Bayesian Analysis of Fusion Plasma Diagnostics.

A review of research completed for Plasma Physics PhD at the Joint European Torus, Culham UK, funded by Imperial College London and the Culham Centre for Fusion Energy.

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Research Plan:

To investigate what information can be extracted from many existing fusion plasma diagnostics at JET, using the analysis techniques of ***forward modelling*** and the principals of ***Bayesian analysis*****.**

1. To infer the plasma state at any instant, making as few as possible assumptions.

2. Achieve a complete and rigorous description of the uncertainty, from:
   diagnostic noise, calibration uncertainty and degeneracy of possible states.

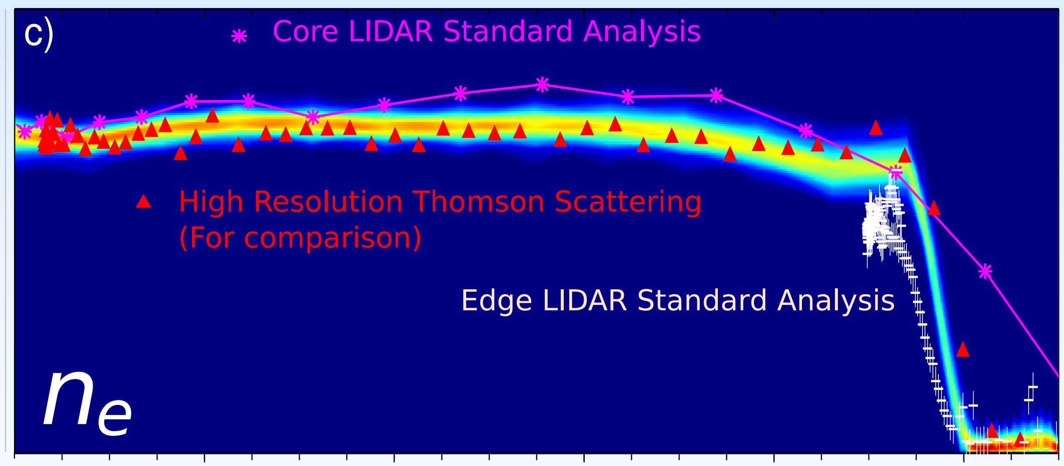
3. To minimise uncertainty by consistently combining data from multiple diagnostics.

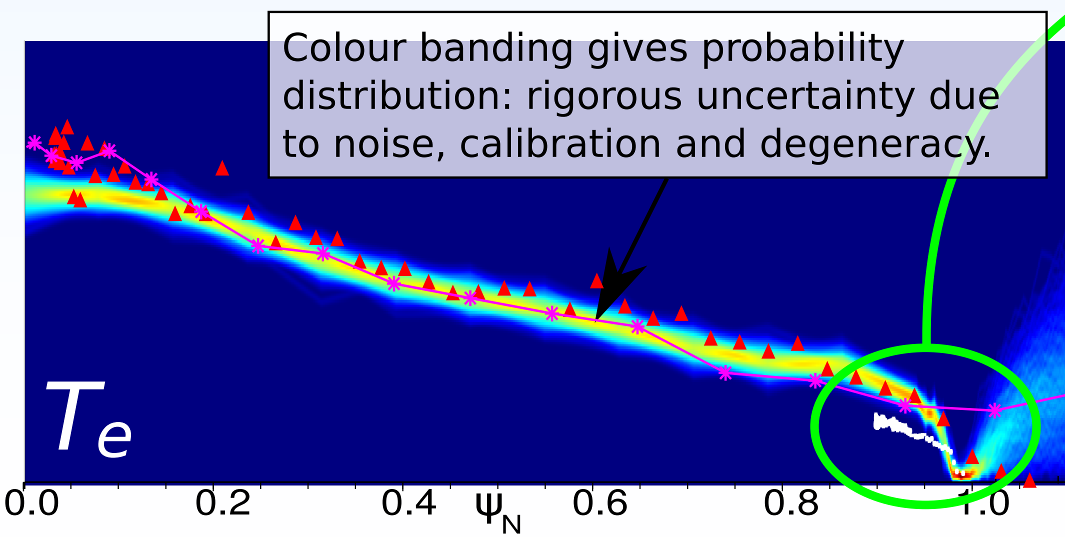
Interferometry + Core LIDAR + Edge LIDAR - Consistent combination of data.

