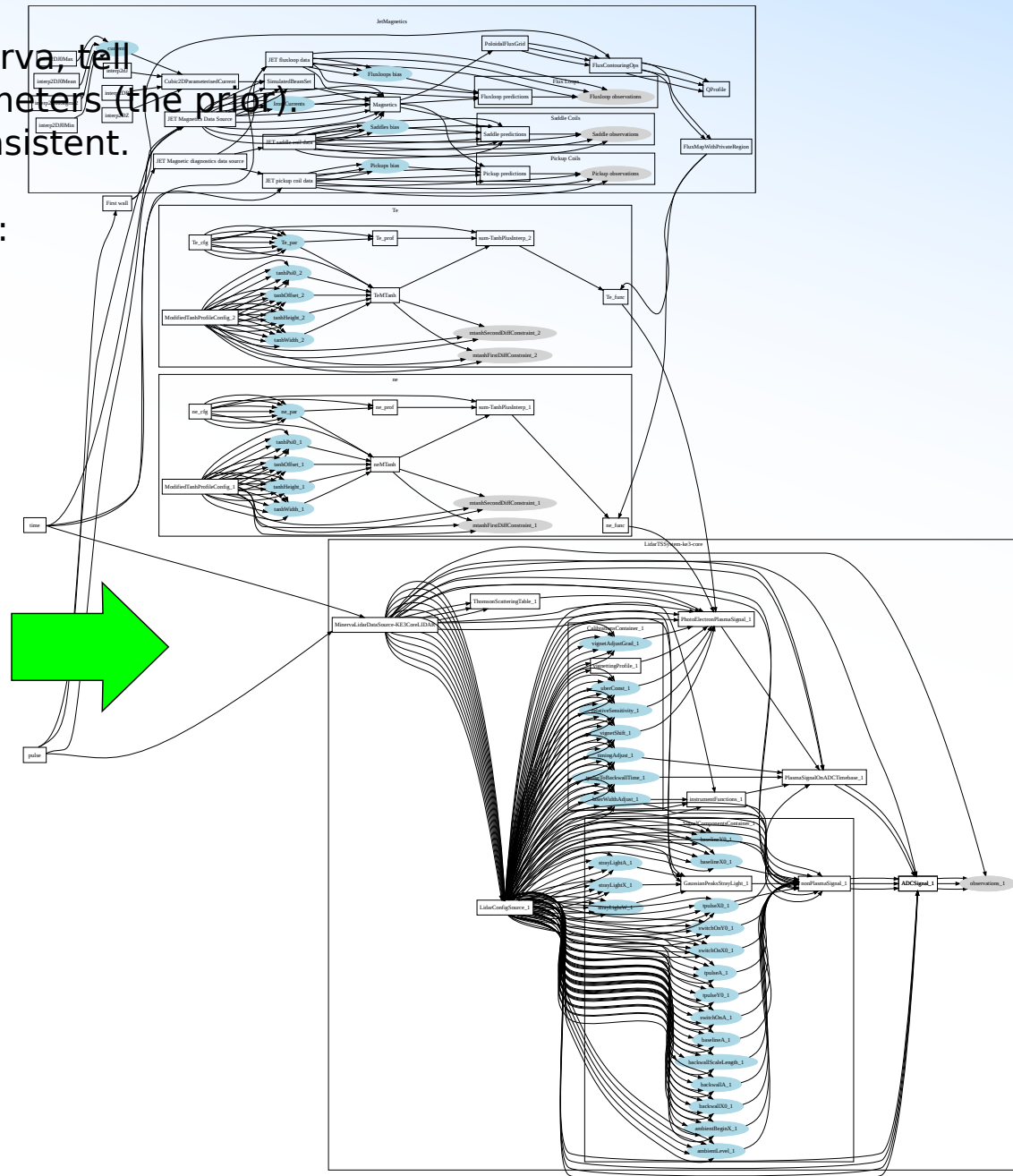
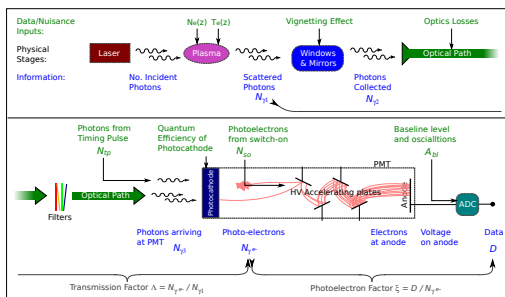
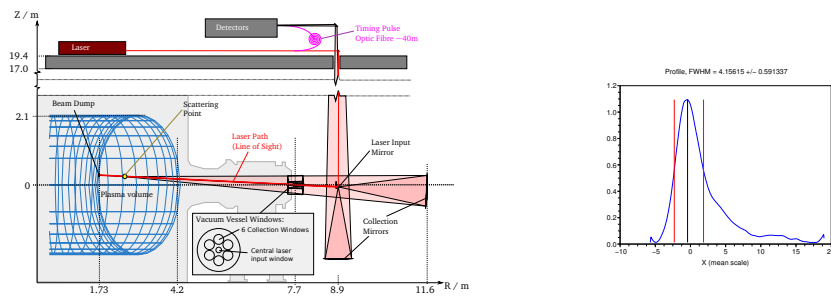
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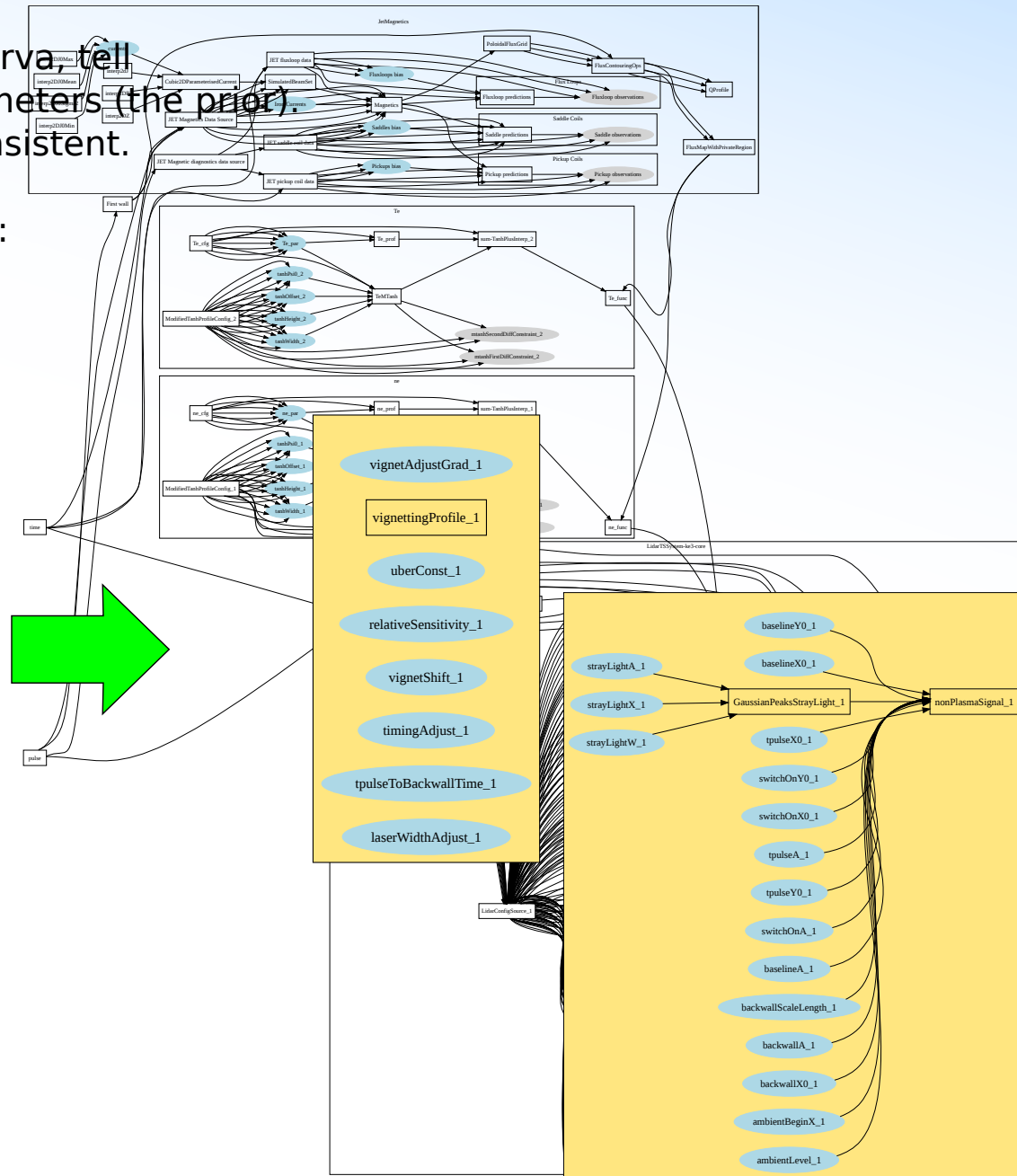
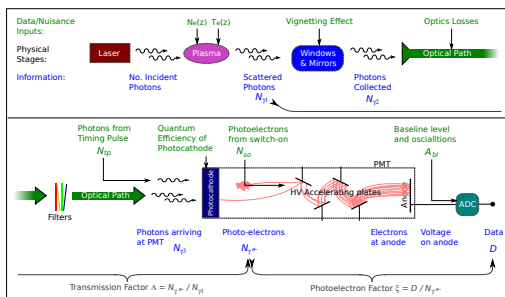
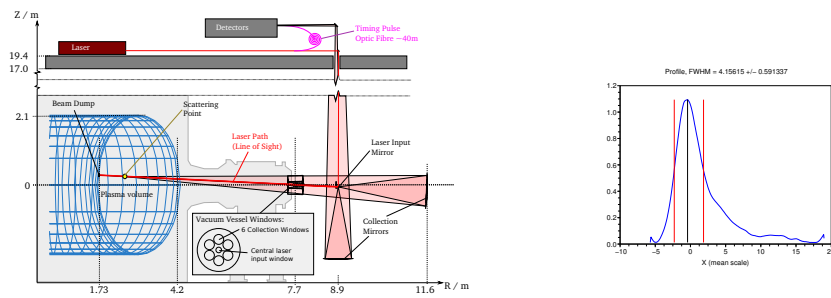
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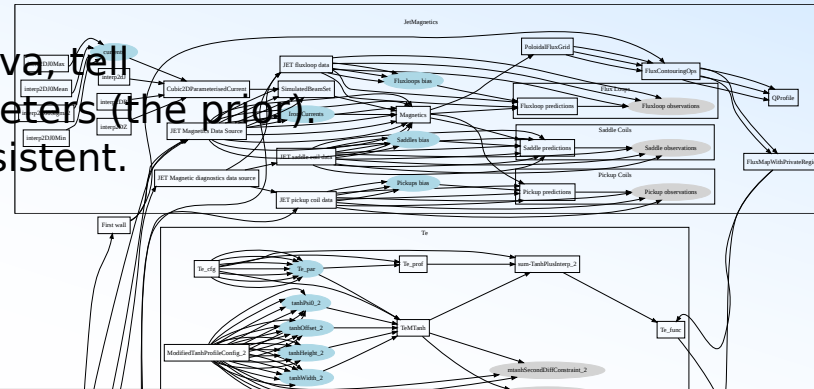


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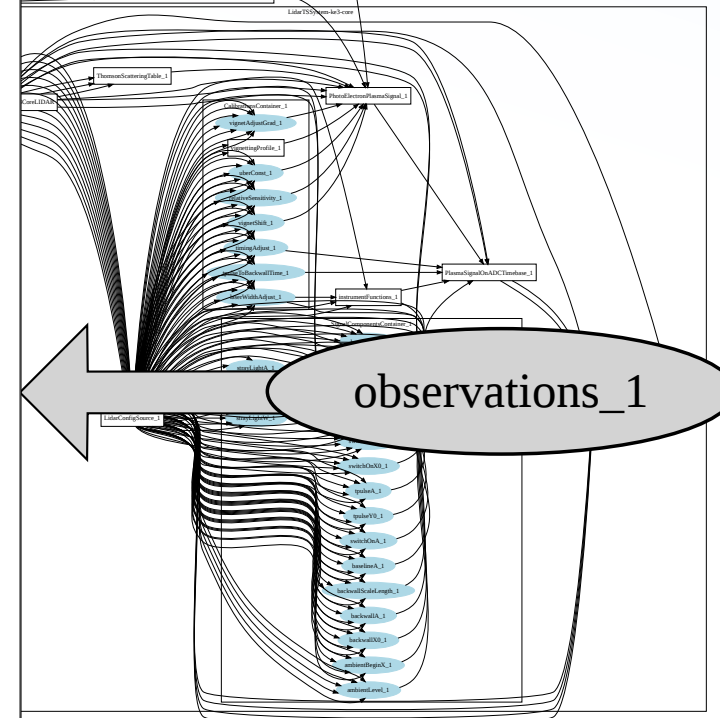
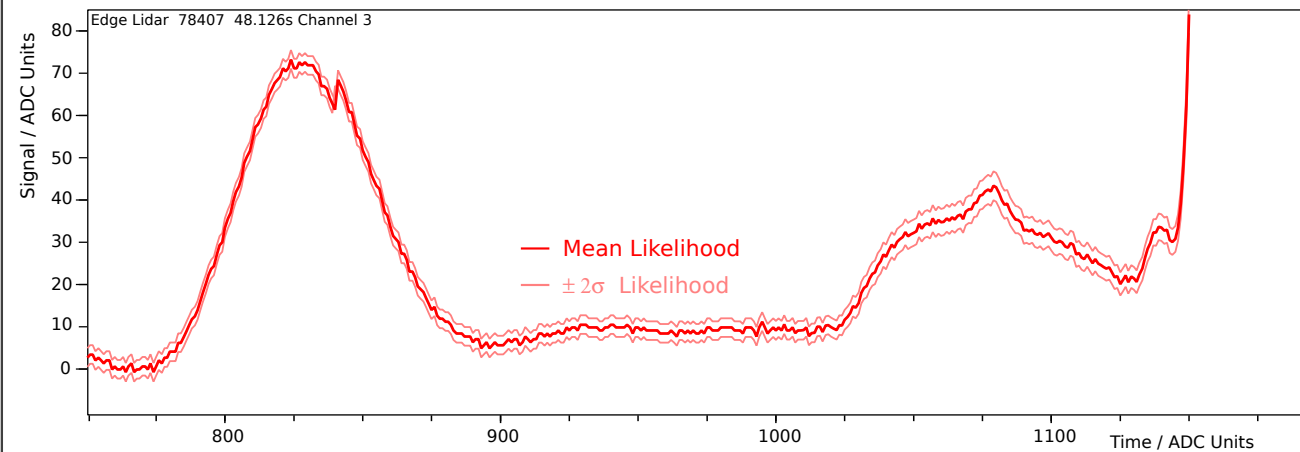
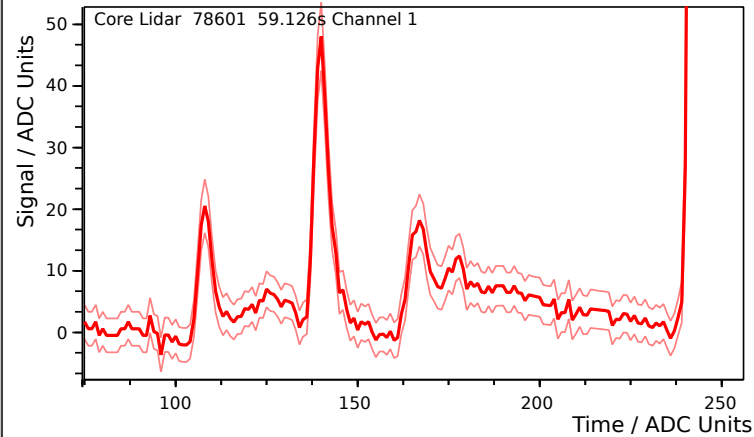
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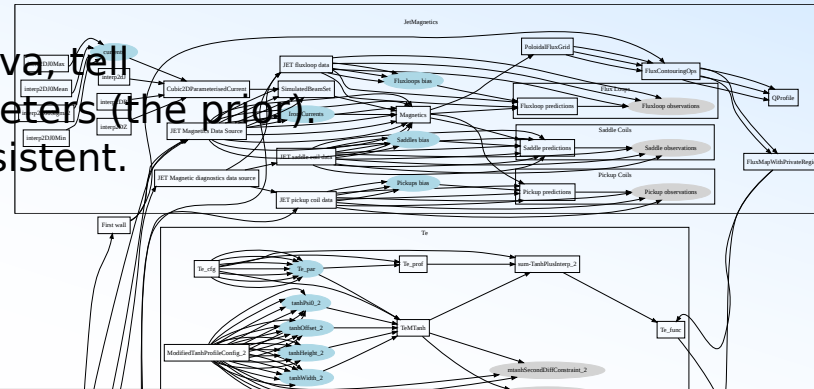


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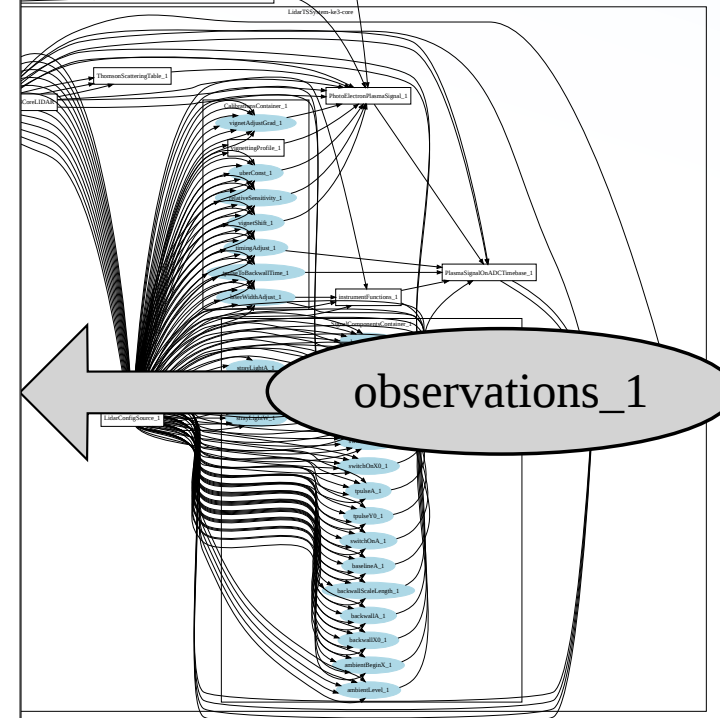
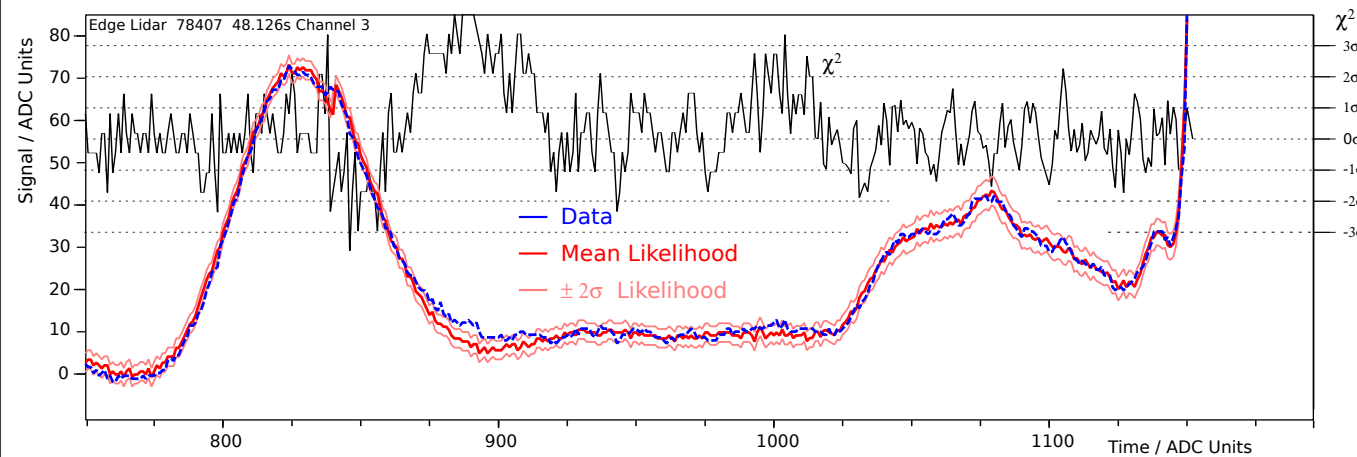
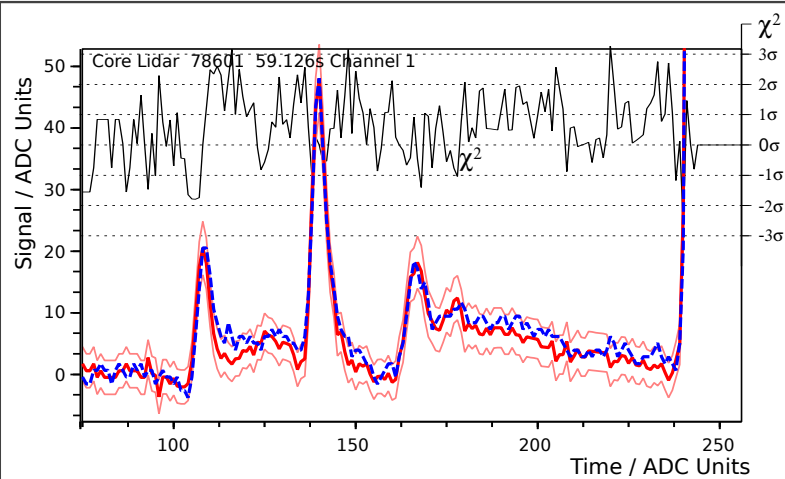
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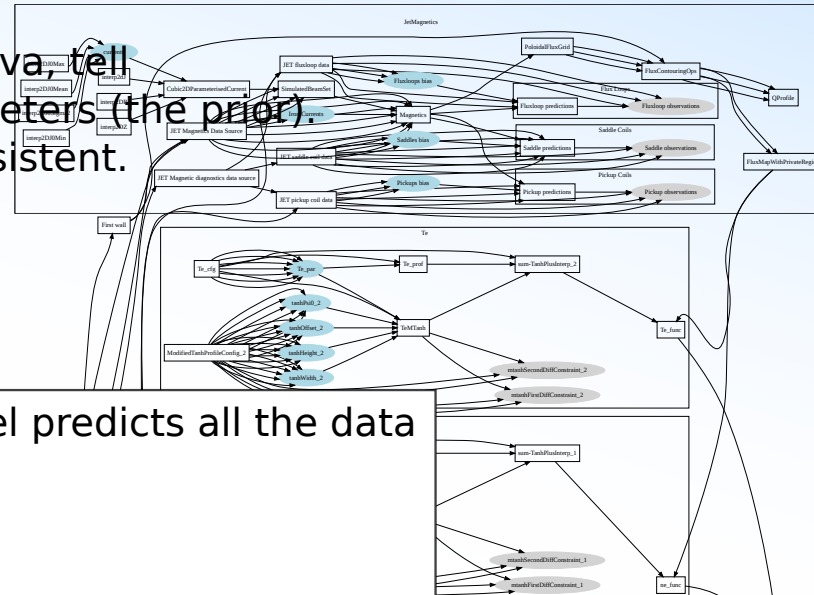