Multiple n_e, T_e diagnostics are available on most Tokamaks and several of these on JET. These quantities are used for interpretation of many other diagnostics, for transport and confinement analysis, pedestal studies, equilibrium constraints, stability analysis, edge modelling and many other Tokamak physics investigations.

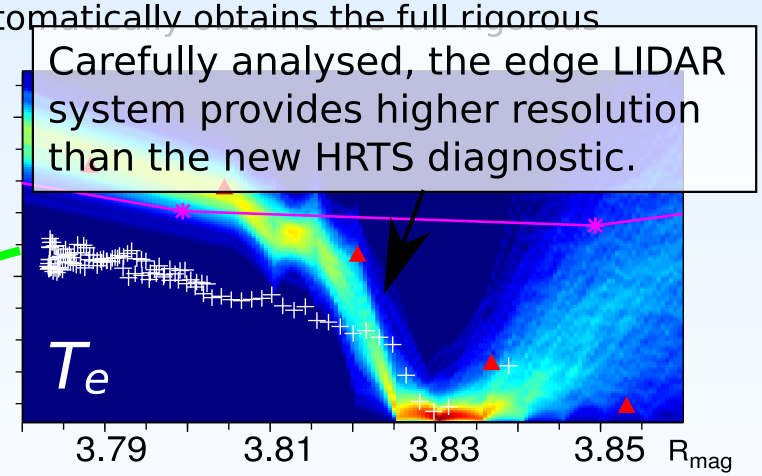
Each diagnostic has strengths and weaknesses. Some calibrations are very accurate, other less so or are unknown. Disagreements and inconsistencies consume a lot of time and systematic uncertainties complicate physics analysis.

With careful modelling of each, diagnostics can be combined easily to infer the most possible information from each.





General genetic and Monte-Carlo algorithms find all possible plasmas and calibration states consistent with data. This automatically obtains the full rigorous uncertainty.



Distribution gains high-resolution edge information from edge LIDAR, accurate T_e calibration from core LIDAR, and absolute n_e information from interferometry.

"*Bayesian Combined Analysis of JET LIDAR, Edge LIDAR and Interferometry Diagnostics*"
P2.150, 36th EPS Conference on Plasma Phys. (2009)

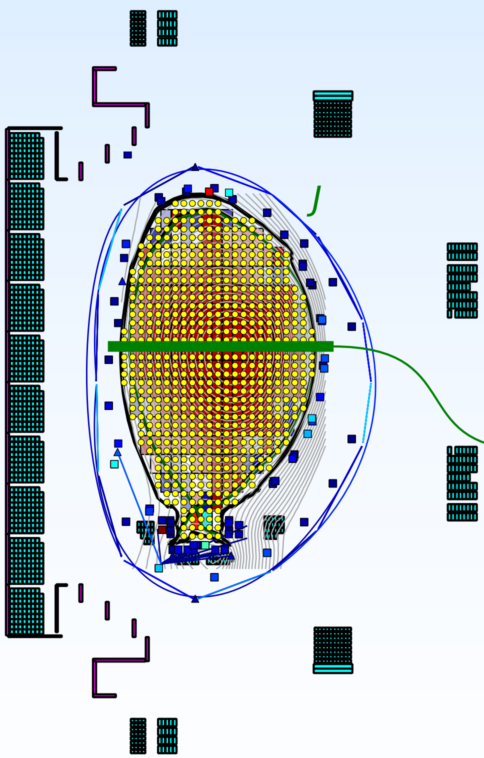
Final result will include uncertainty from uncertain flux surfaces/equilibrium.

Current Tomography and Equilibrium

The inferred plasma current and magnetic field topology effects also all Tokamak experimental physics, from mapping of other quantities (assumption of constancy on flux surfaces) through to ELM models and stability analysis.

For many, the uncertainty in B, ψ_N etc would probably be large factor but normal equilibrium codes rarely calculate the uncertainty from the data and even less, the due to any degeneracy in the solutions, which is removed with assumptions instead.