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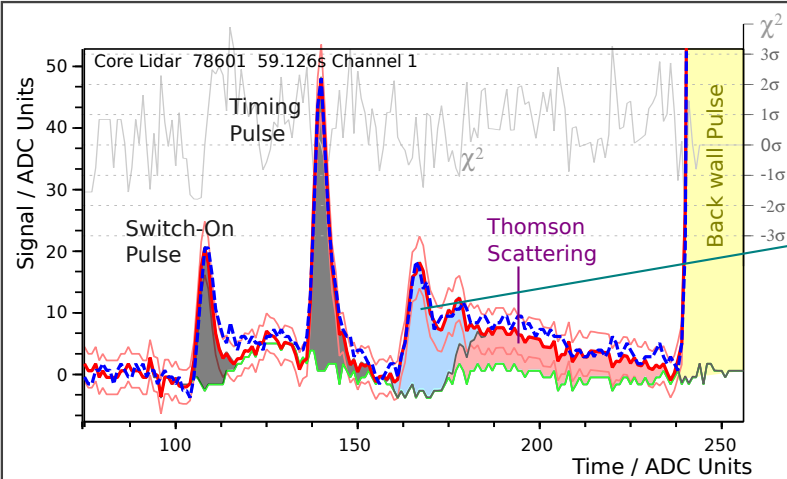
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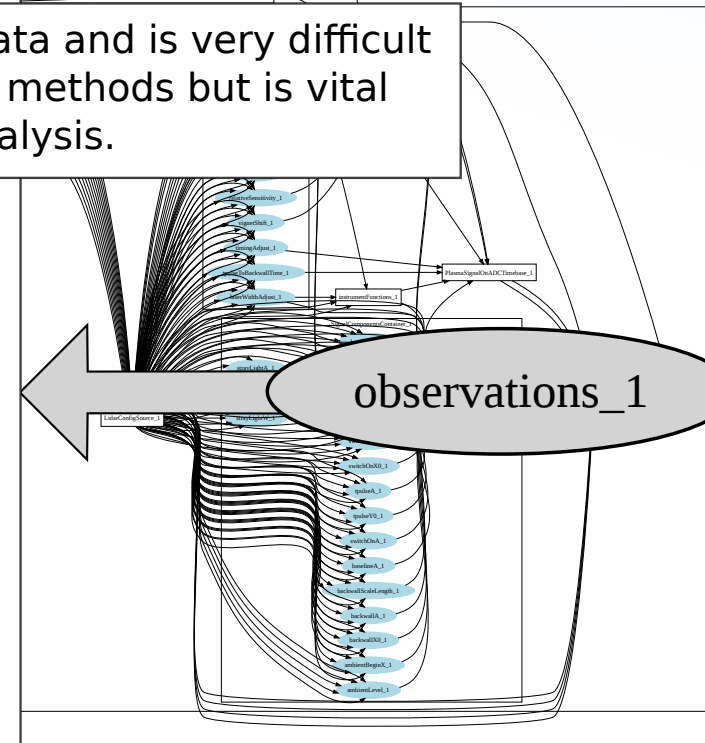
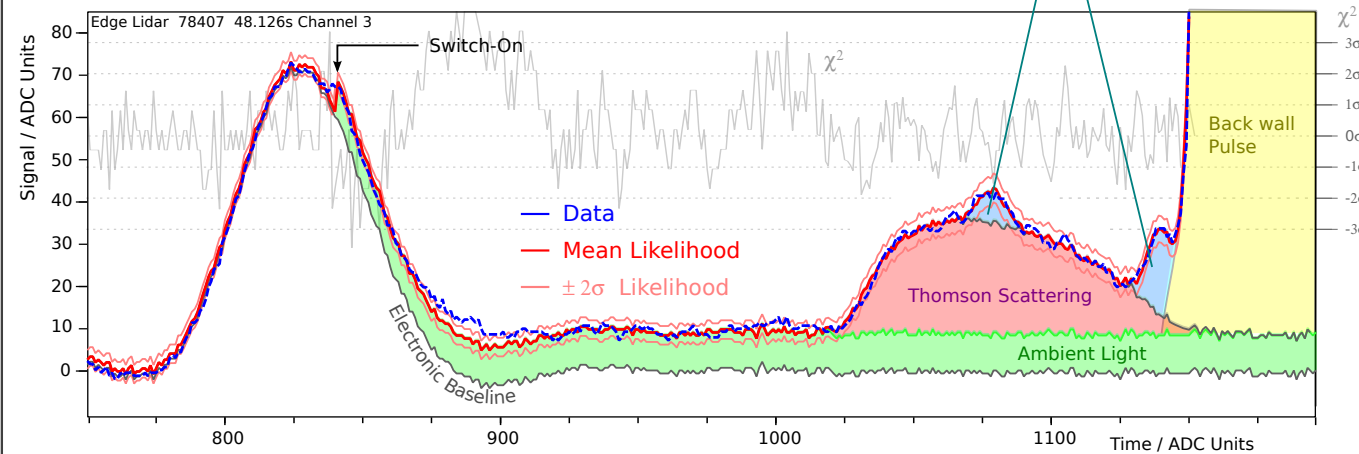
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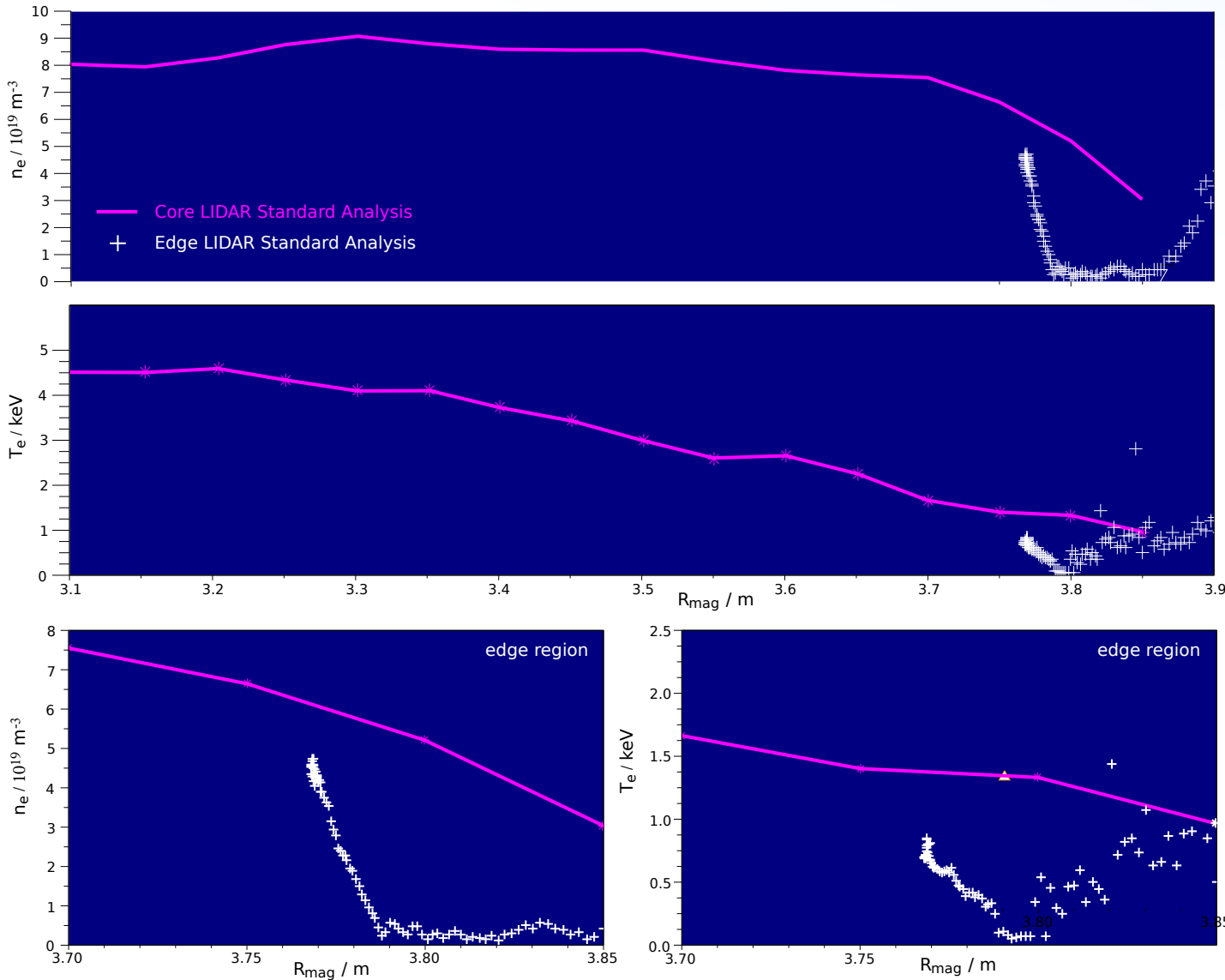


Stray light obscures TS data and is very difficult to handle with traditional methods but is vital for proper edge LIDAR analysis.



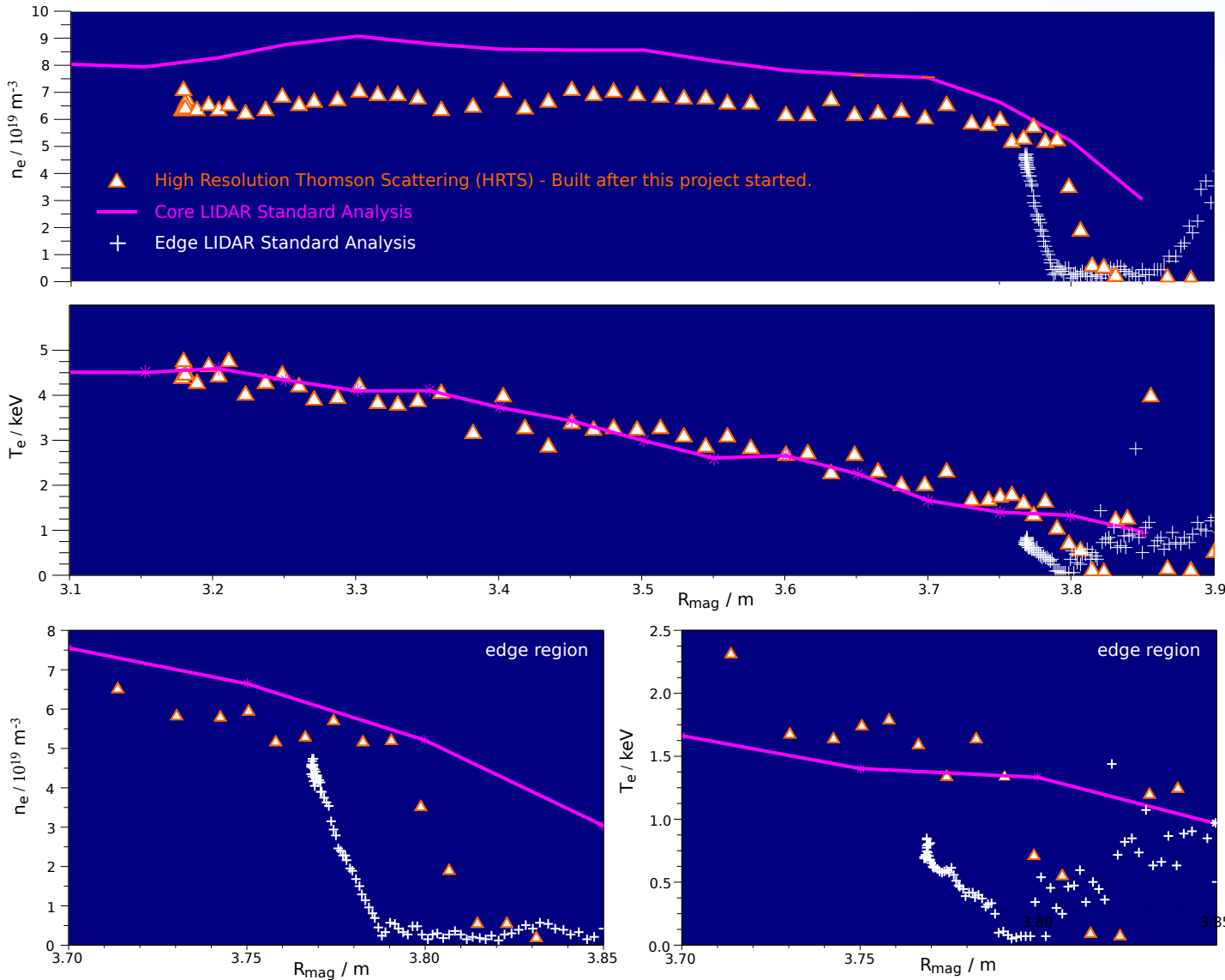
Core LIDAR + Edge LIDAR + Interferometry

A typical high density H-mode pulse:



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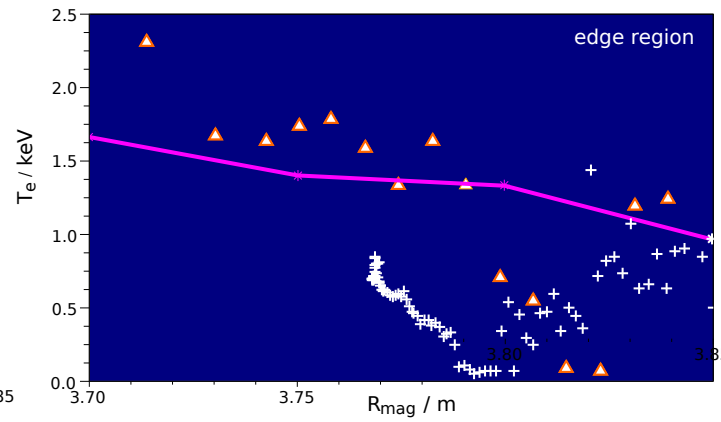
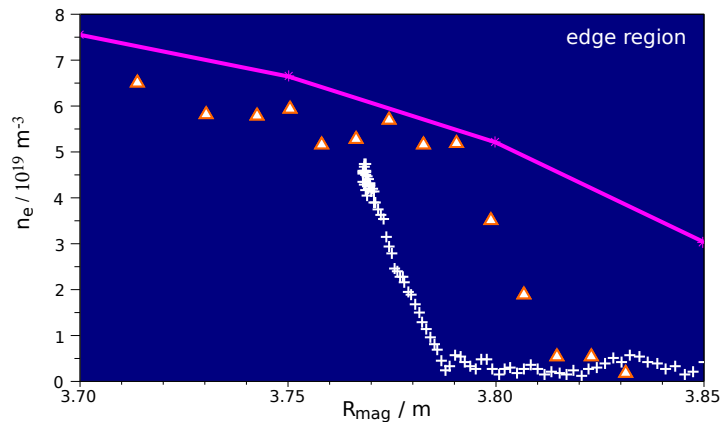
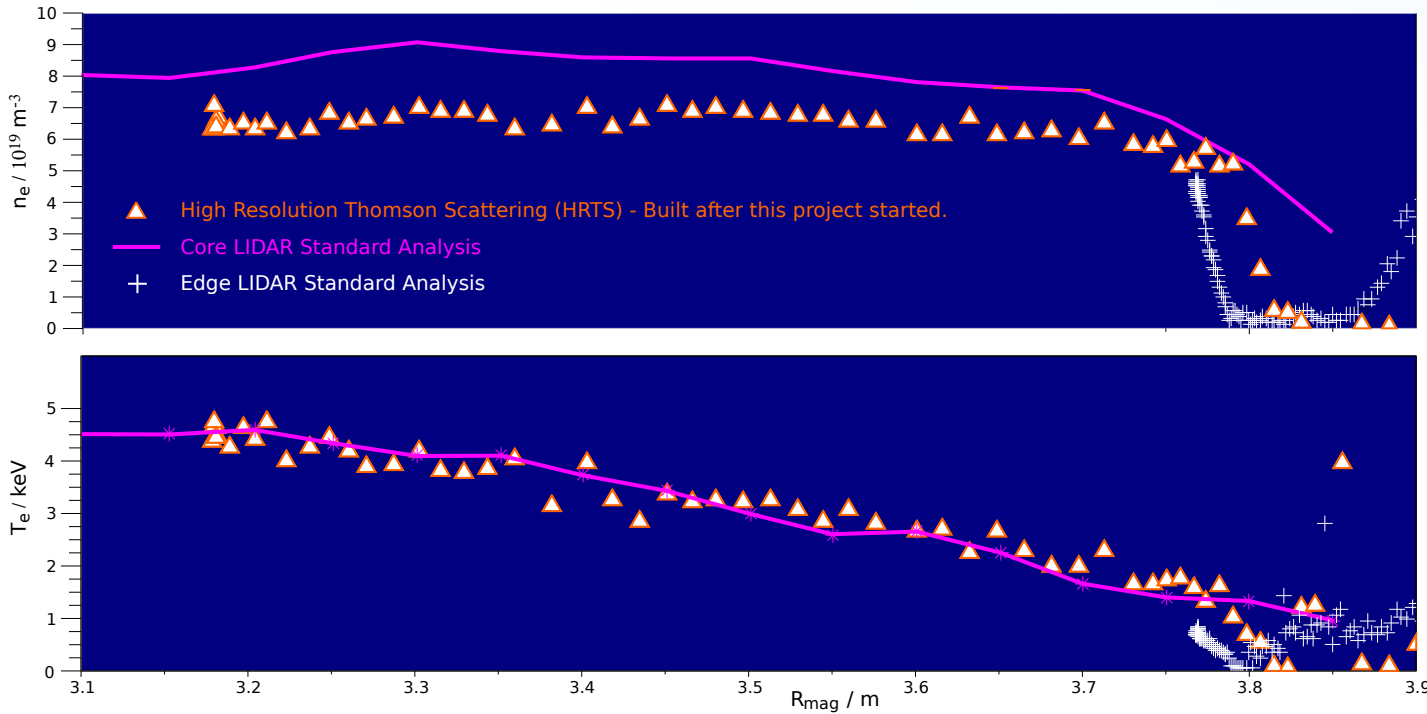
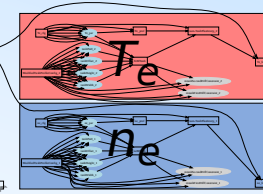
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A typical high density H-mode pulse:
- Connect up the model.

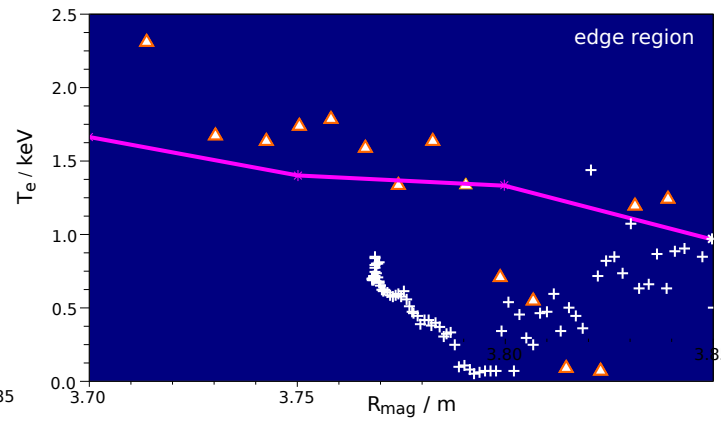
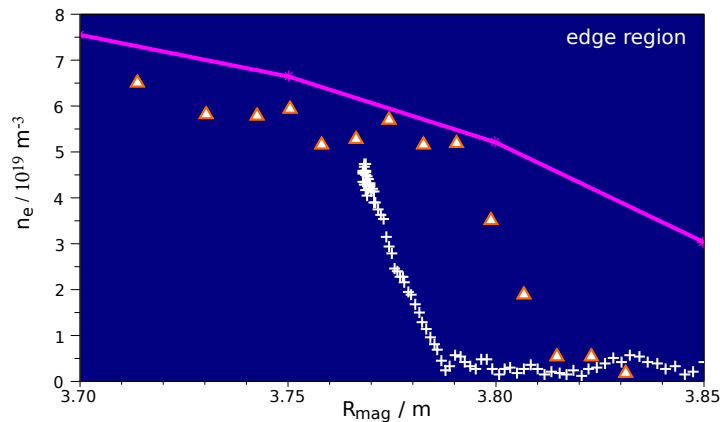
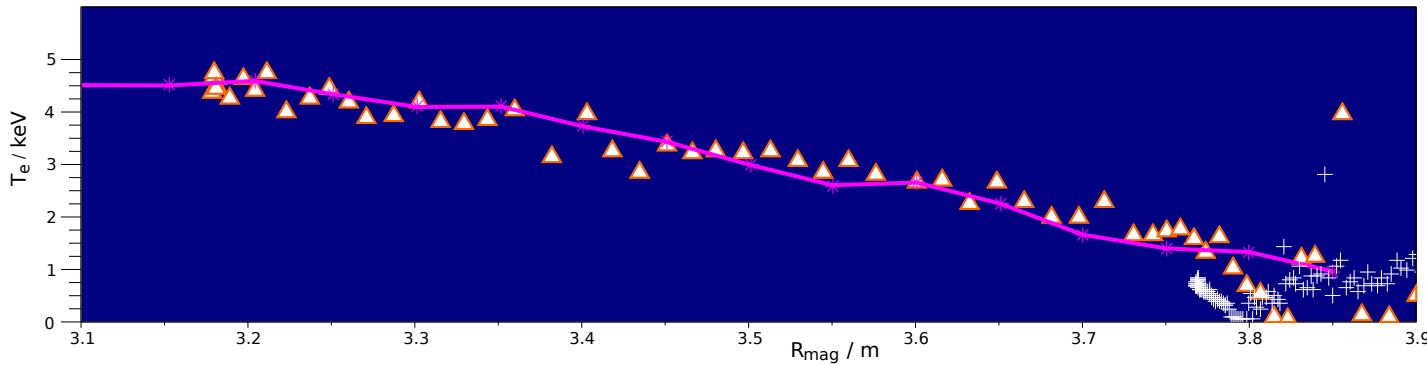
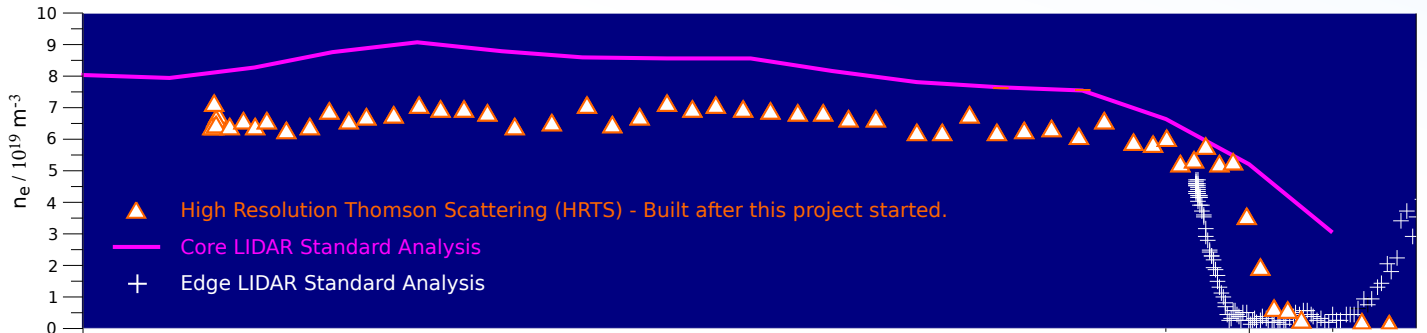
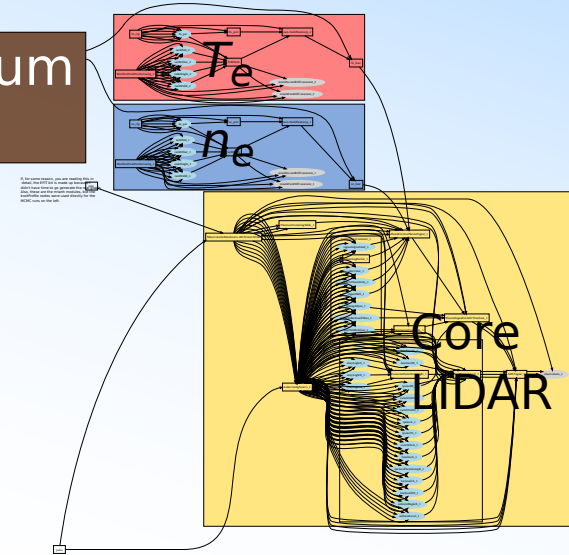
Equilibrium
Code



Core LIDAR + Edge LIDAR + Interferometry

A typical high density H-mode pulse:
- Connect up the model.

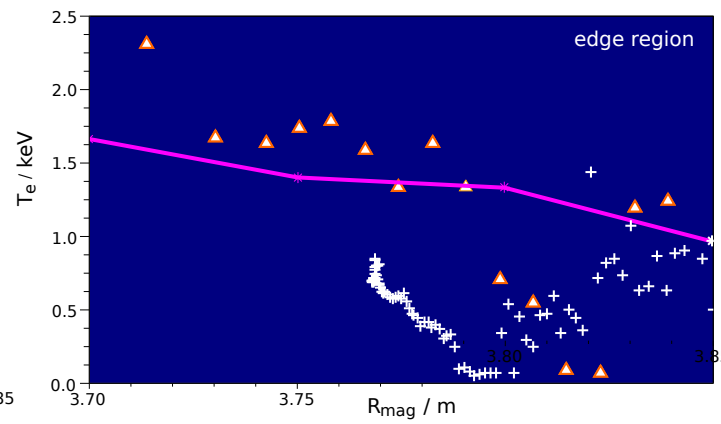
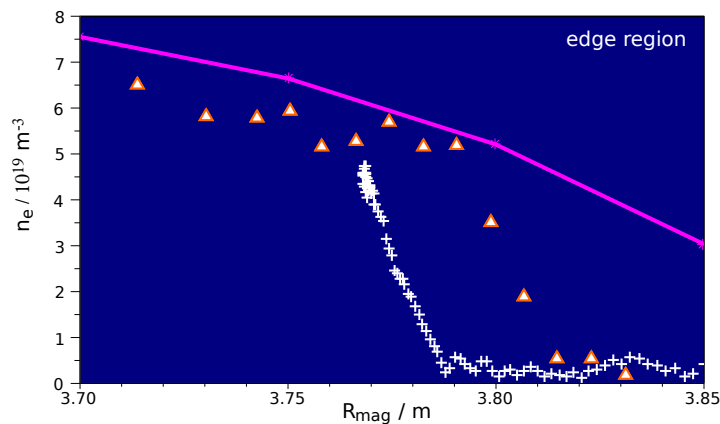
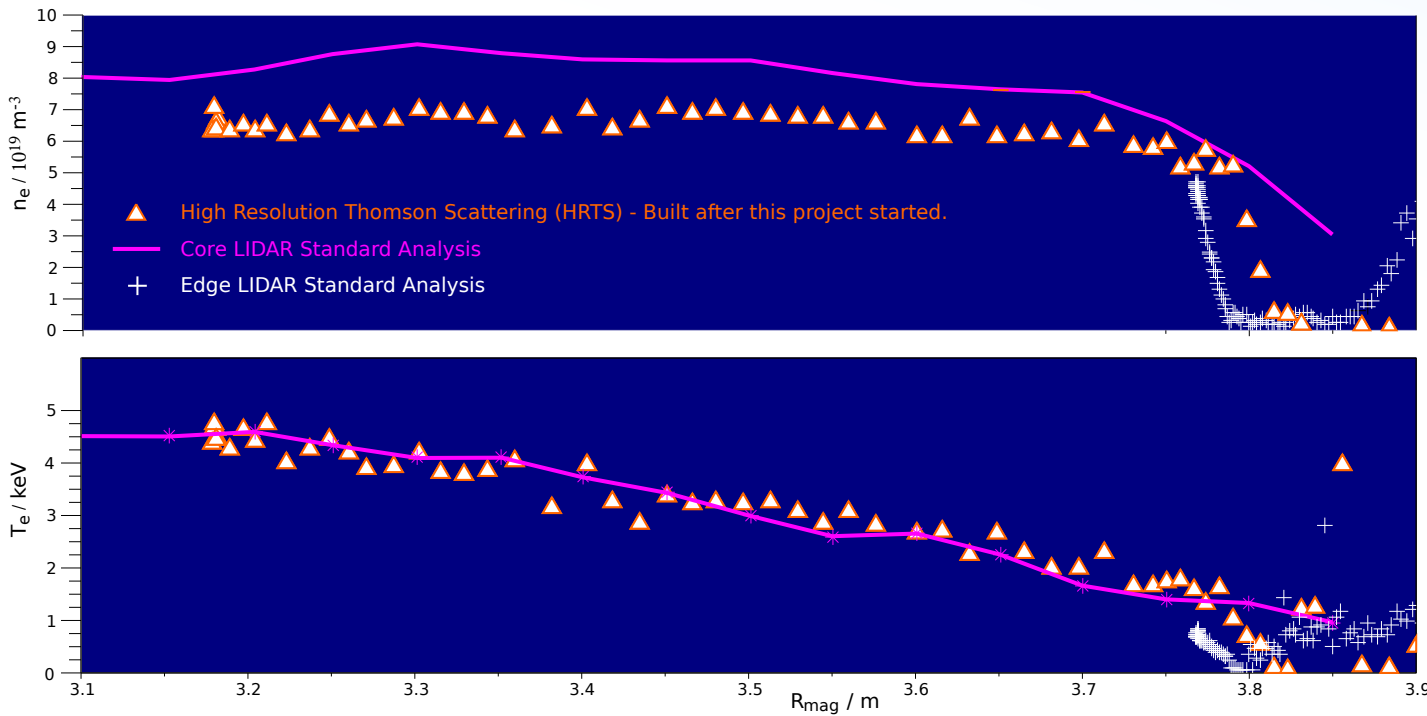
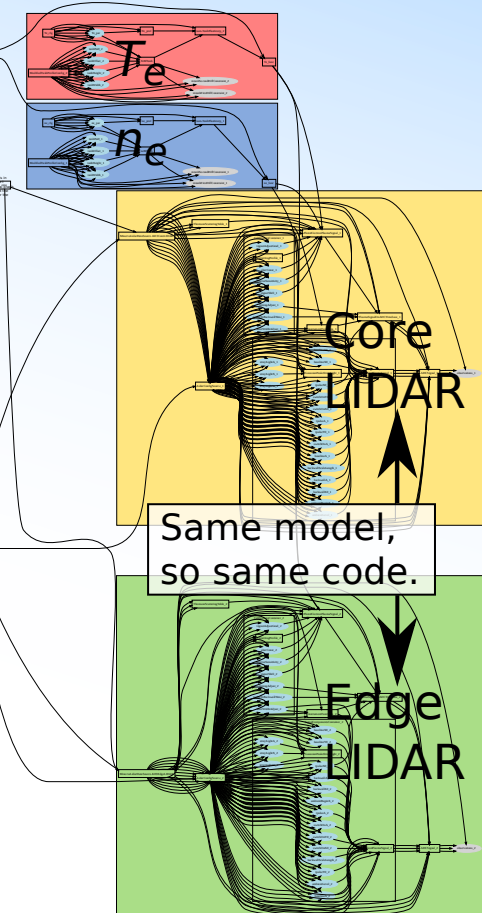
Equilibrium
Code



Core LIDAR + Edge LIDAR + Interferometry

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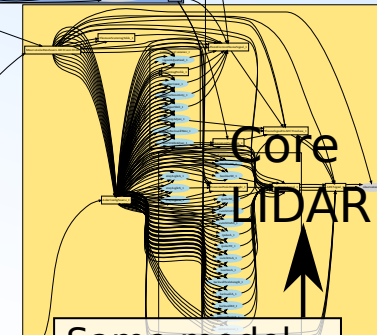
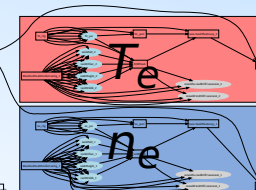
Equilibrium
Code



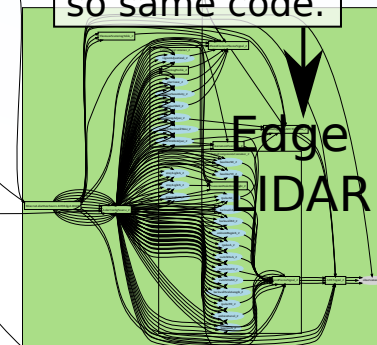
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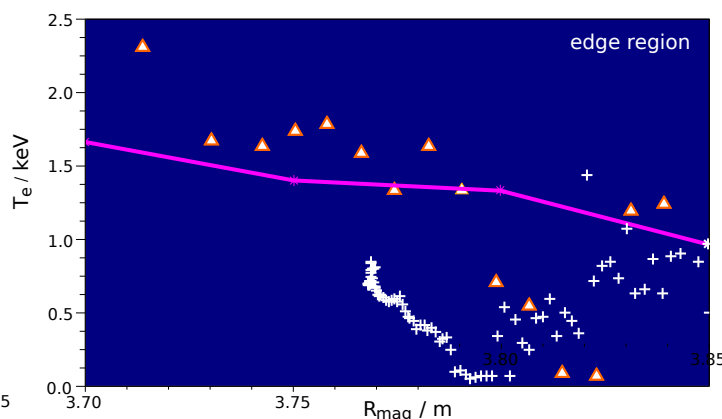
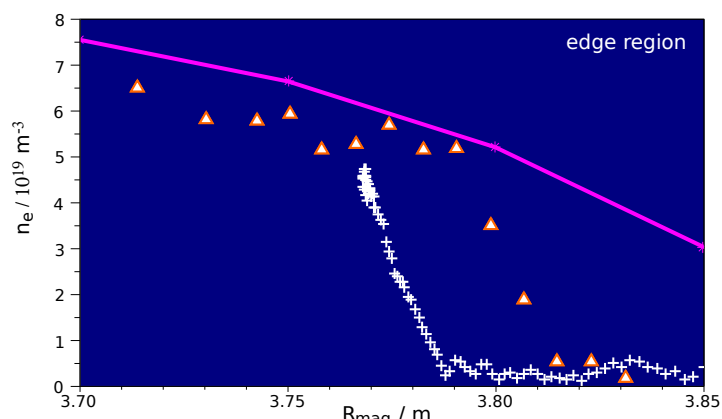
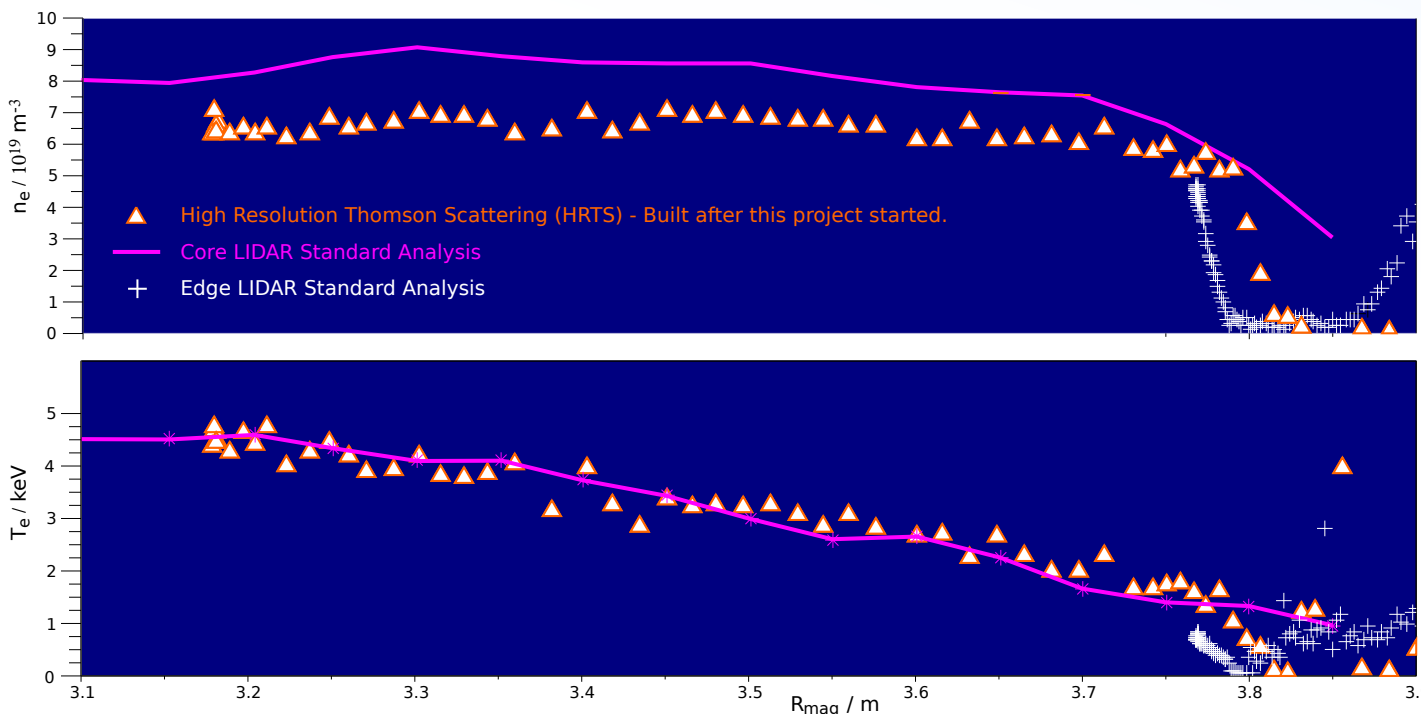
Equilibrium
Code



Same model,
so same code.



Interferometry

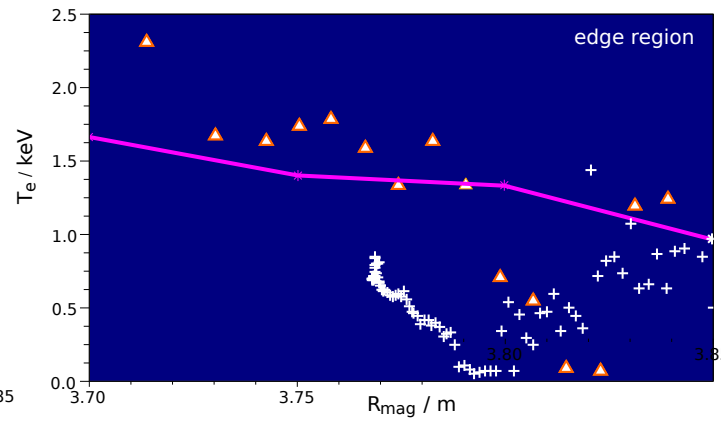
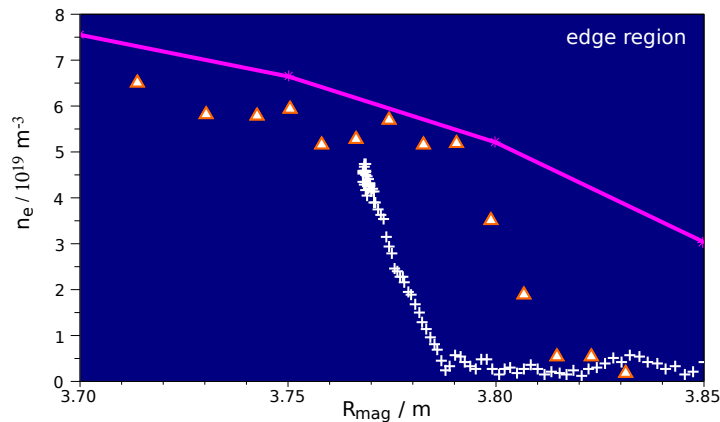
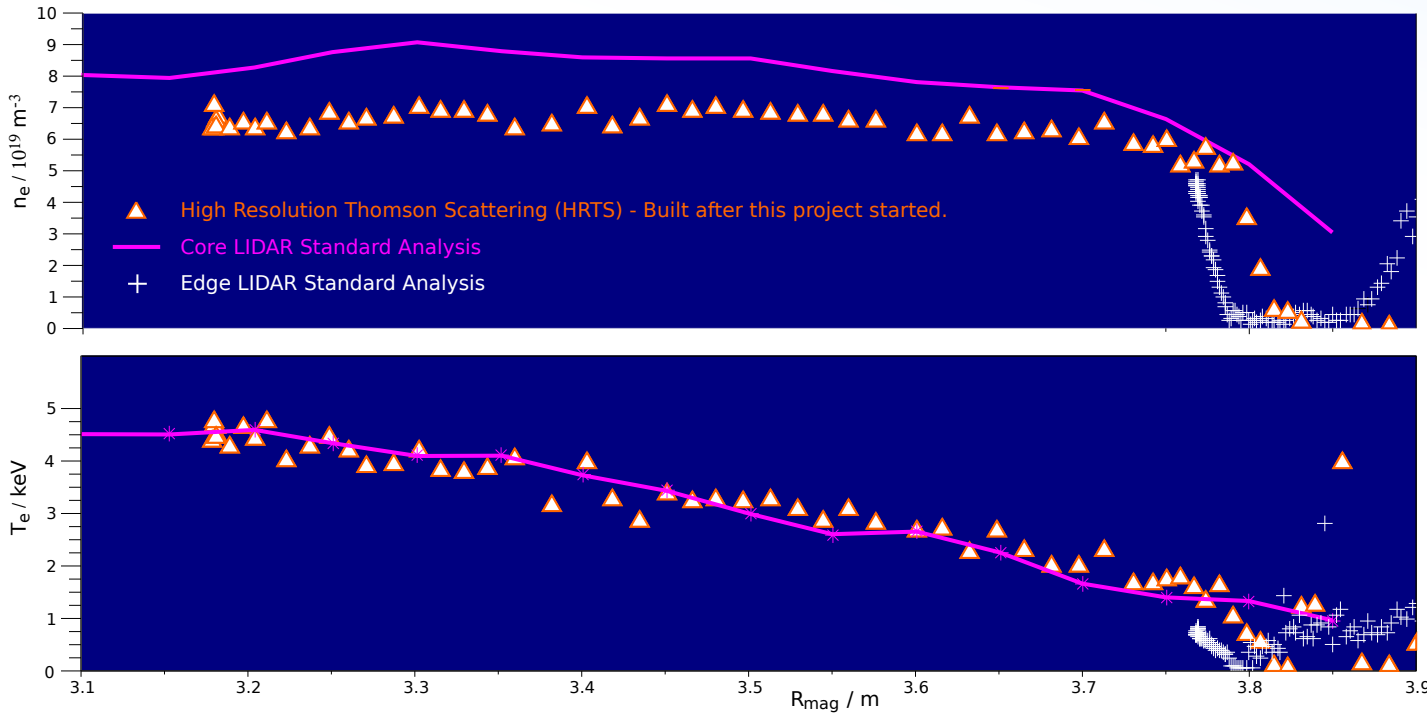
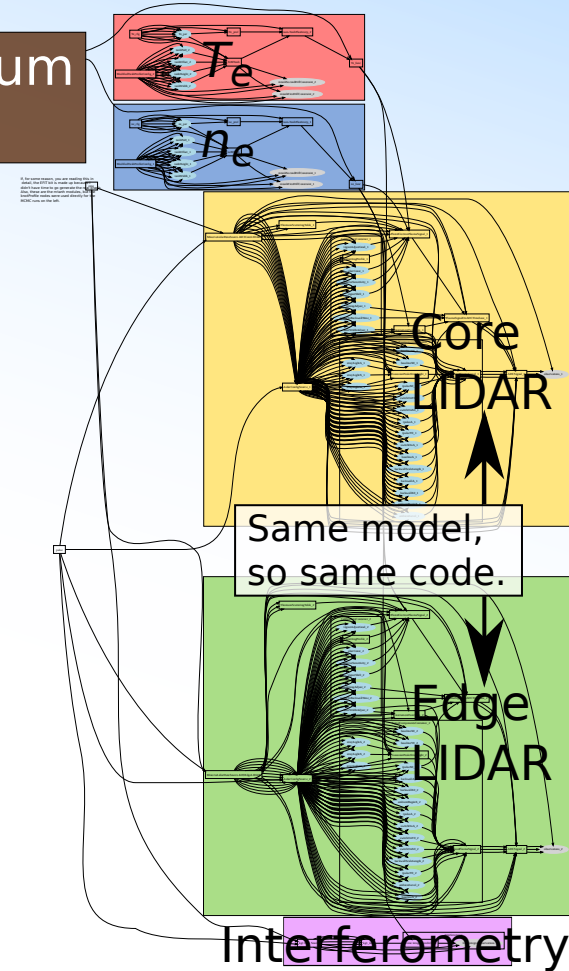


Core LIDAR + Edge LIDAR + Interferometry

A typical high density H-mode pulse:

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Equilibrium
Code

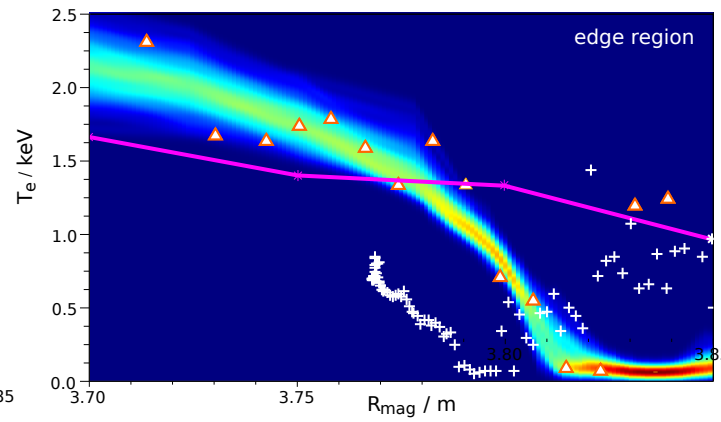
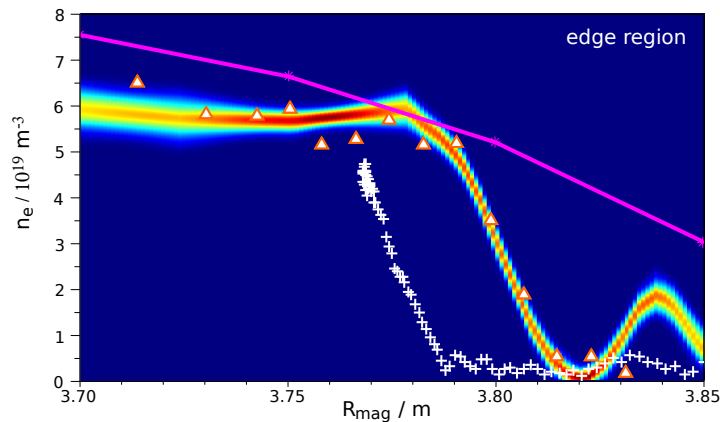
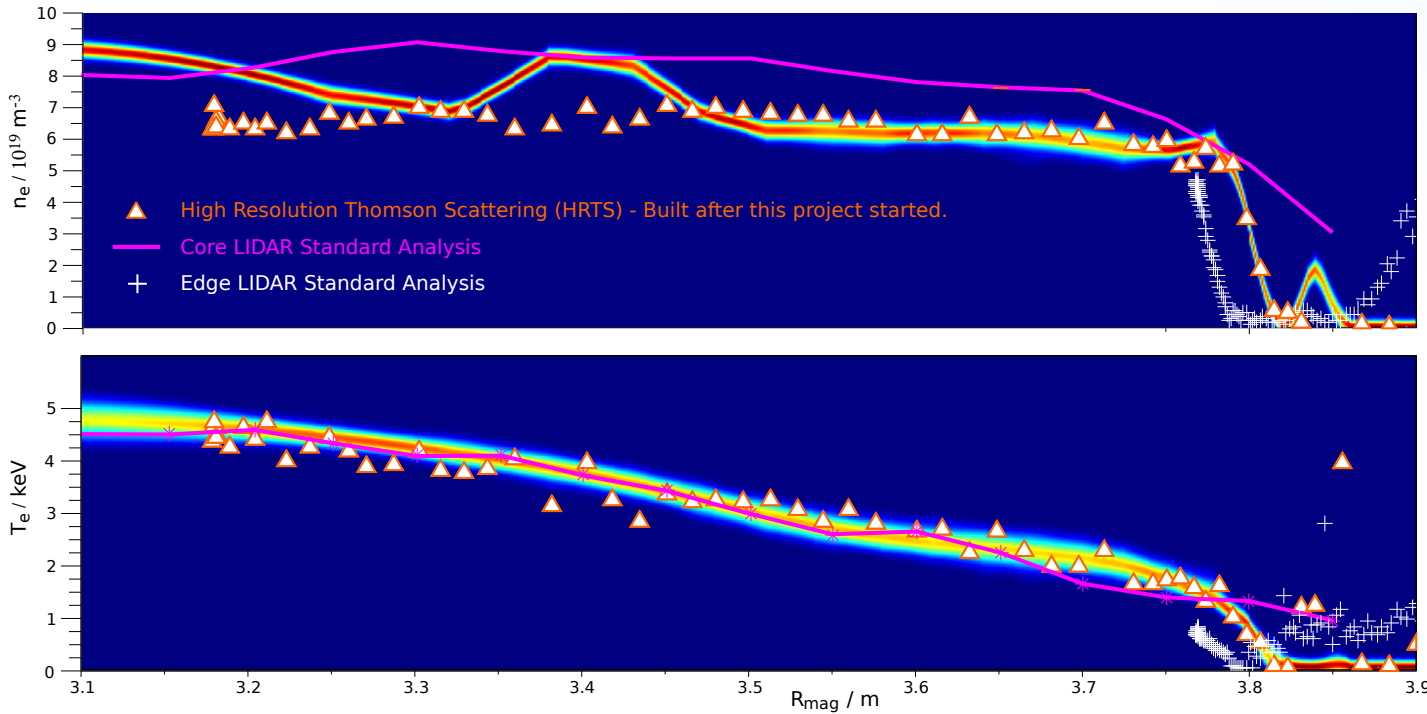
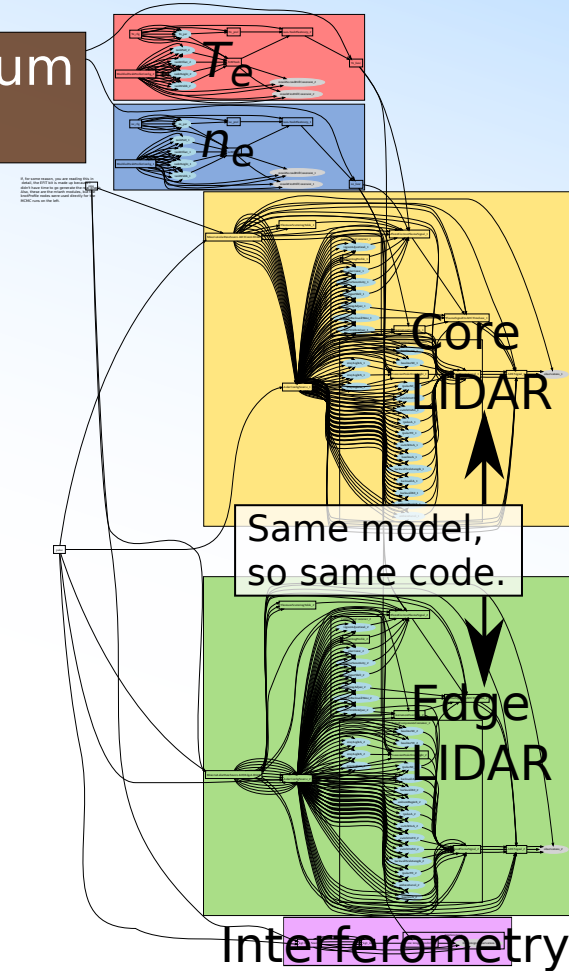