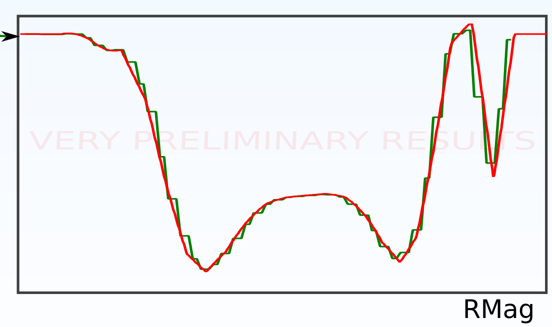
The Bayesian approach, 'Current Tomography': [J. Svensson et. al. PPCF **50** 085002 (2008)].



- Parametrise plasma current (Uniform beams, 2D linear/cubic interpolation etc)
- Predict B, ψ_N, magnetic sensor data etc.
- Linear inversion to distribution of all possible plasma currents.

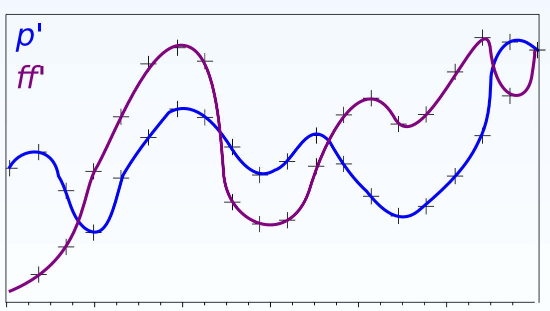
In this research:
- Parametrise pressure p or $p'(\psi_N)$ and poloidal current f as $ff'(\psi_N)$, but do not apply rigid functions - instead add *weak prior distributions* (smoothing etc).

- Add probabilistic comparison of either side of Grad-Shafranov equation over (R,Z) plane:



$$P\left(J_\phi \stackrel{?}{\approx} Rp' + \frac{\mu_0}{R} f f'\right)$$

(For now: Assume isotropic pressure and low flow)



The problem (now a non-linear 1000D+ distribution) is difficult for the algorithms to handle.

- Parallelise the linear solver and iterate to find most probable answer.
- Parallelise MCMC algorithms and explore the posterior.

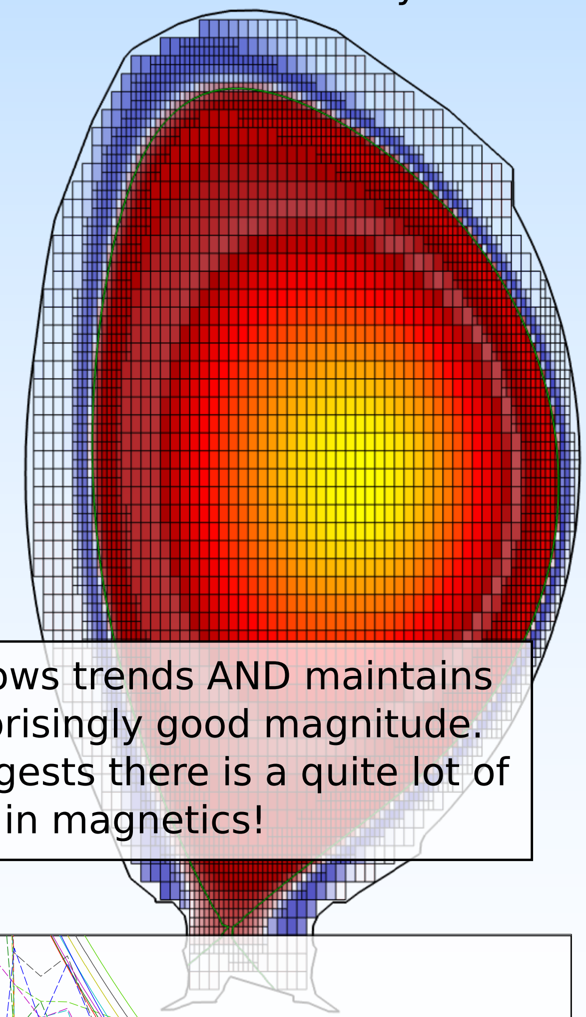
Imperial College London

Magnetics and Equilibrium: Extraction of information.