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Equilibrium
Code

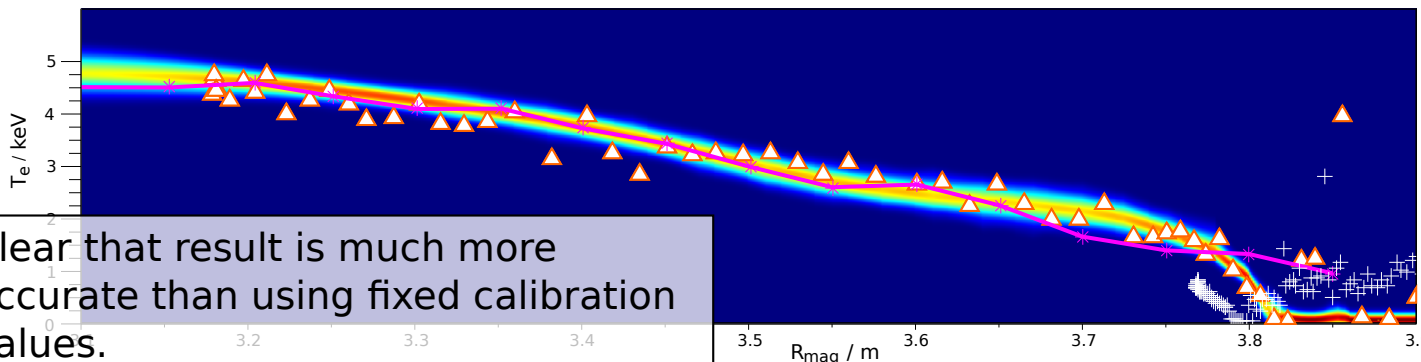
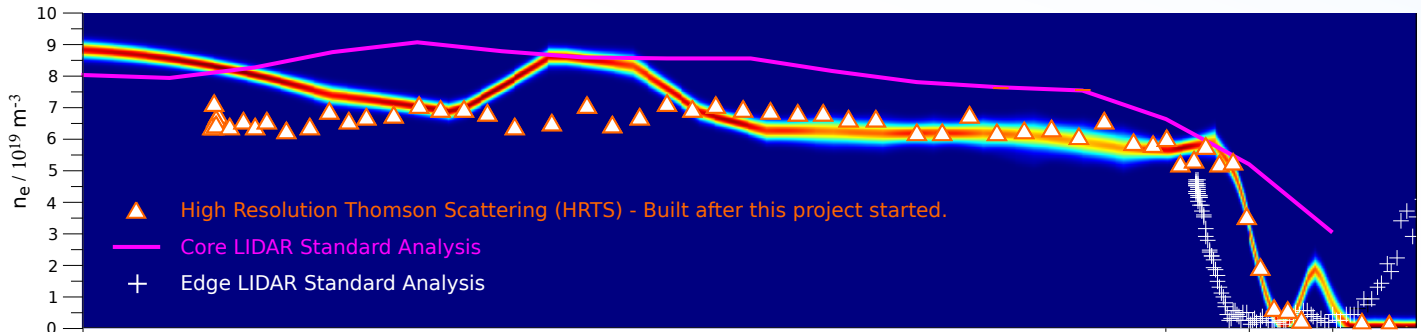
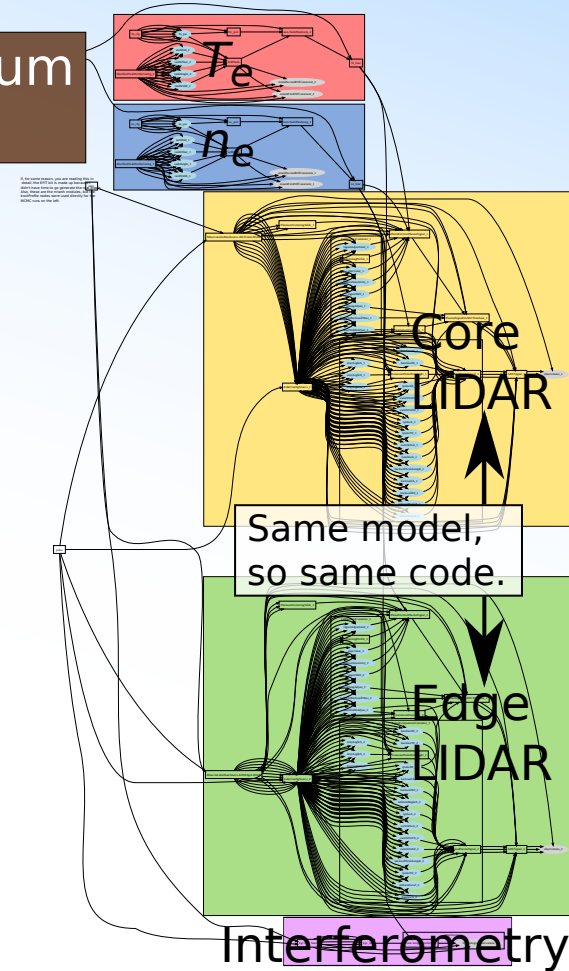


Core LIDAR + Edge LIDAR + Interferometry

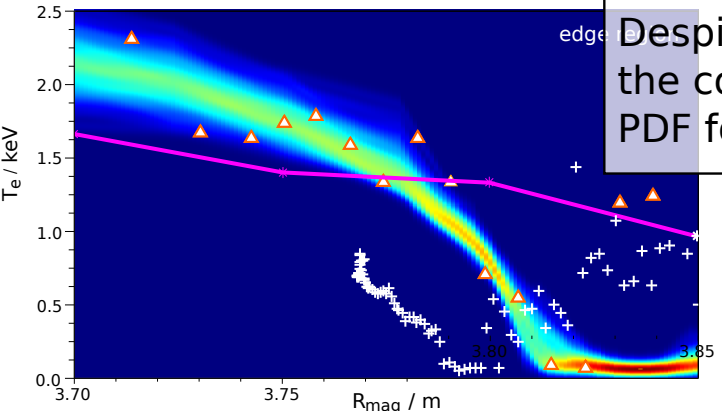
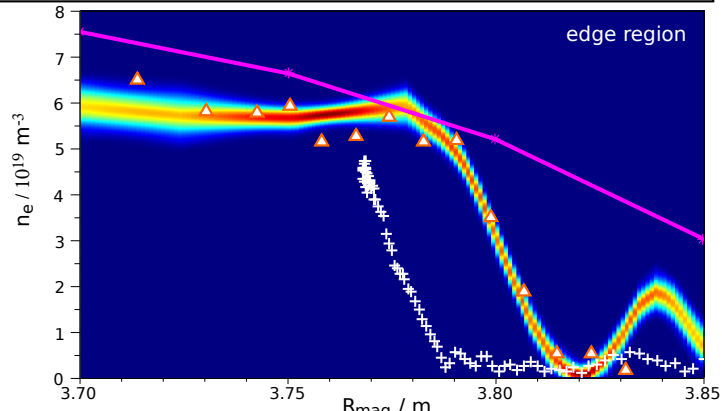
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Equilibrium Code



Clear that result is much more accurate than using fixed calibration values.



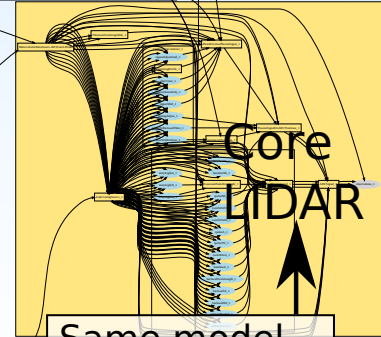
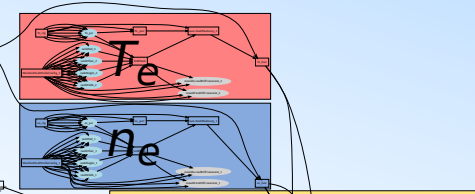
Despite completely free T_e calibration, the combination can fix T_e and gives a PDF for the calibration values.

Core LIDAR + Edge LIDAR + Interferometry

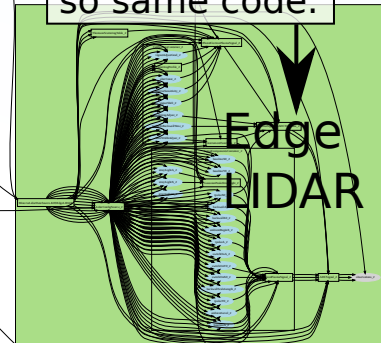
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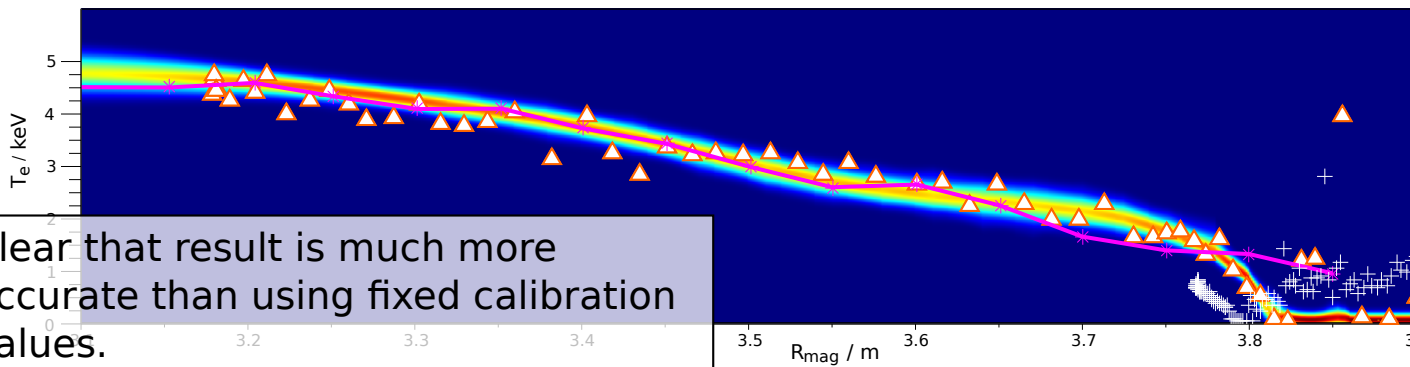
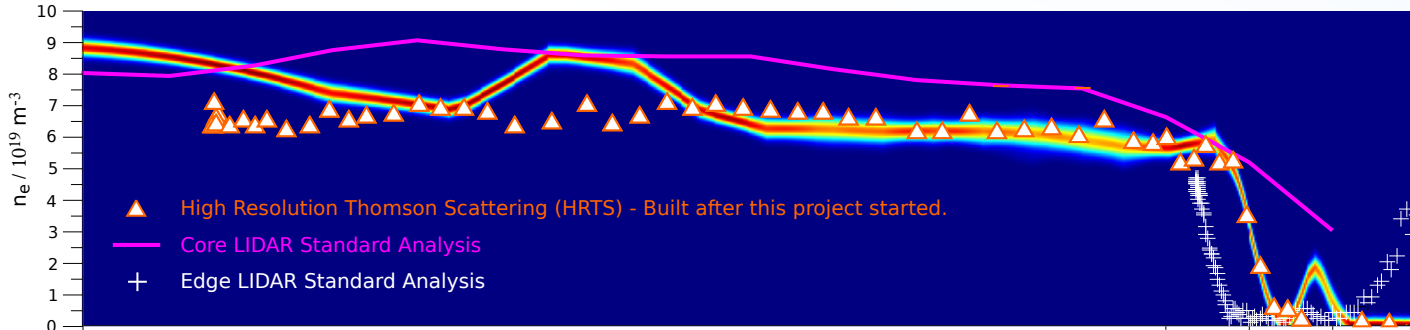
Equilibrium Code



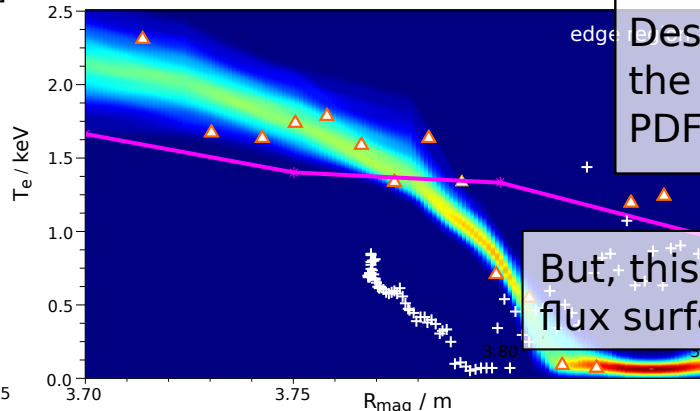
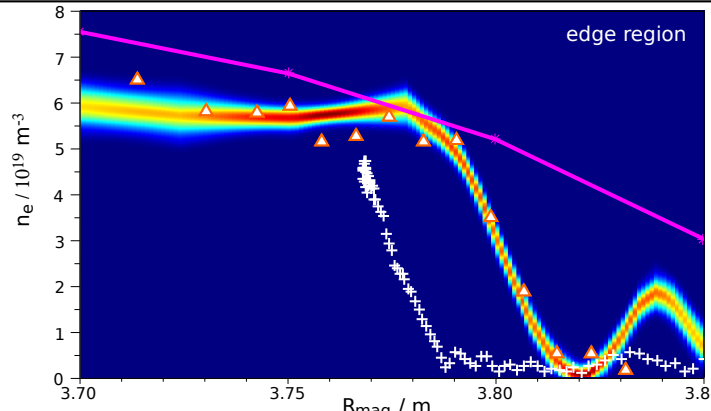
Same model, so same code.



Interferometry



Clear that result is much more accurate than using fixed calibration values.

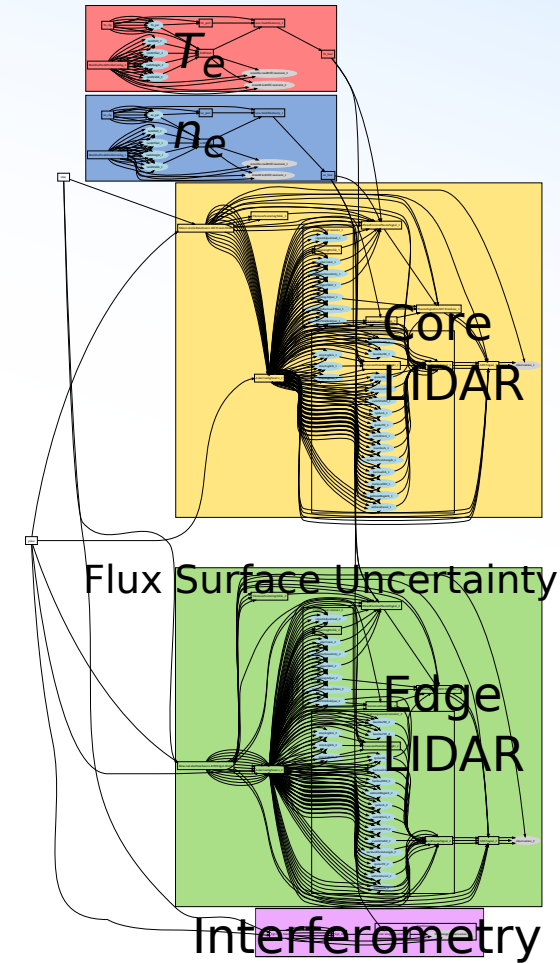


Despite completely free T_e calibration, the combination can fix T_e and gives a PDF for the calibration values.

But, this isn't complete - we are still using flux surfaces fixed to the equilibrium code.

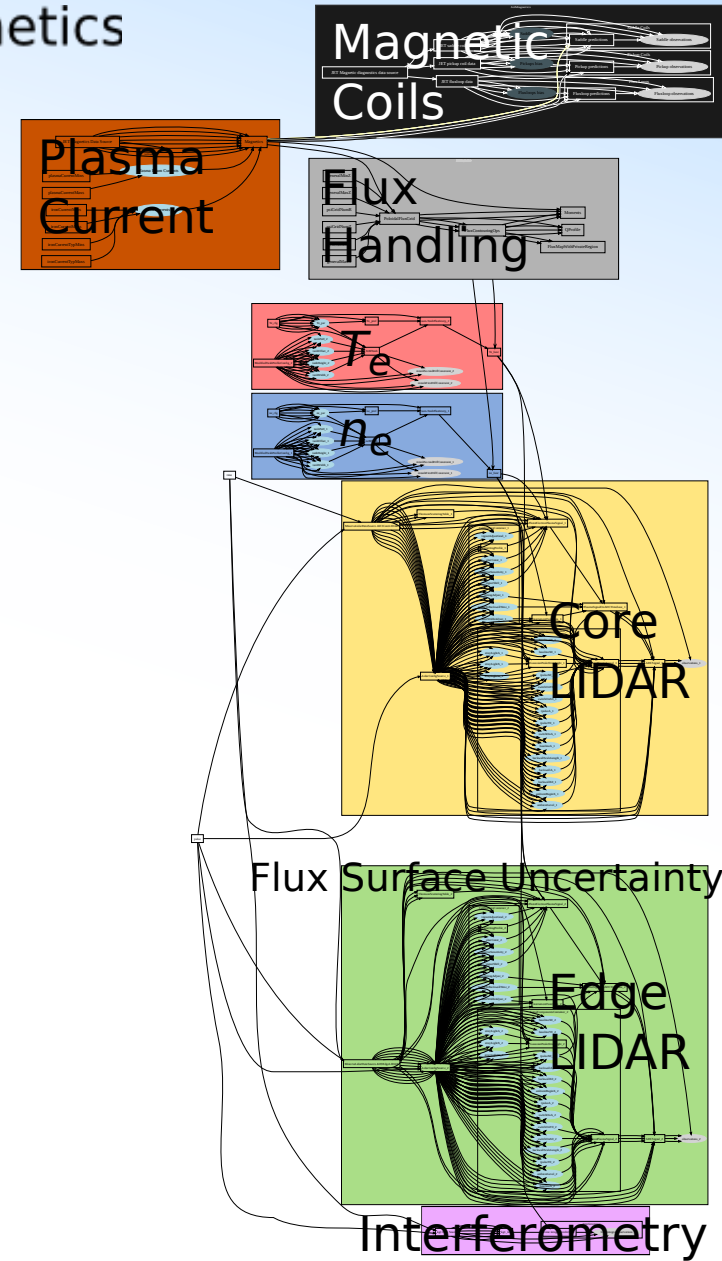
Core LIDAR + Edge LIDAR + Interferometry + Magnetics

Connect magnetics model and run inversion.



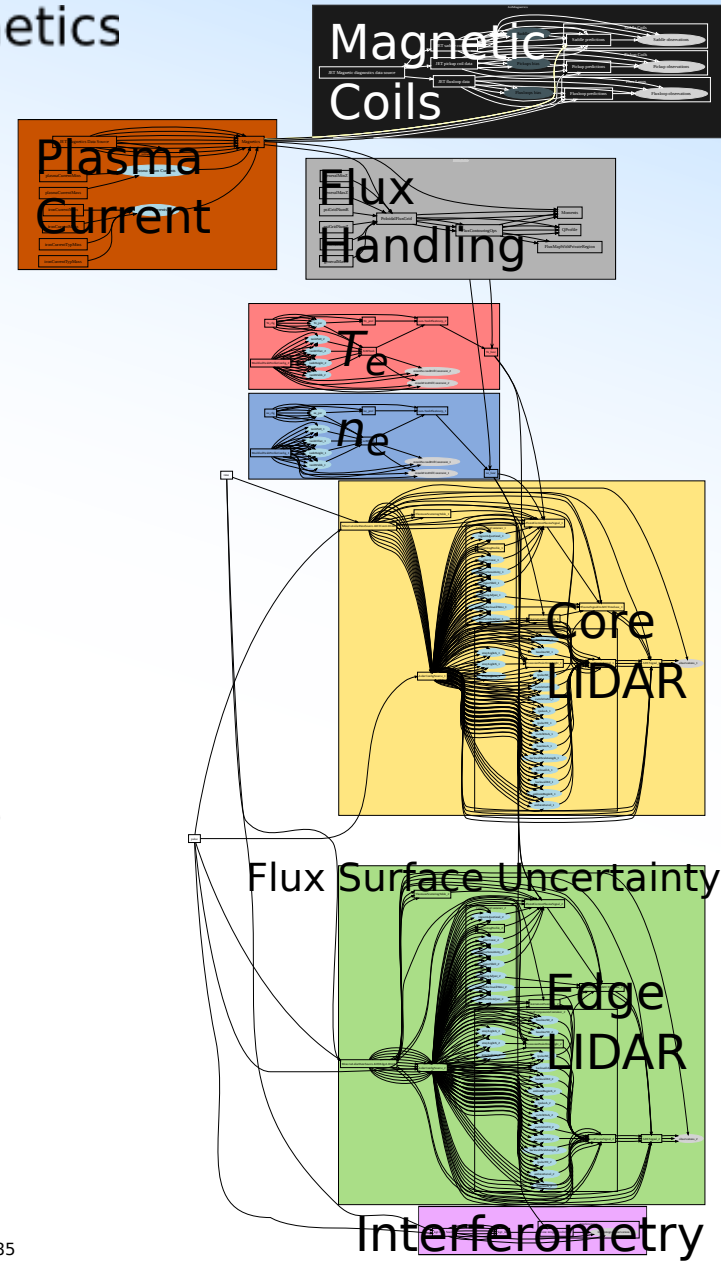
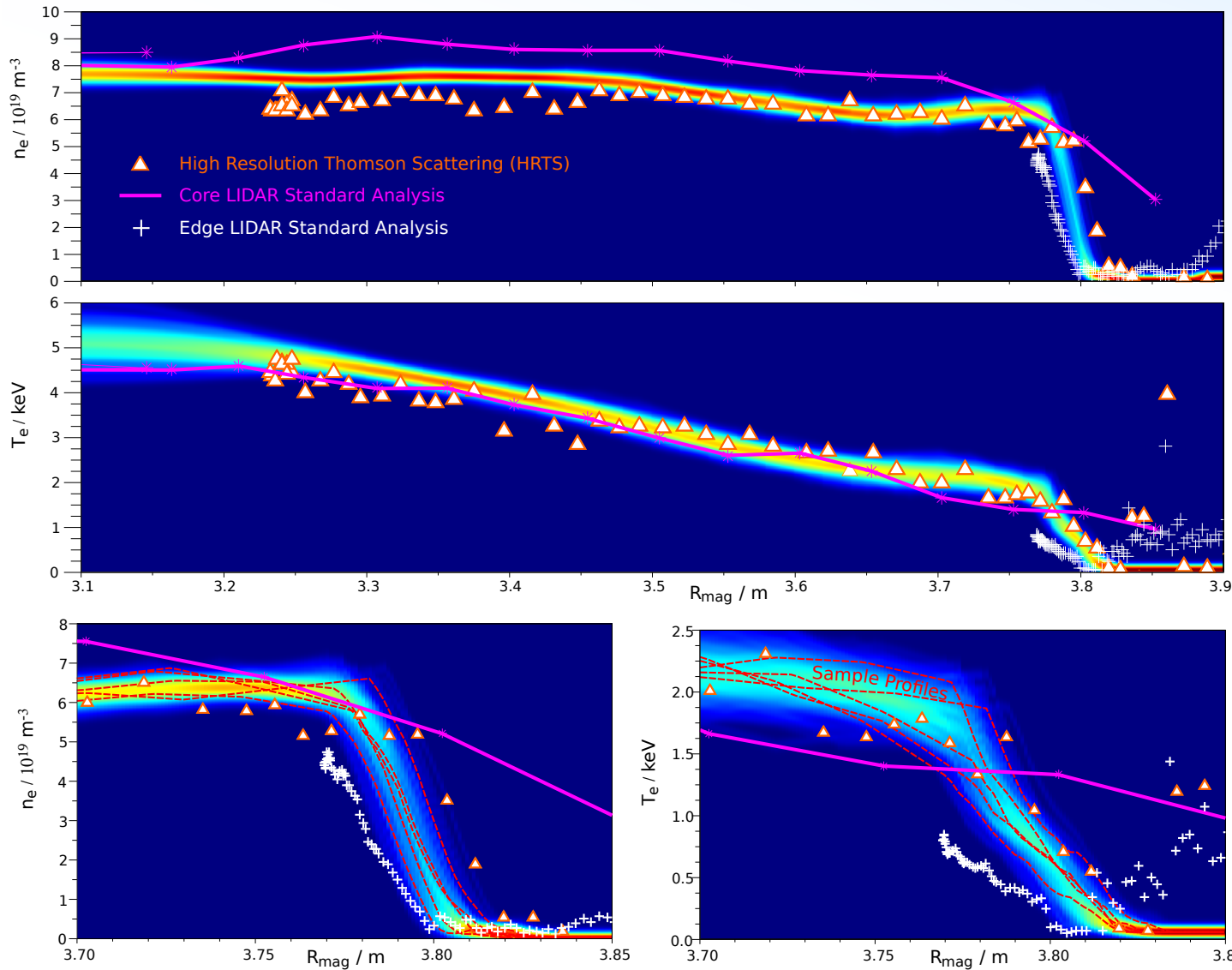
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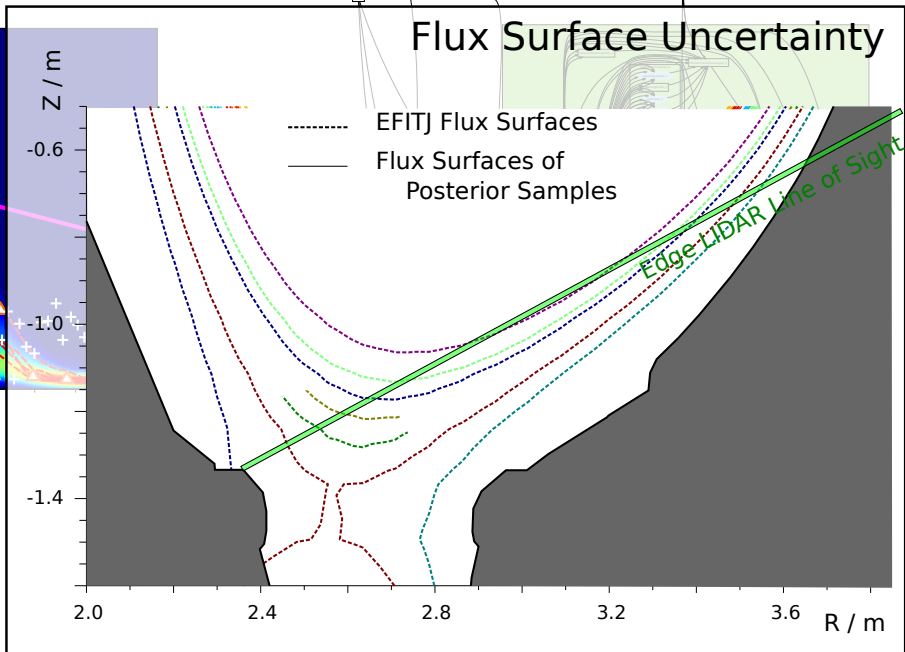
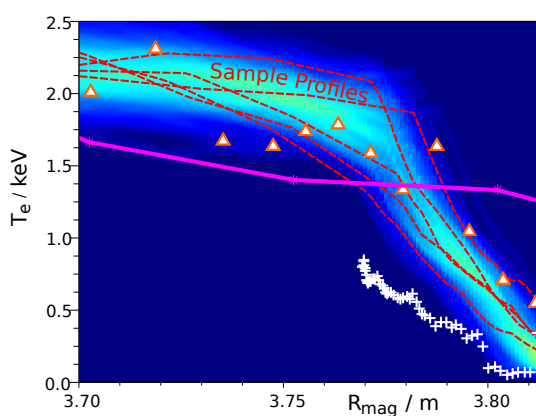
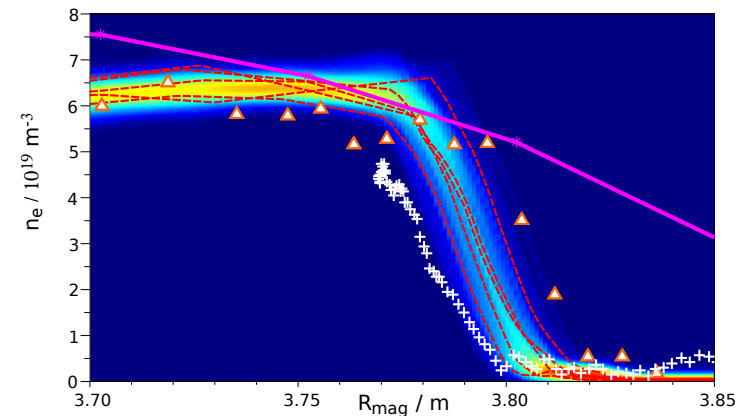
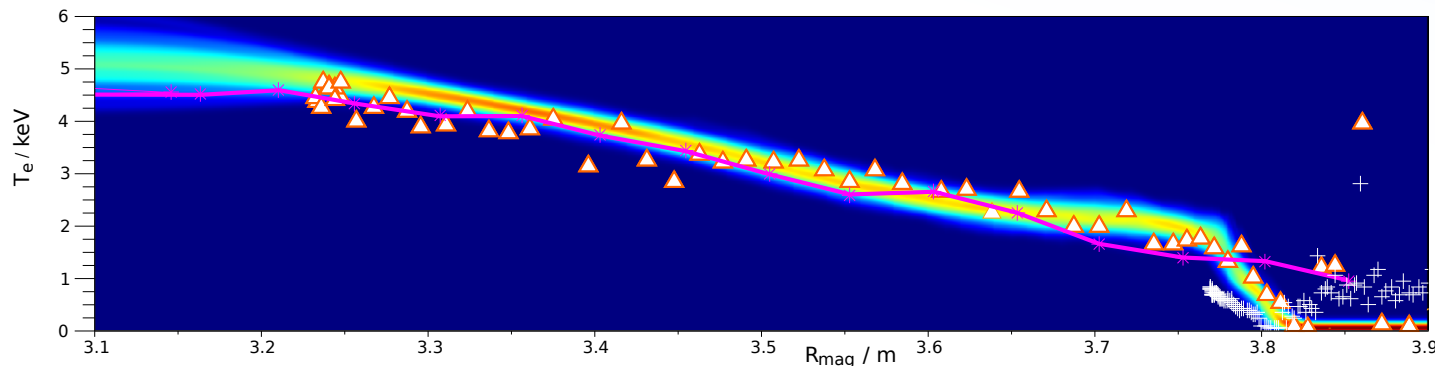
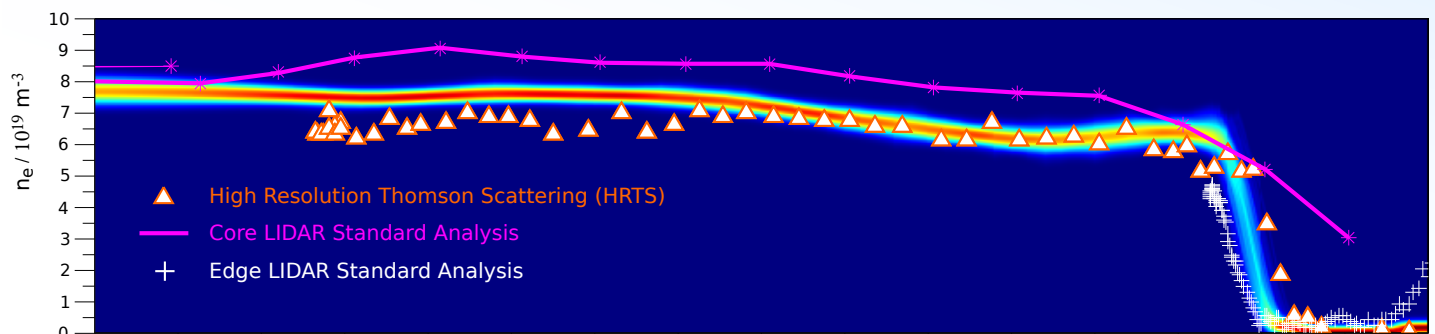
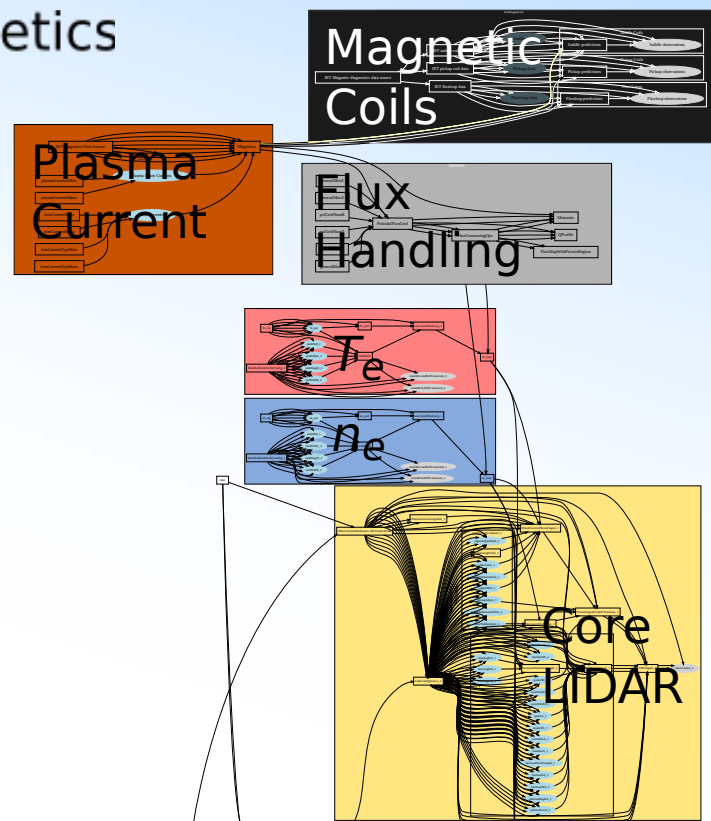
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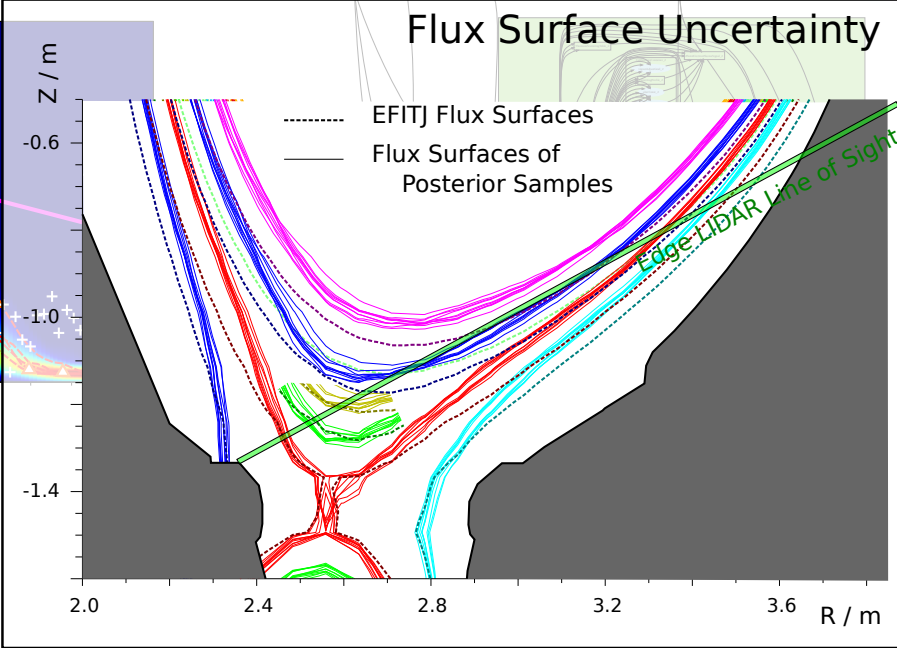
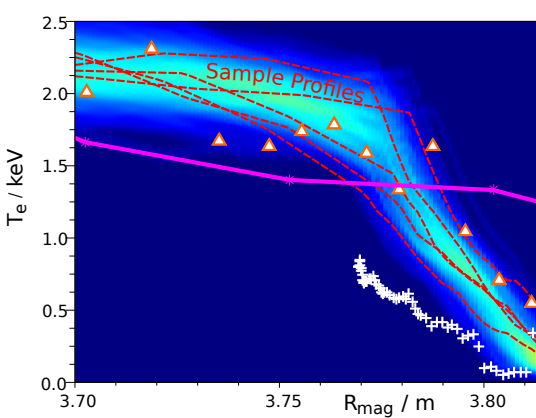
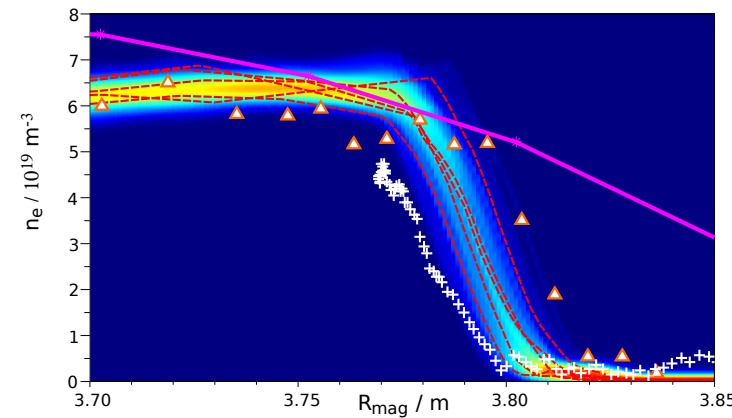
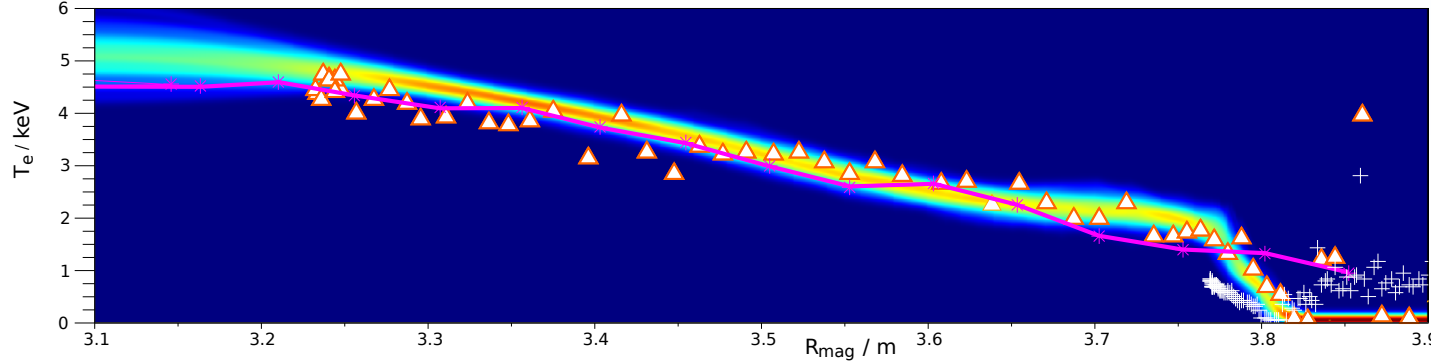
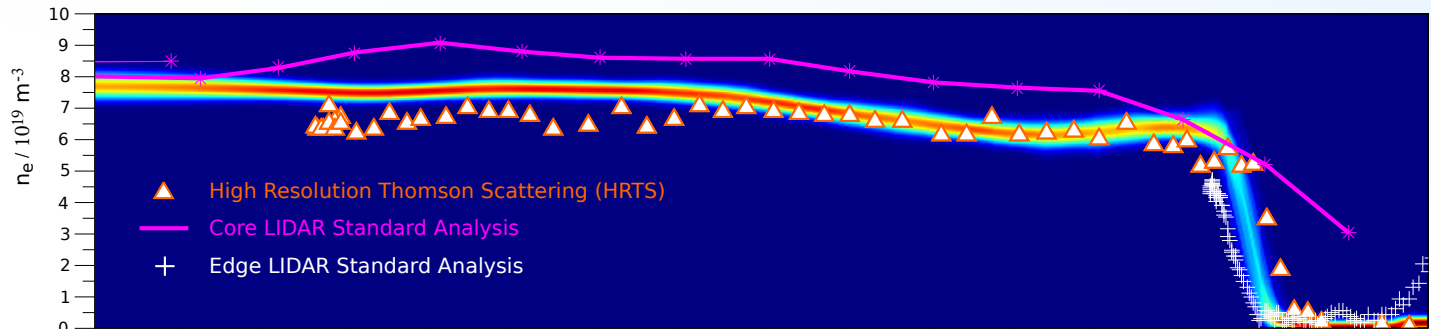
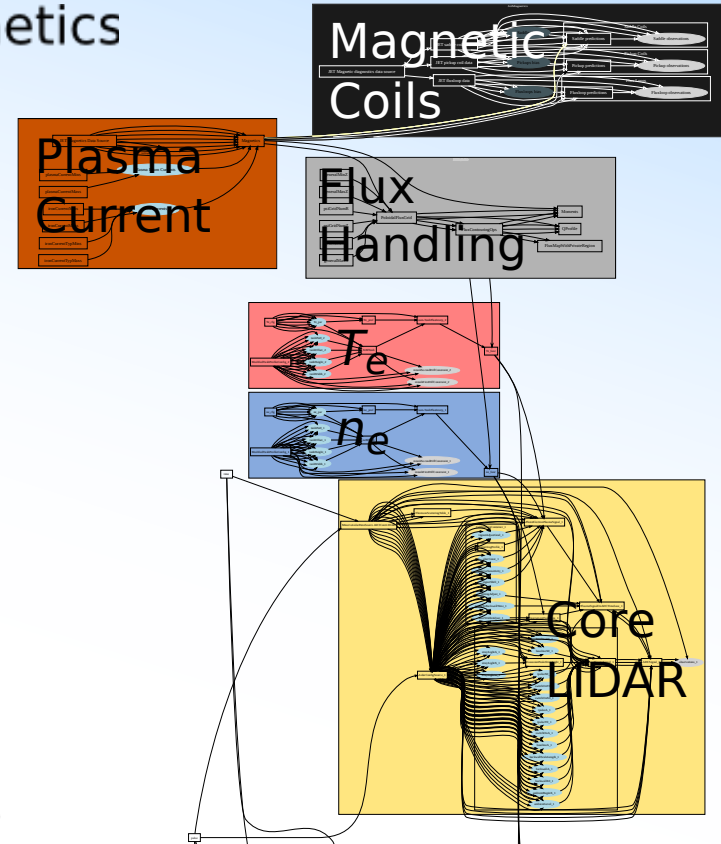
Flux Surface Uncertainty

..... EFITJ Flux Surfaces
—— Flux Surfaces of Posterior Samples

Edge LIDAR Line of Sight

Core LIDAR + Edge LIDAR + Interferometry + Magnetics

Connect magnetics model and run inversion.



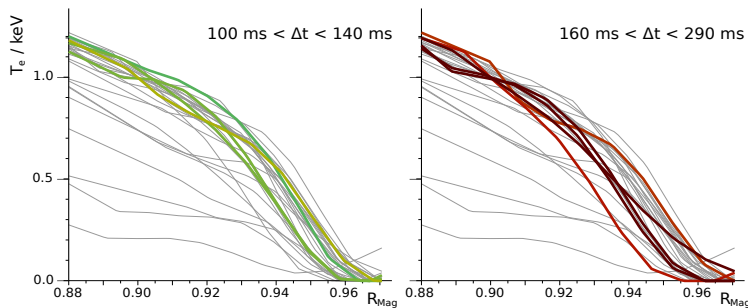
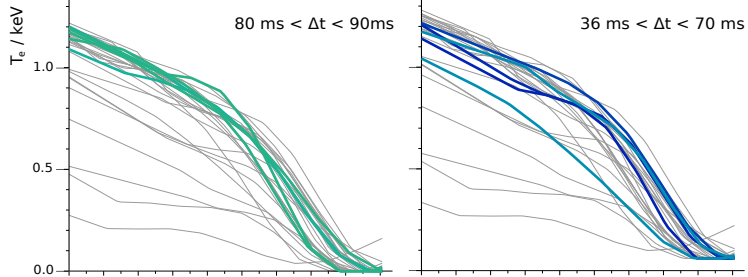
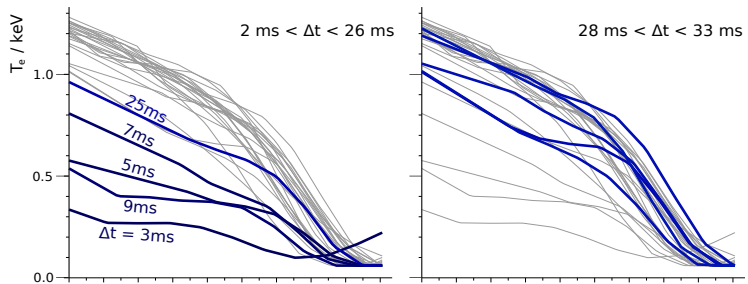
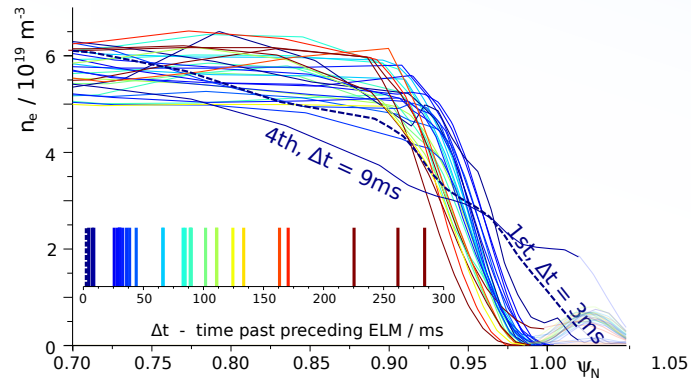
The TS diagnostics provide information on plasma current near line of sight.
Plasma current one of the most important and least diagnosed parameters in Tokamaks.