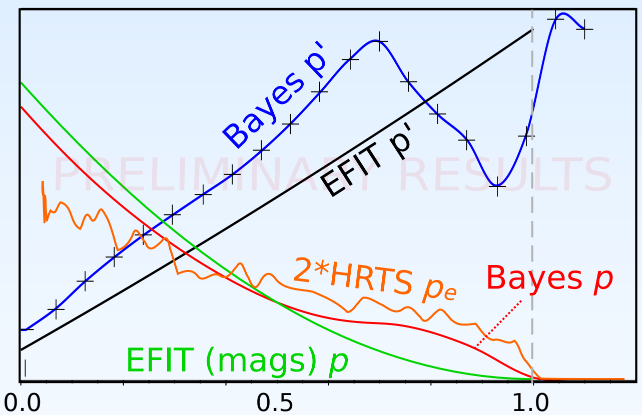
78601 High ne H-Mode (pellets)

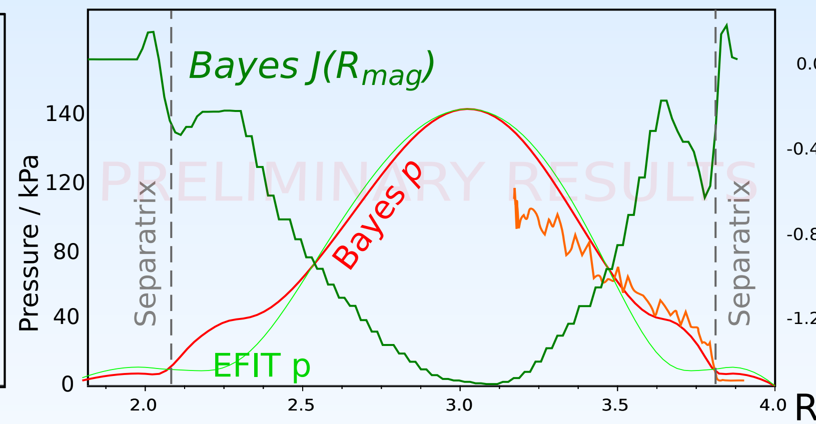
Because of modularity, we can switch parametrisation and priors of *J*, *p'* and *ff'* at will and on-the-fly.
For H-Mode, fast changes at edge so:

- J_ϕ: Current beams with higher resolution near edge (~**1cm**, ~5cm in core).
- $p'(\psi_N)$, $ff'(\psi_N)$: 20 knots, weak smoothing priors.
  Assume small in SOL (but not fixed to 0)



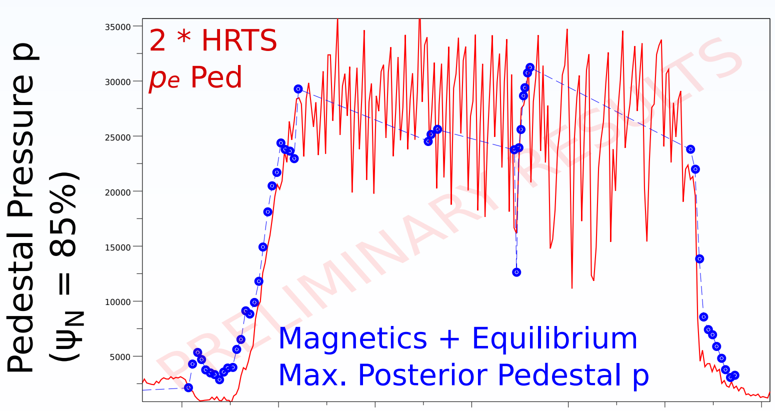
As suspected, with such weak priors a huge range of plasmas are possible.
Adjust *p'* and *ff'* priors to get something sensible **for 1 time slice**:

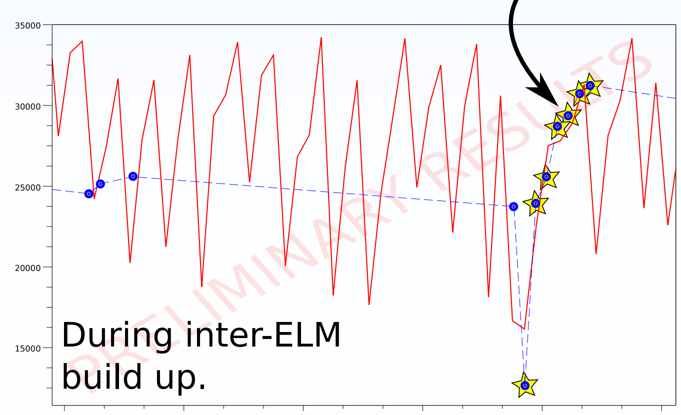


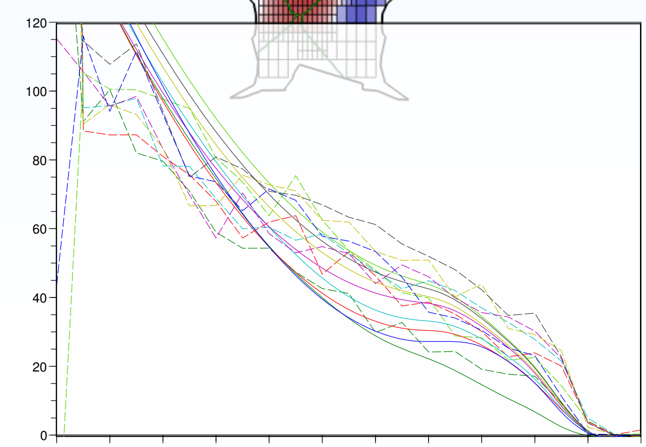


Follows trends AND maintains surprisingly good magnitude.
Suggests there is a quite lot of info in magnetics!

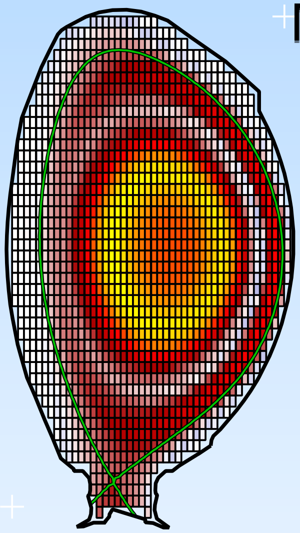
Magnetics data seems to see edge current (and hence some *p'*).
Exact shape does depend on priors - information is weak.
But... **Hold priors** and run across H-mode pulse. Is there any vague trend?







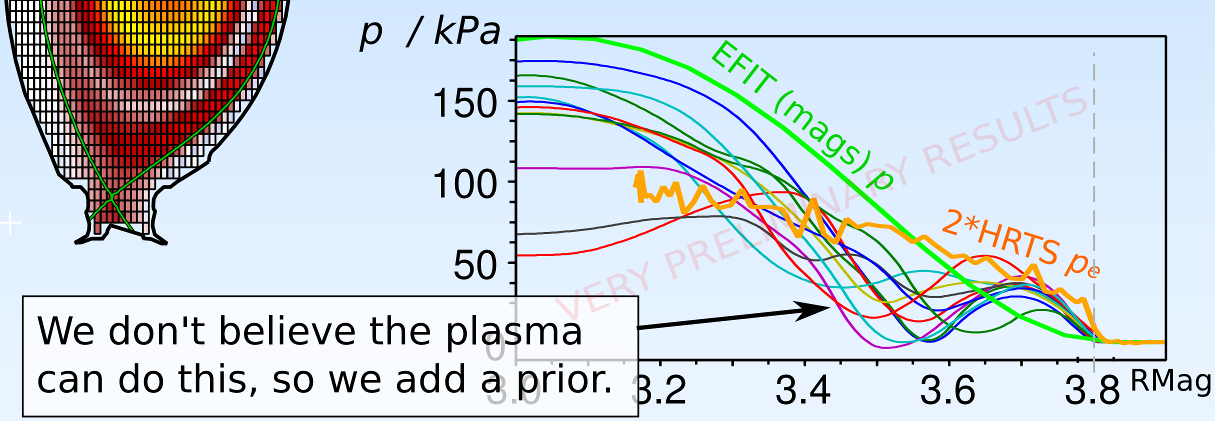
Magnetics and Equilibrium Exploration: Equilibrium uncertainties.