Core LIDAR + Edge LIDAR + Interferometry: Pedestal Evolution Study

Looked in detail at evolution of ne/Te pedestals through the ELM cycle. 28 time points over 6 almost identical pulses.

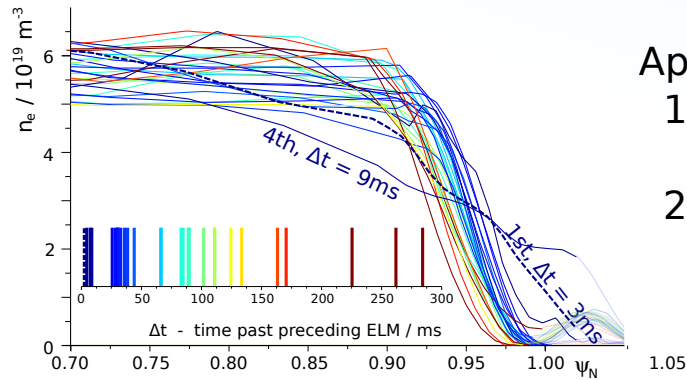
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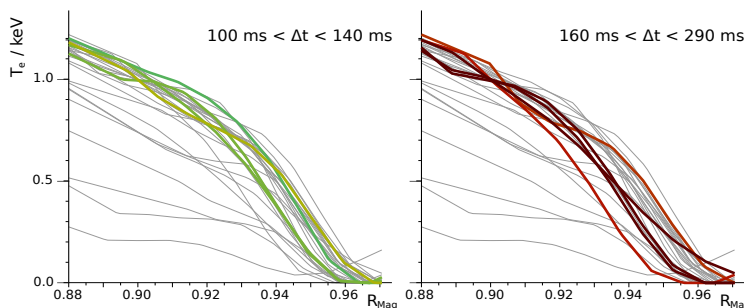
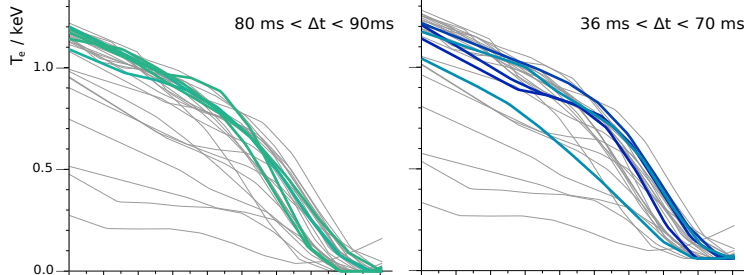
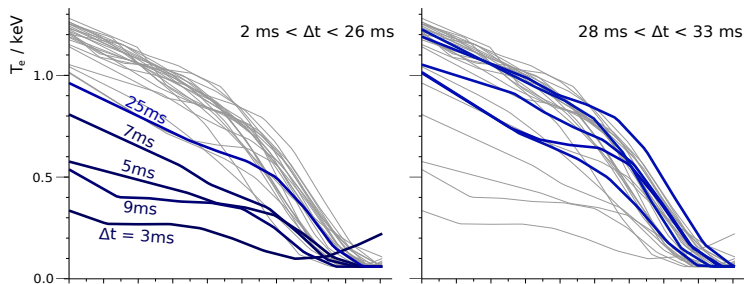
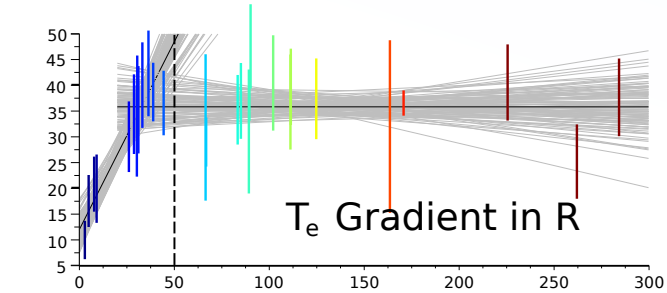
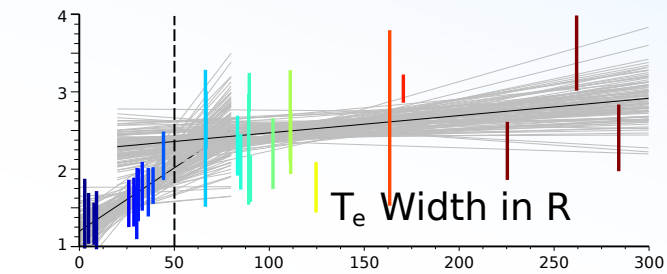
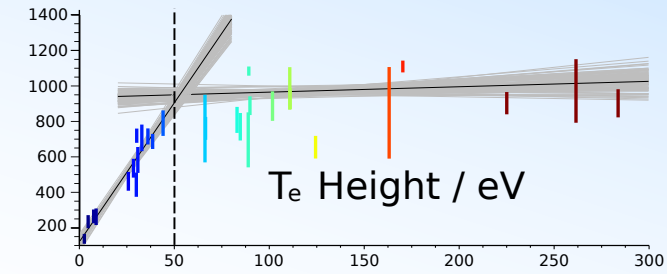


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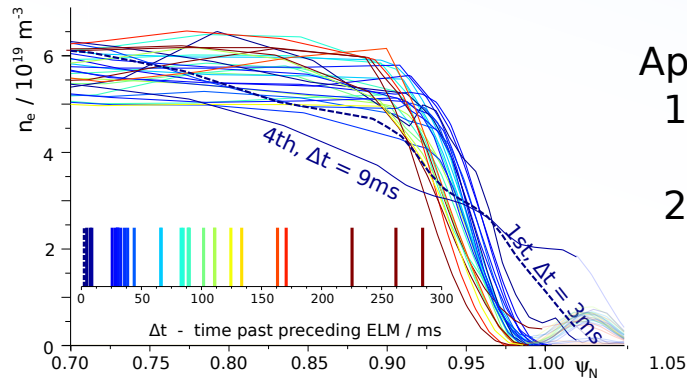


- Appears to be two distinct phases for T_e :
- 1) Rapid rise in height and gradient during first 50ms.
 - 2) Slow rise in height and width at fixed gradient until next ELM.

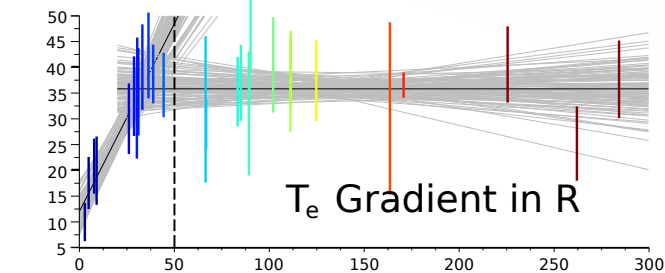
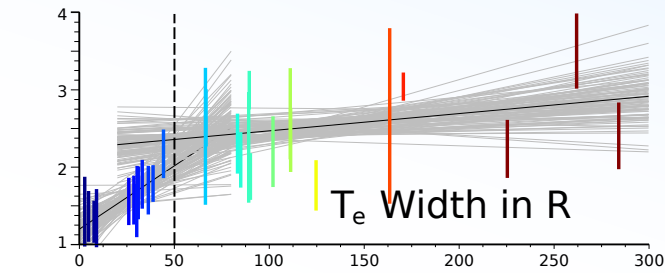
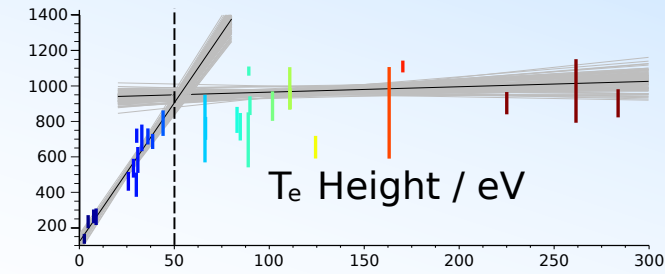


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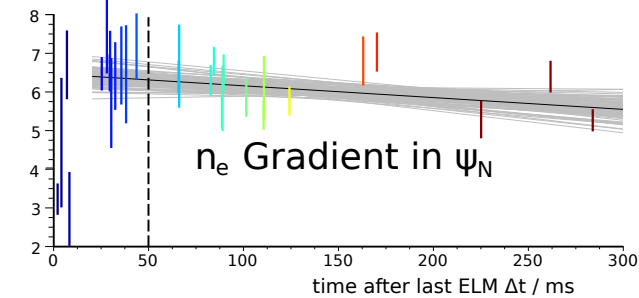
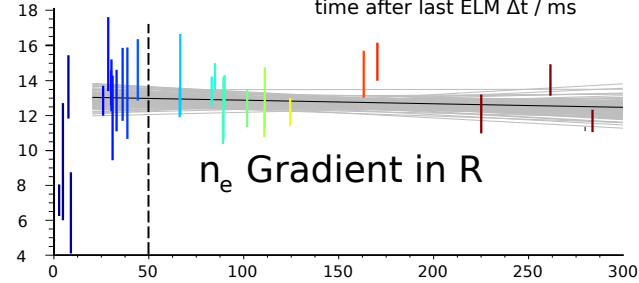
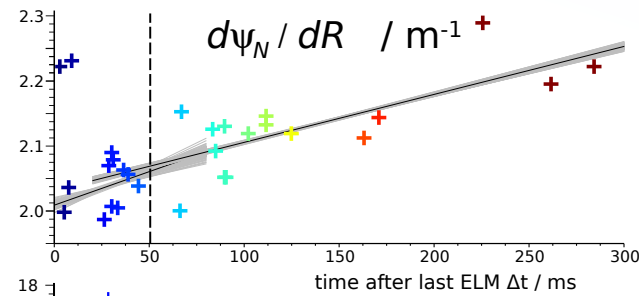
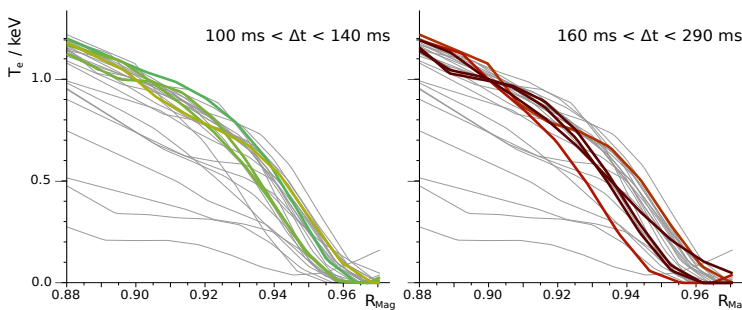
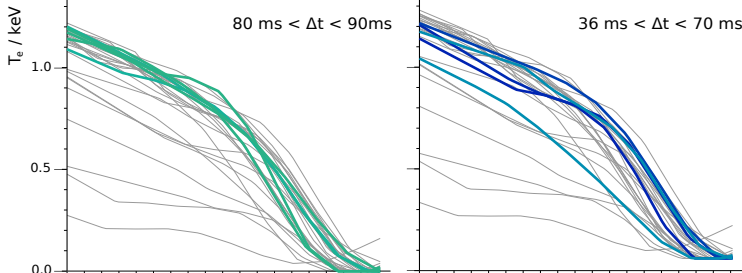
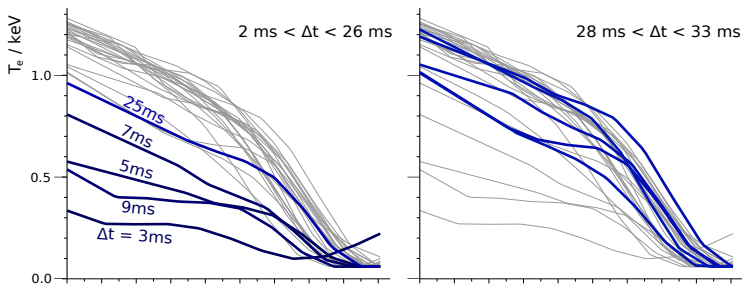
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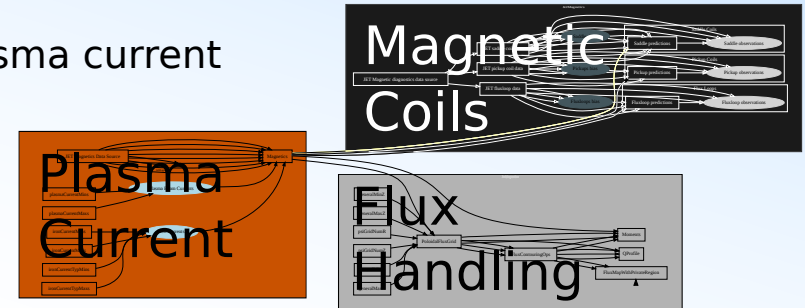
Some indication that n_e pedestal has a fixed gradient in real space despite compression of flux surfaces.



Equilibrium I

Inference of plasma current and flux surfaces $P(\psi_N | \dots)$ is the big problem.

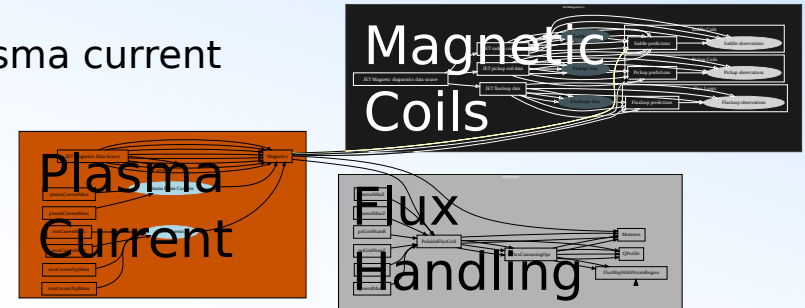
With enough extra diagnostics, it might be possible to infer plasma current accurately, entirely from data.



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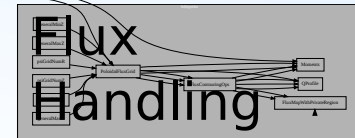
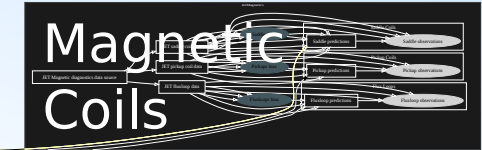
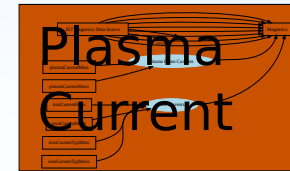
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For now, we can add the prior assumption of Equilibrium.
(Isotropic and no flow)

$$J_\phi = Rp' + \frac{\mu_0}{R} f f'$$

NB: It's not immediately clear how restrictive force balance (GS equation) actually is, since it is almost always used with strong prior constraints on p' (or p - the equilibrium pressure) and ff' (or f - the poloidal current flux). With weak constraints on p' and ff' , the space of possible solutions is still very large.



Equilibrium I

Inference of plasma current and flux surfaces $P(\psi_N | \dots)$ is the big problem.

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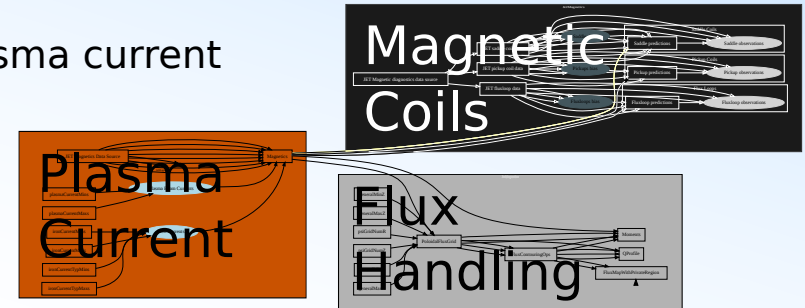
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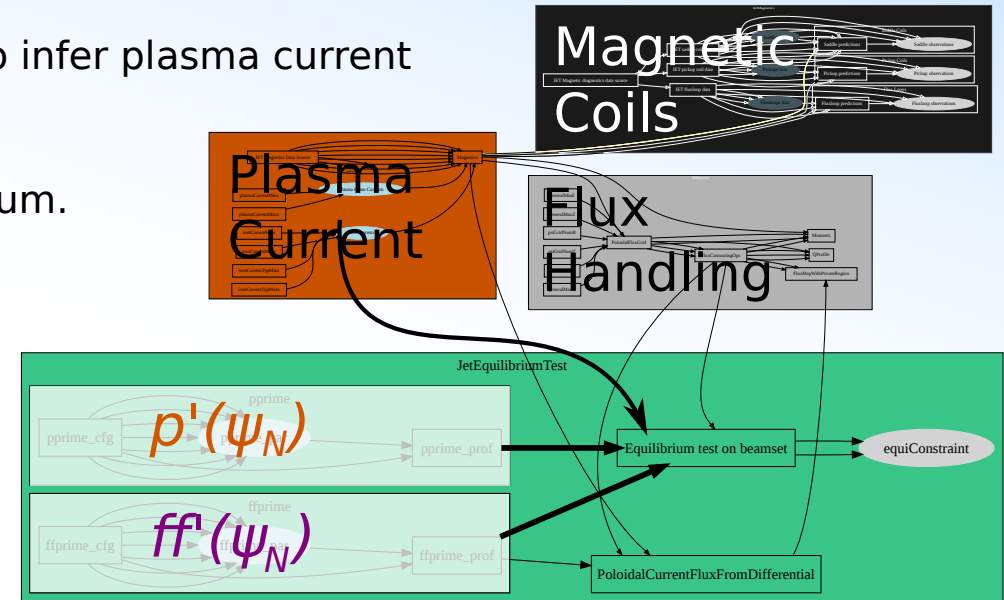
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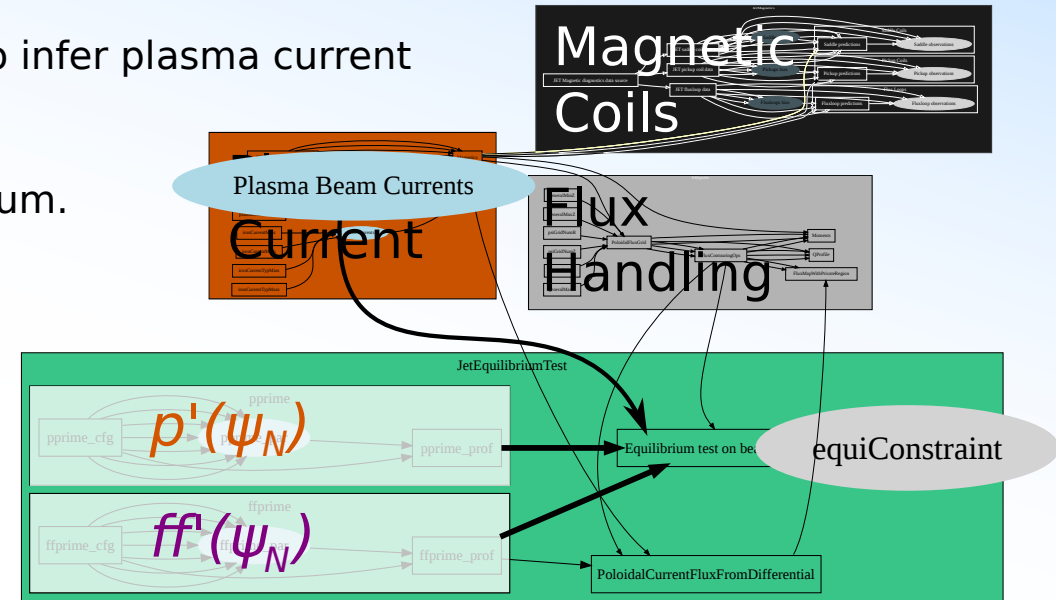
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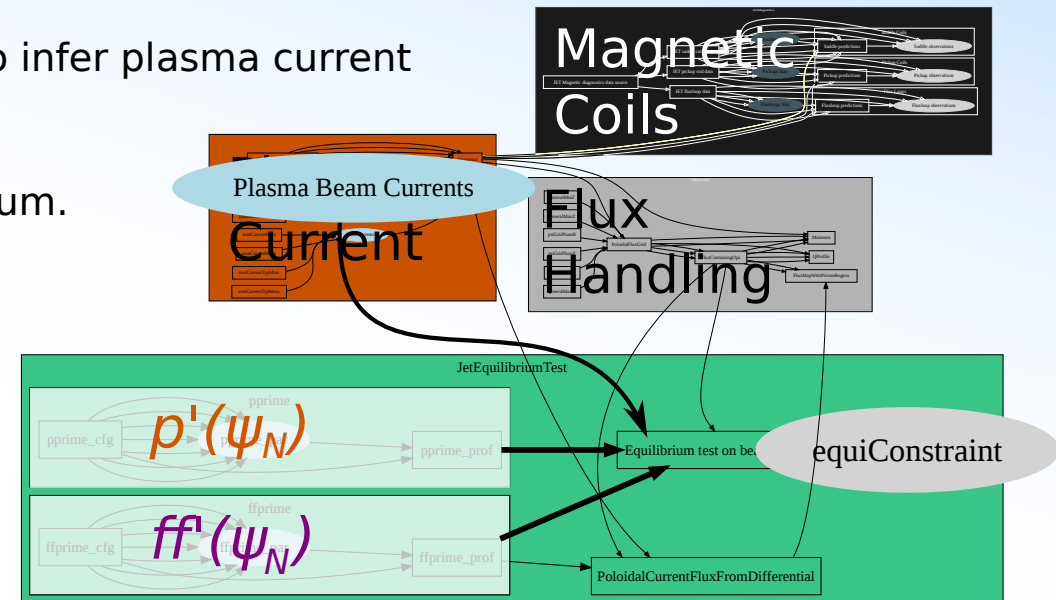
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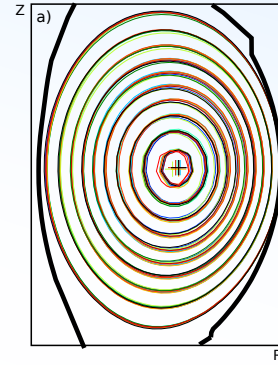
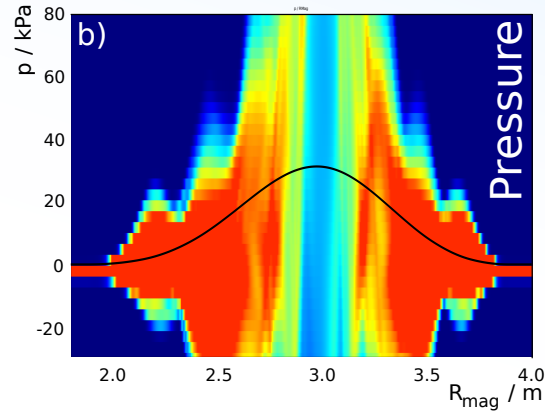
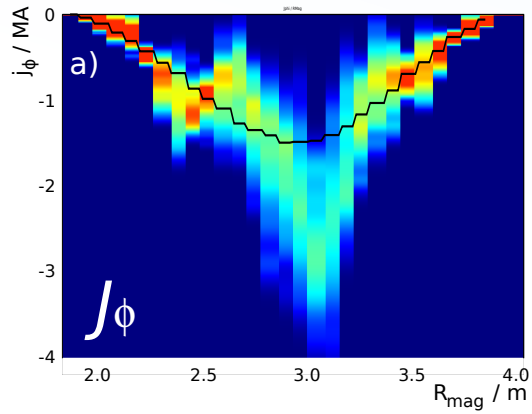
1) Parallelise the linear solver and iterate to find MAP
(slower but more stable than EFIT).

2) Exploring the PDF only just possible for simpler current profile shapes.



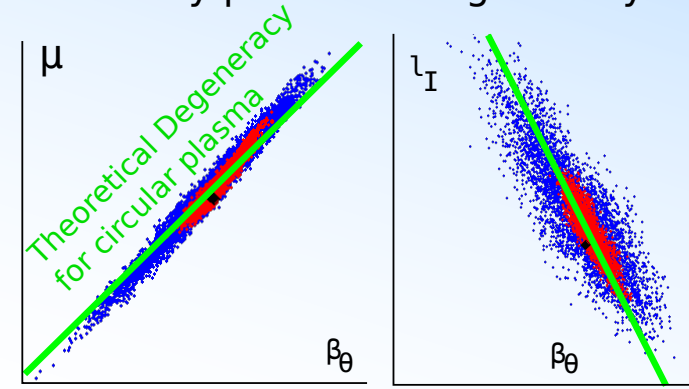
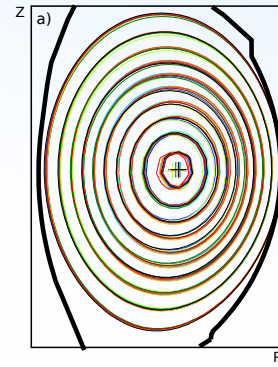
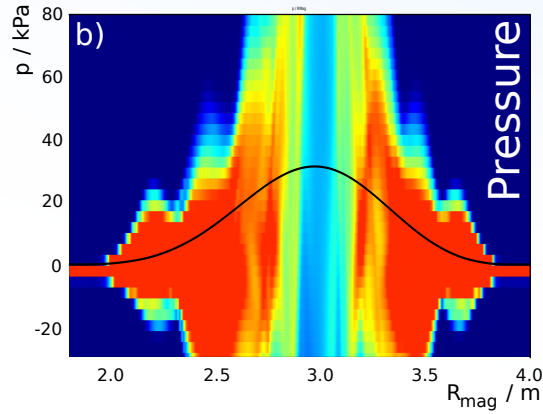
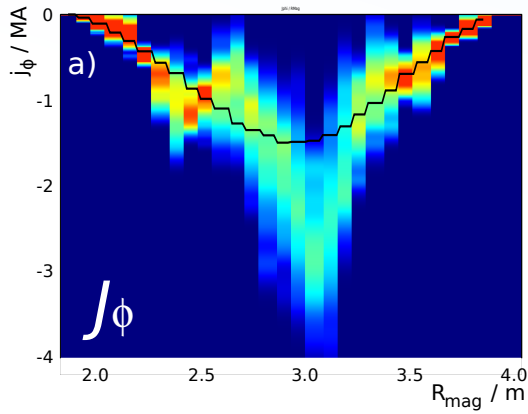
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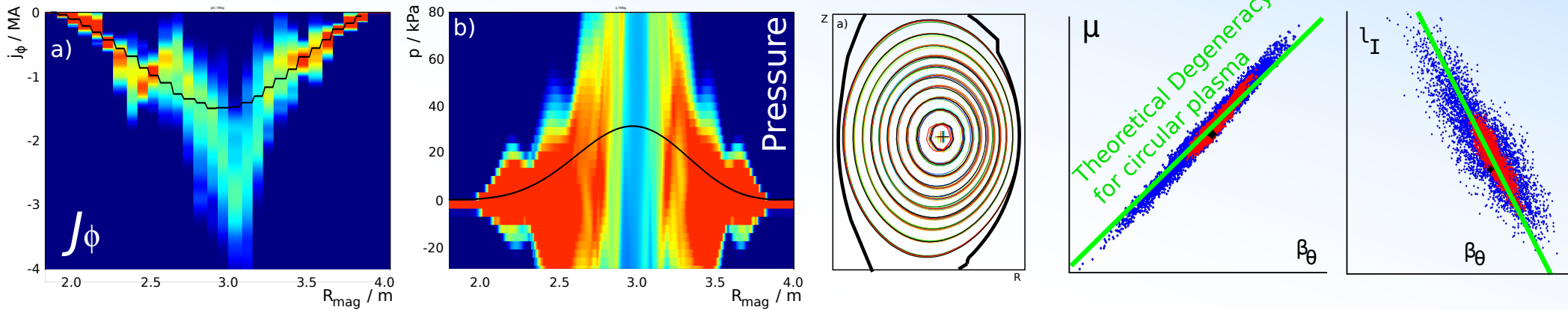
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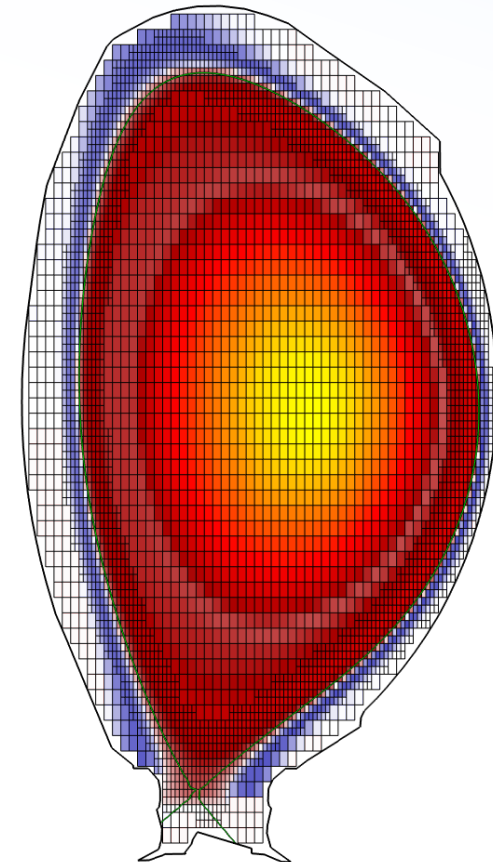
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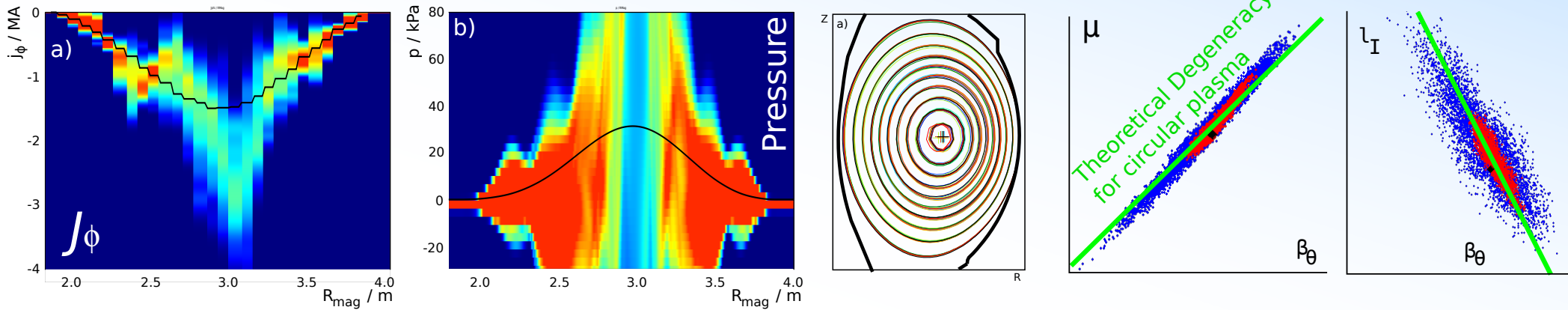
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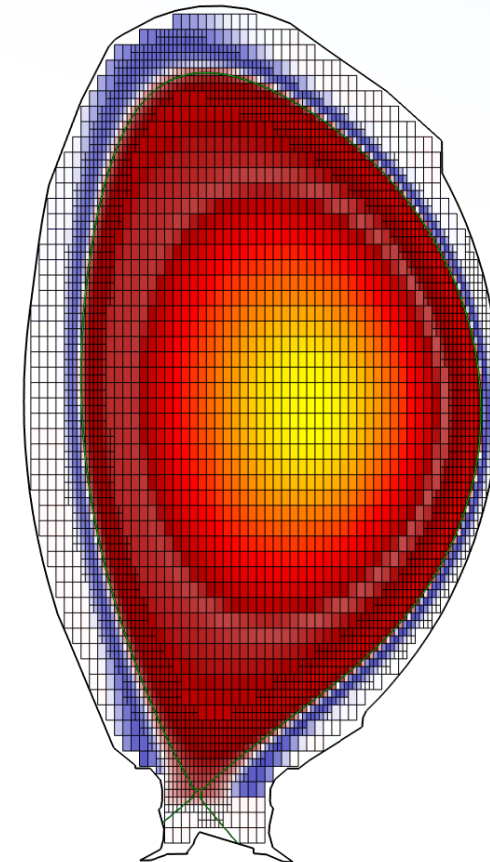
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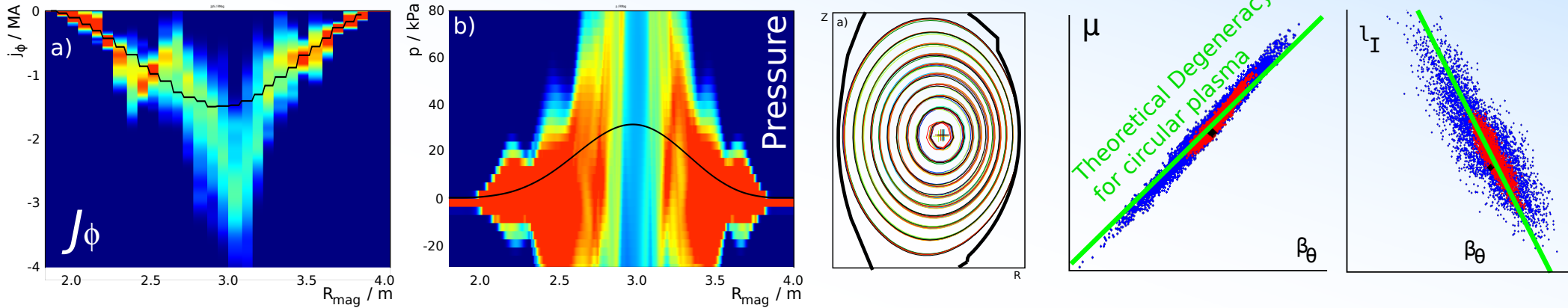
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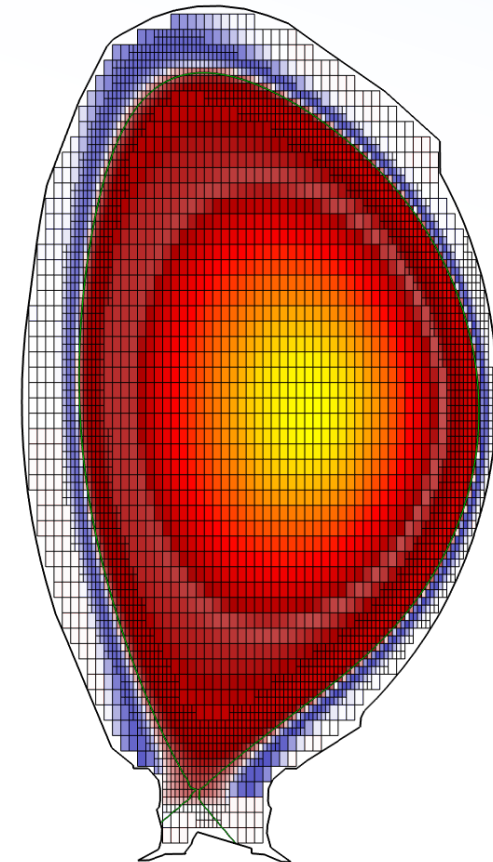
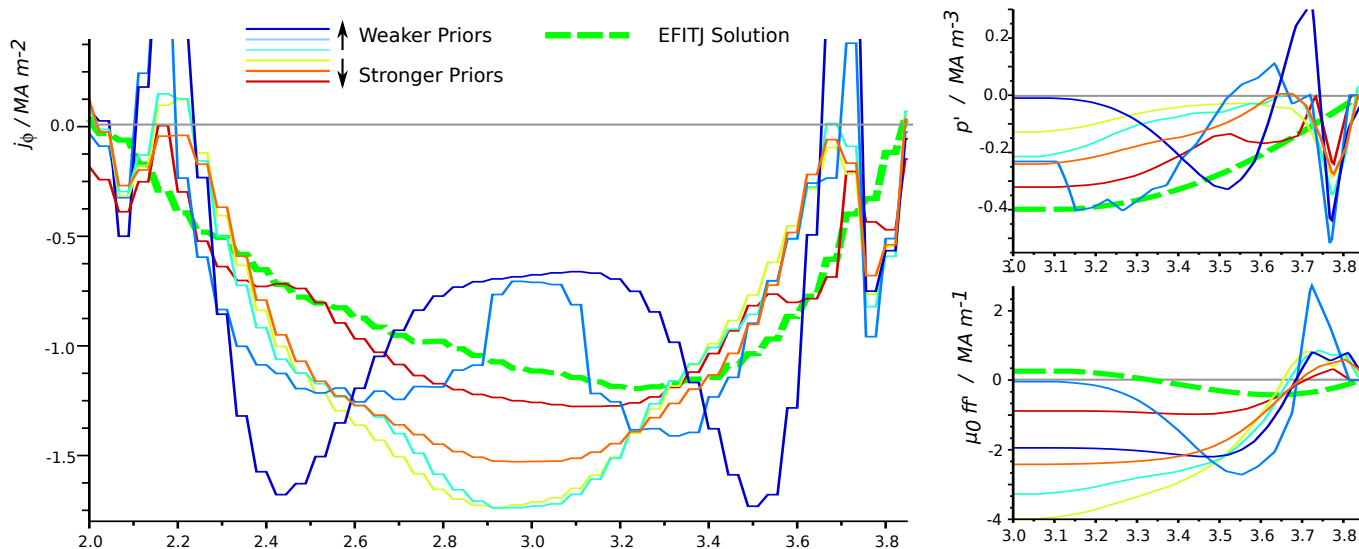
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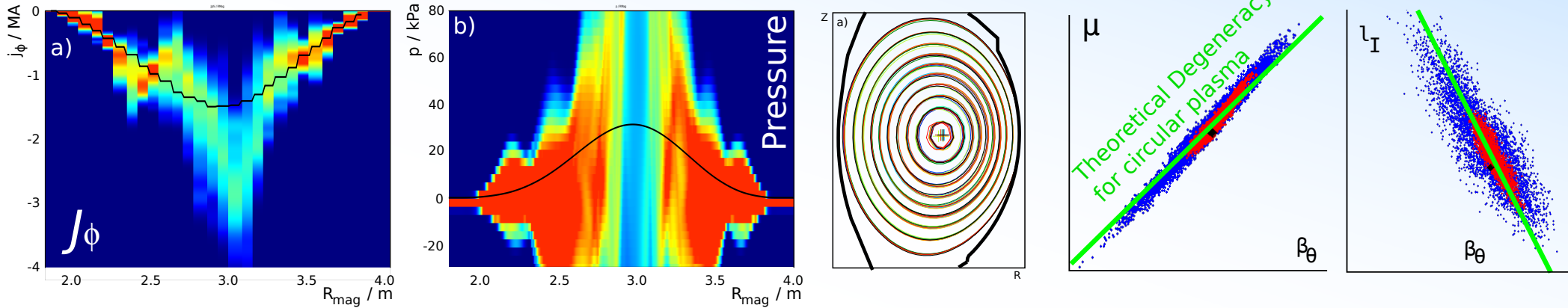
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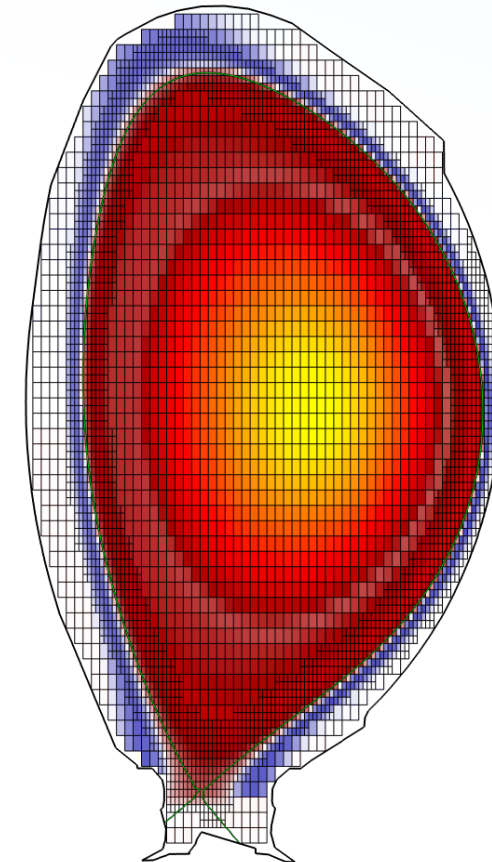
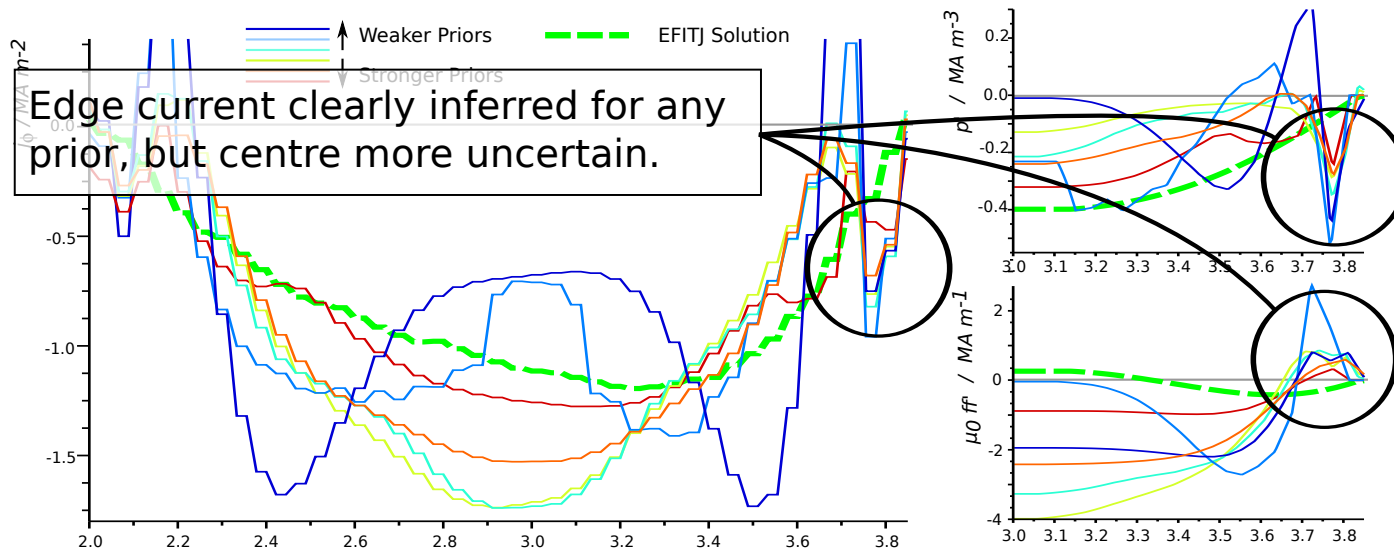
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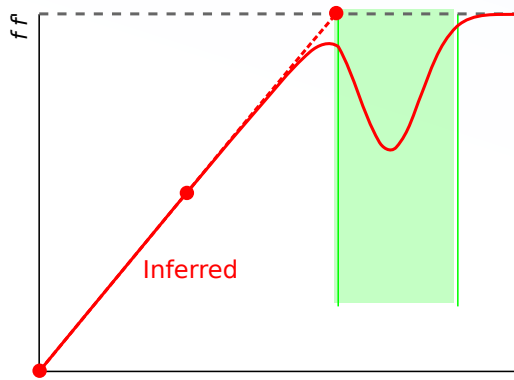


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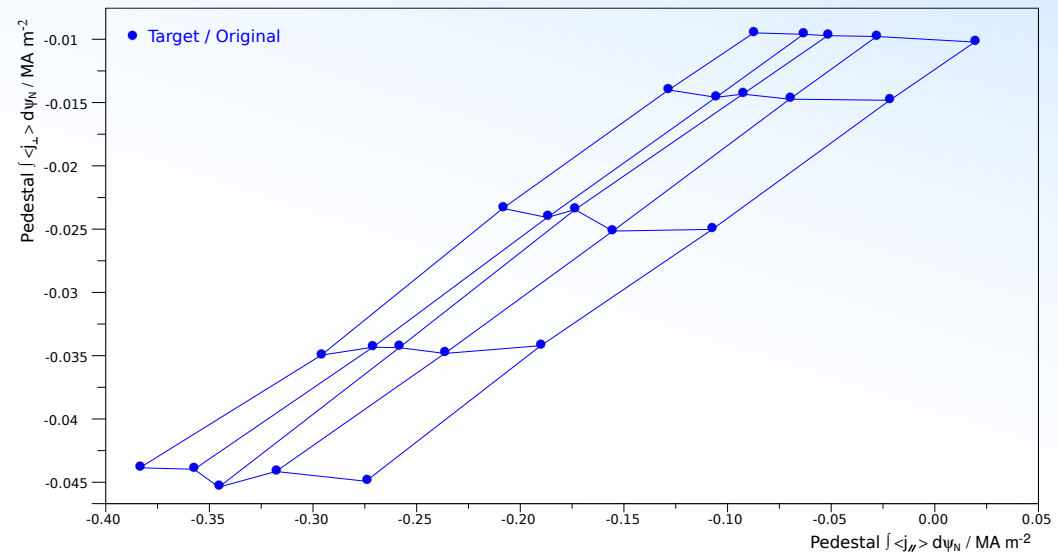
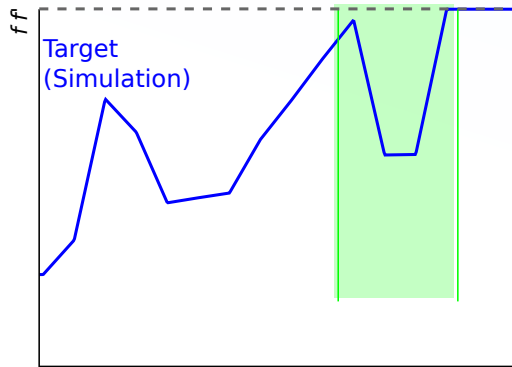
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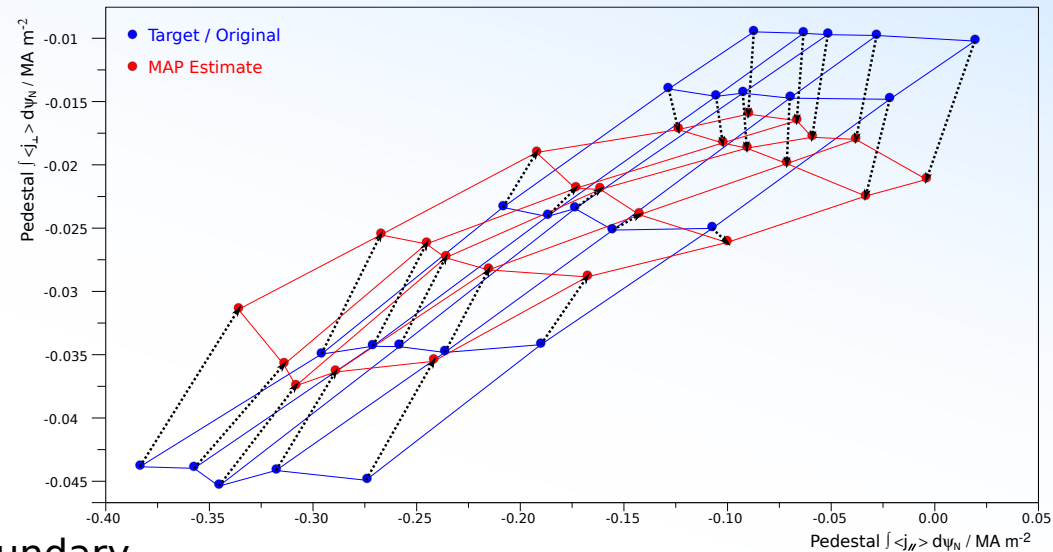
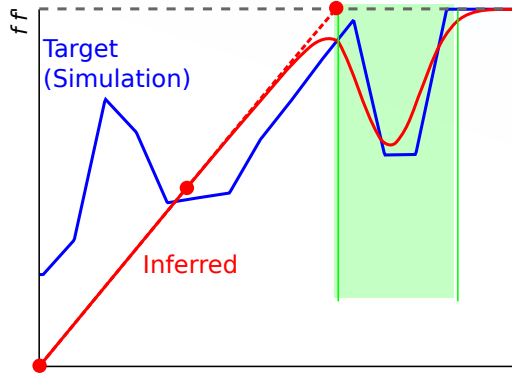
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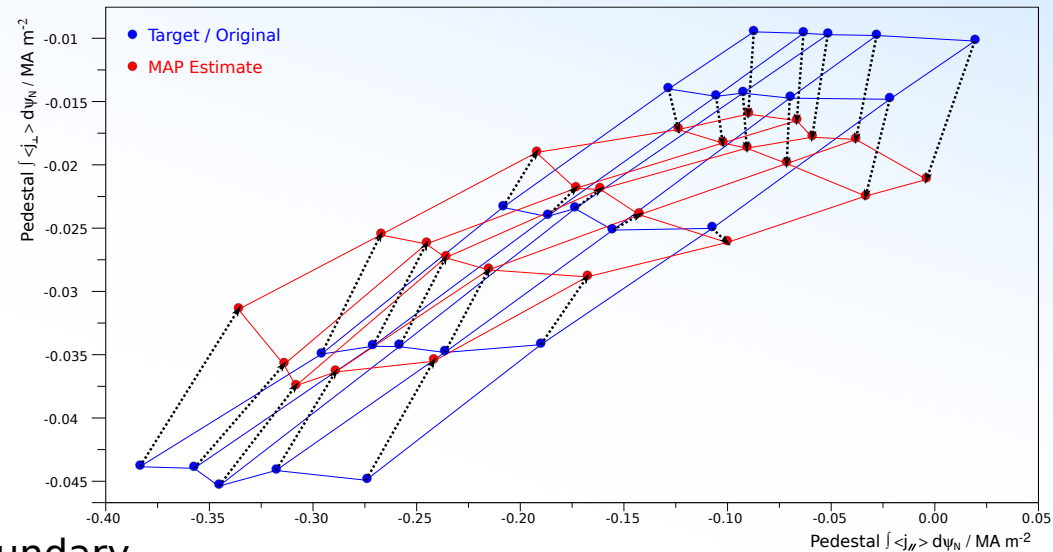
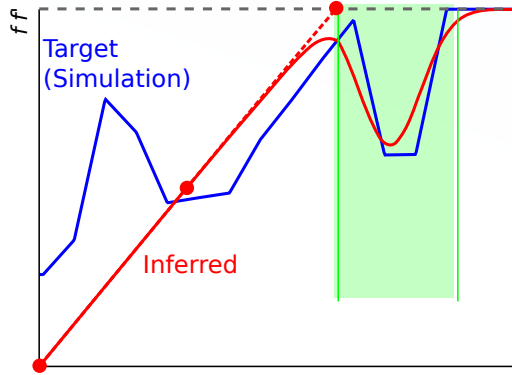
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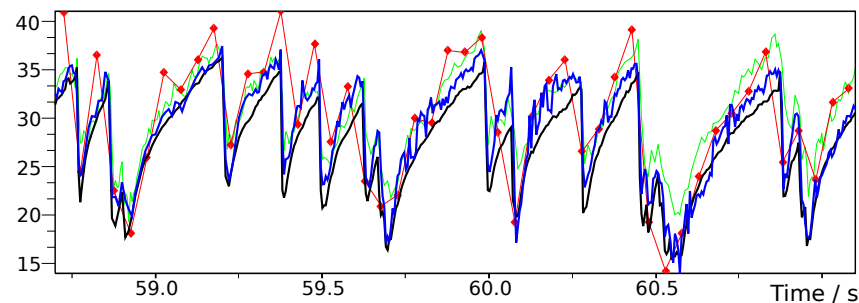
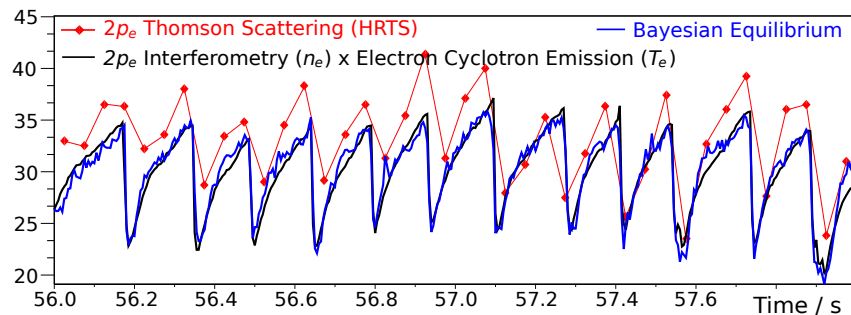