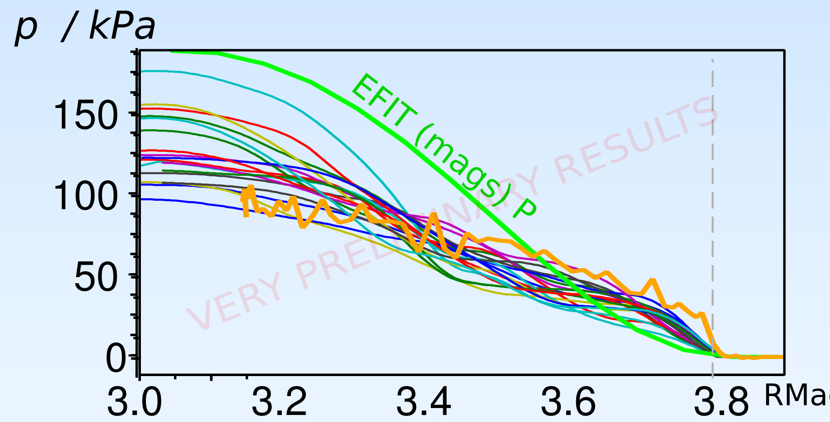


78601 High ne H-Mode (pellets)

Explore the PDF ***P(*** *J*, *p*', *ff*' | *Magnetics, equilibrium, priors* ***)***... (Reduced beam resolution to 5cm).

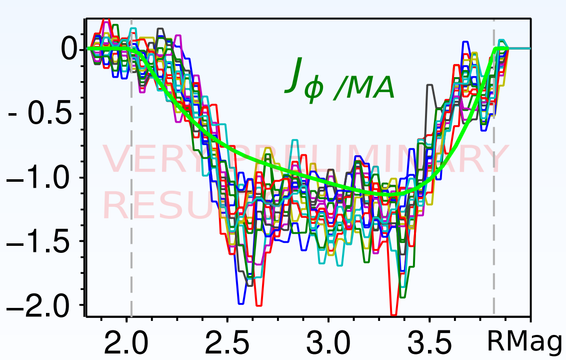
PDF shows many possible consistent answers.
We can add strong priors, e.g. require monotonic *p* (-ve *p*'):



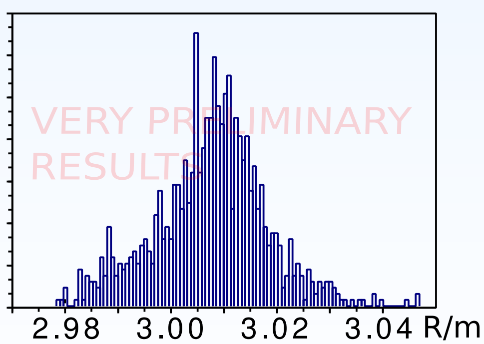


Uncertainties in everything derived is also automatically or easily calculated...

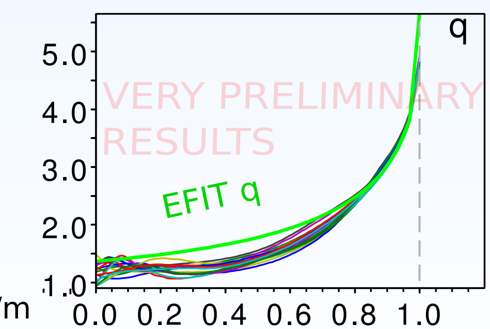
Toroidal current



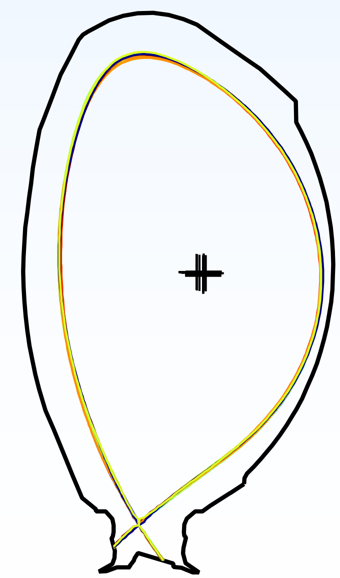
Magnetic axis position



q Profiles.



Flux surfaces and separatrix.



Simple to add diagnostics (Polarimetry, Diamagnetic loop, MSE, LIDAR-TS etc) and to modify parametrisation, priors and equilibrium model.

All directly transferable to MAST as it is part of the common code base for Bayesian analysis on JET, MAST, ANU, W7-AS, and will work directly from the MAST magnetic model.

[To be submitted as part of PhD thesis and for publication later this year]

Possible future extensions and uses.