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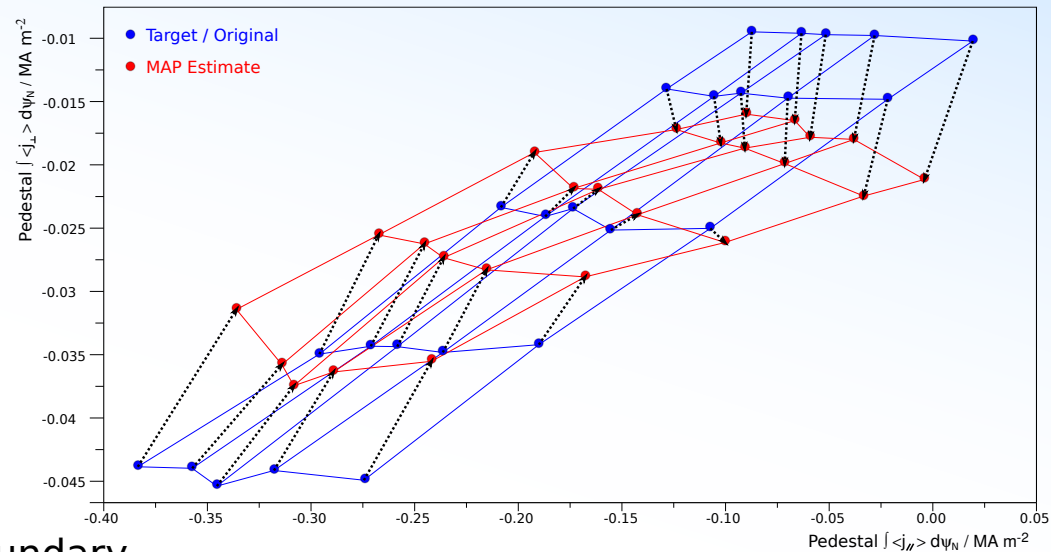
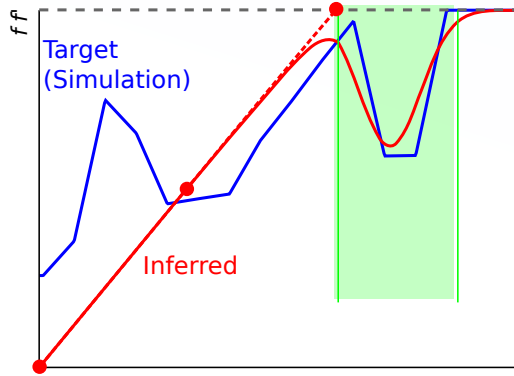
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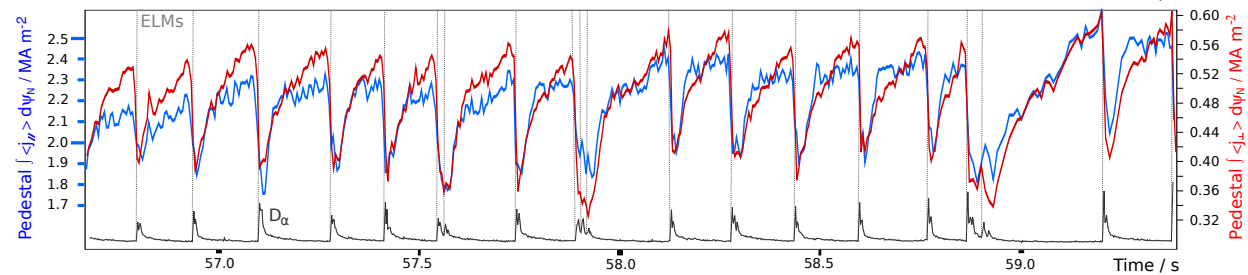
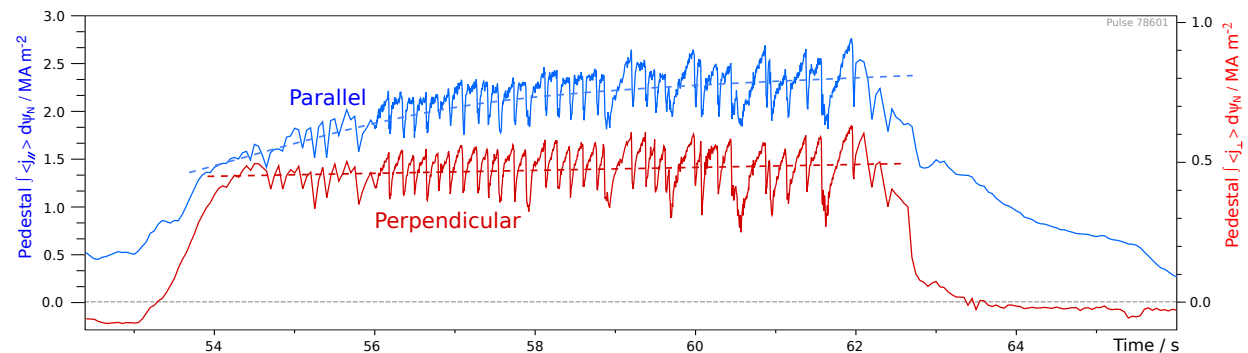
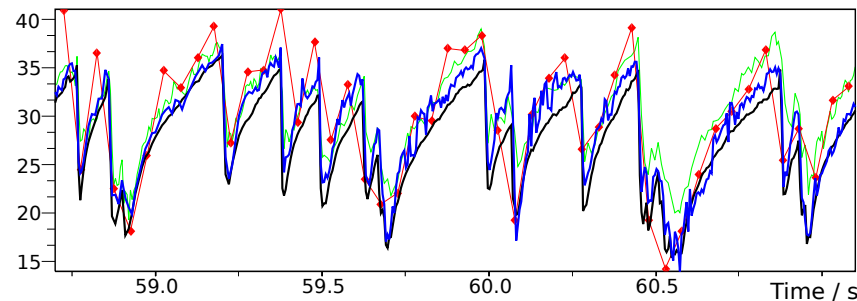
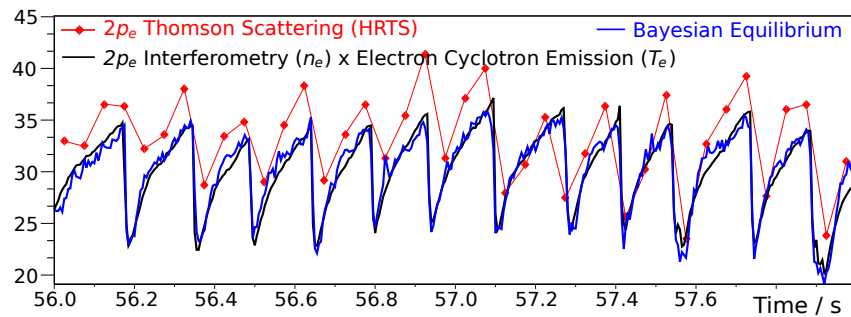


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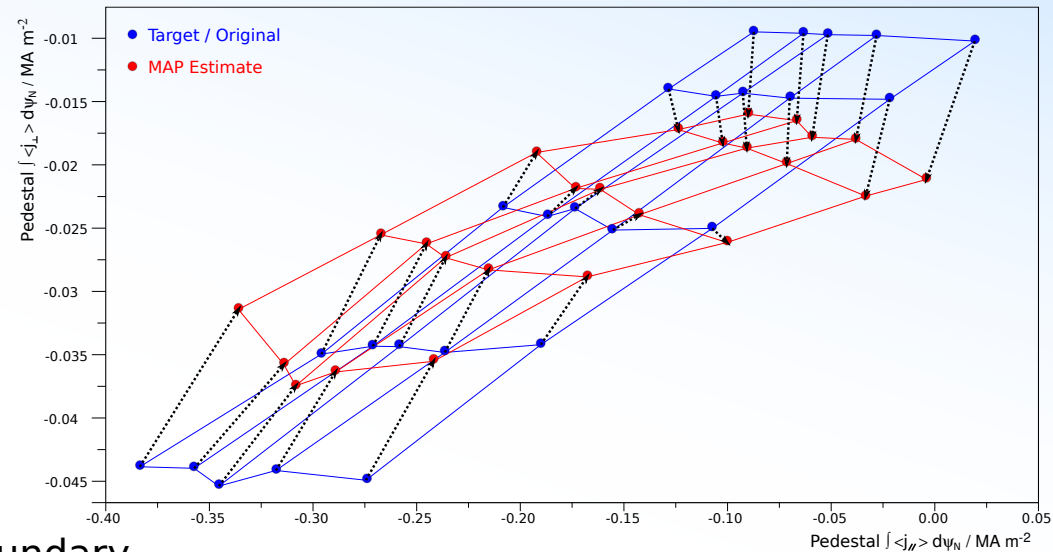
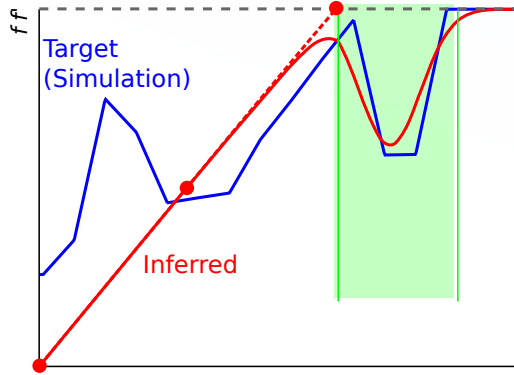
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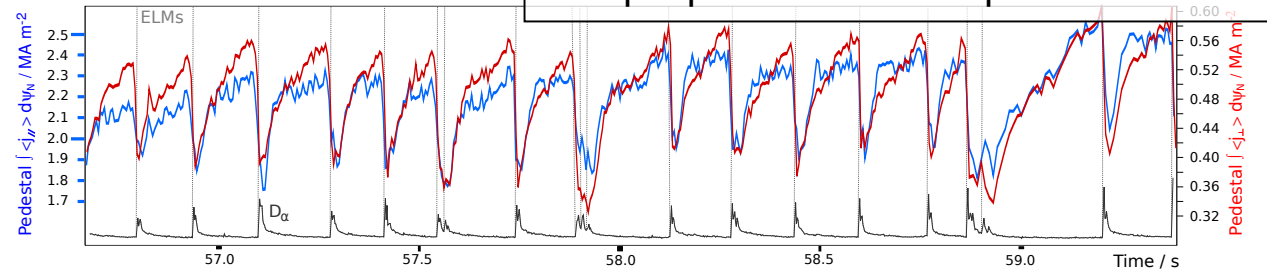
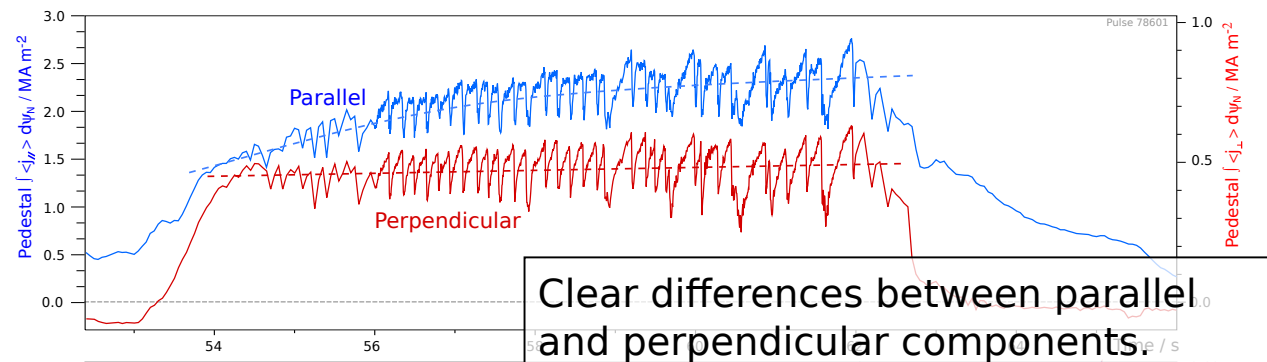
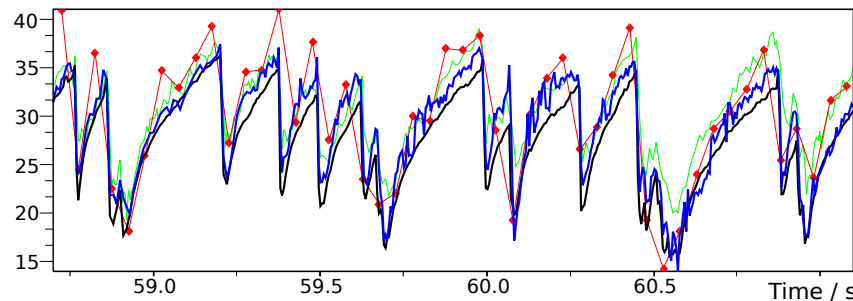
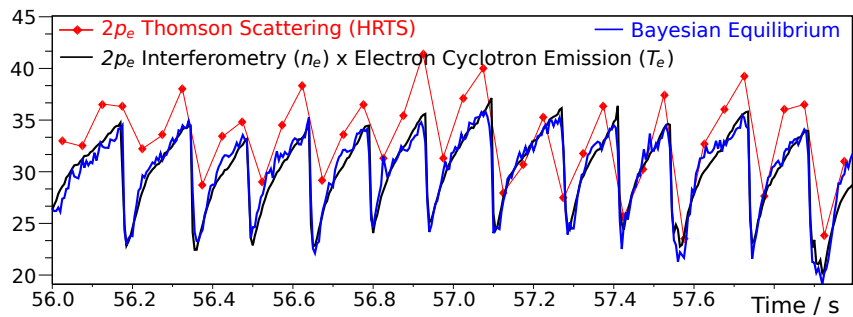


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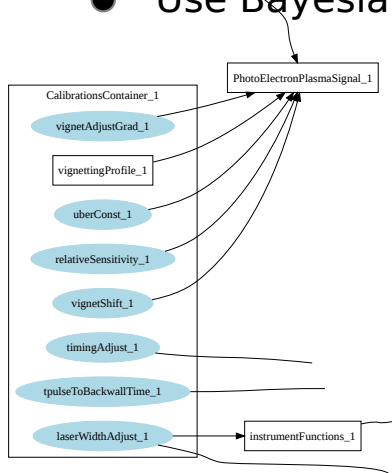
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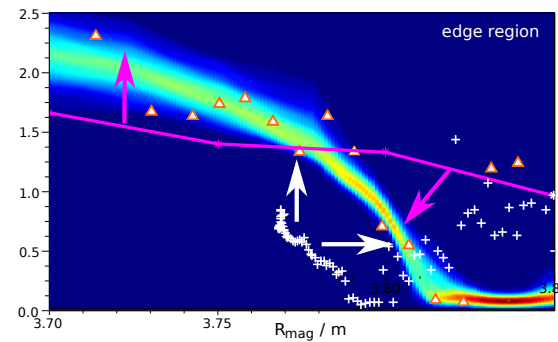
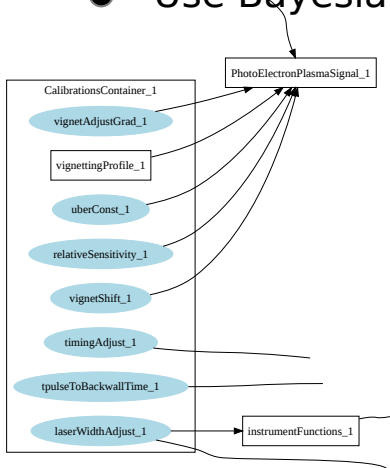
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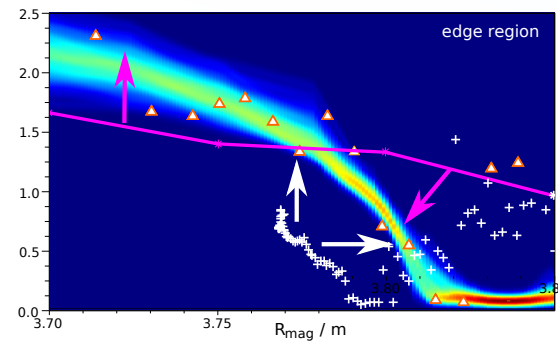
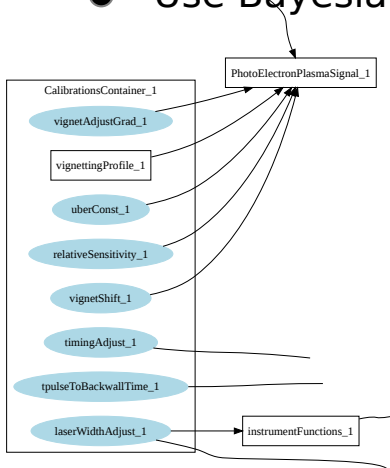


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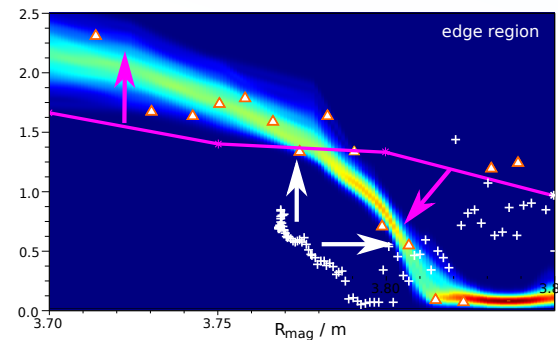
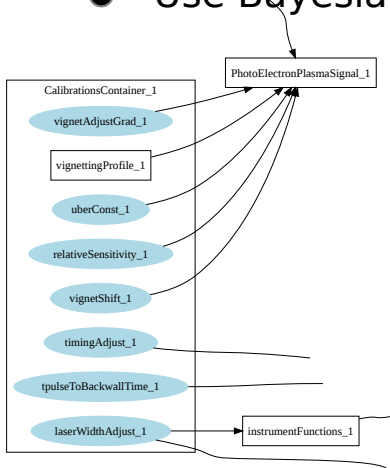


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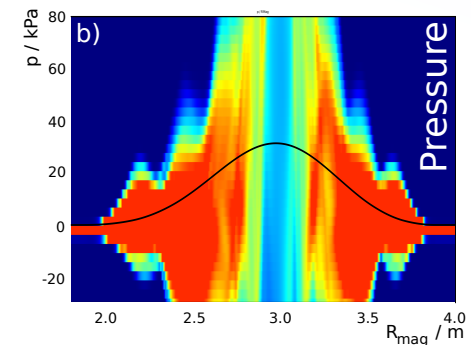
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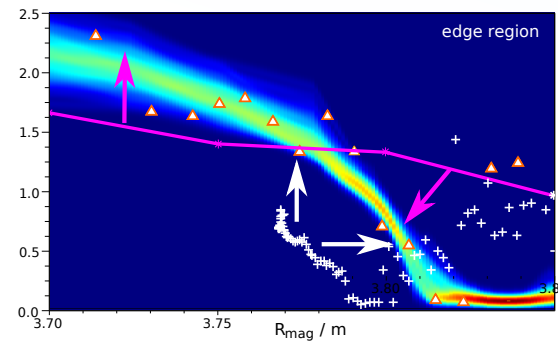
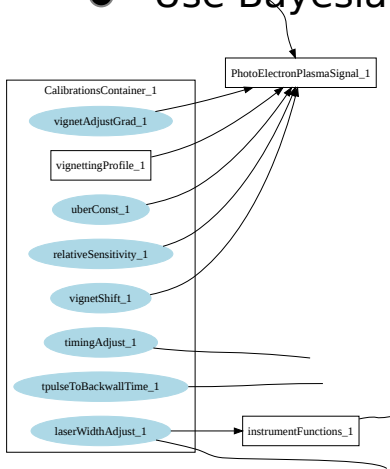
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