The recent advances in the work and consequent capability of treating the equilibrium under Bayesian principals opens a wide range of possibilities:

- Fully explore the surprising amount of information apparently available from only magnetic diagnostics and investigate what is added by other diagnostics (MSE, Polarimetry etc, for which the modelling is already done).
- Examine the certainty with which the edge parallel current can be inferred from the Bayesian equilibrium with only magnetics and with the other diagnostics and possibly its evolution during the ELM cycle.
- Test the validity of the parallel current models at the edge, e.g. bootstrap approximations etc .

- Combine LIDAR and equilibrium work to examine the n_e / T_e pedestal gradients/widths with flux surface uncertainty. Examine their scaling with global parameters in support of the work being performed with the independent HRTS diagnostic.
  [7000 time points in type-I ELMy H-Mode, marked and clear of ELMS since Edge LIDAR upgrade C20-C27]

- Add the necessary parametrisation and equilibrium model to include flow and/or anisotropic pressure.

Polarimetry - Verification of finite T_e effects and relativistic polarisation theory.

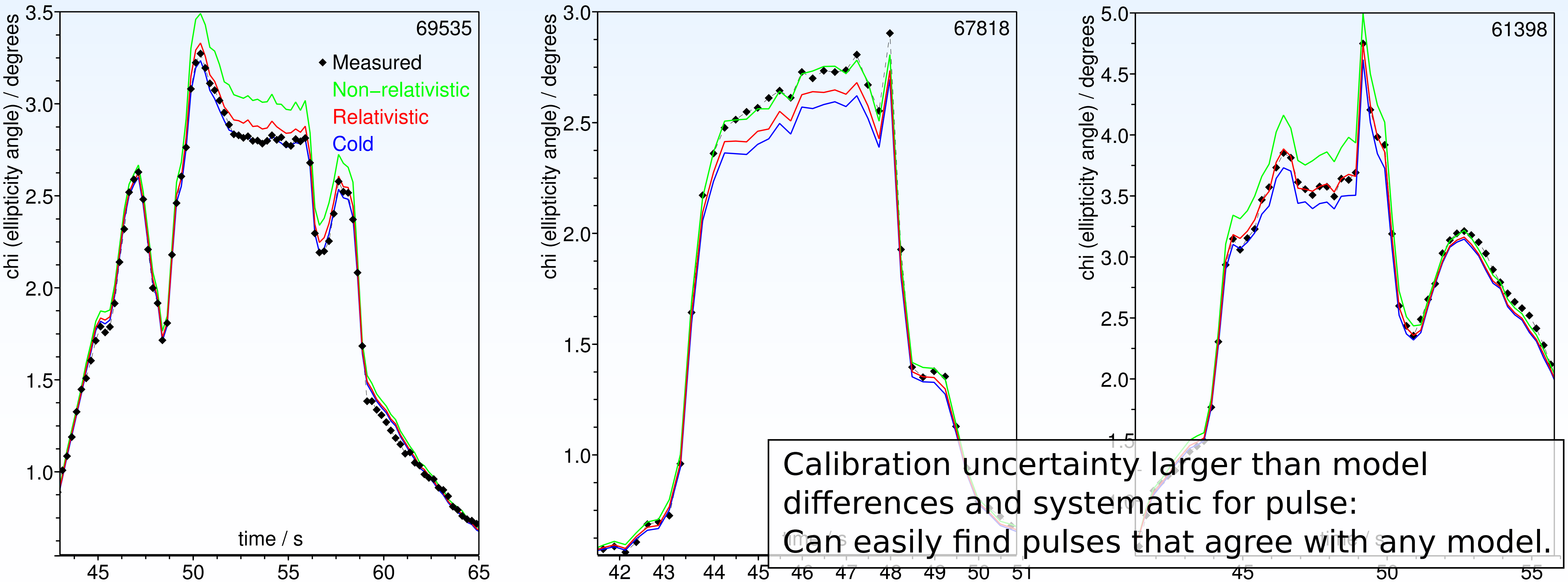
- Detailed modelling of diagnostics allows extraction of a plasma physics results, from existing data, and from far below the noise level.

Plasma polarimetry usually treated using 'cold plasma' model based on fluid approximation.
Two papers gave corrections for finite-T_e effects derived from kinetic theory:

a) S.E. Segre (2002): Argues non-relativistic kinetic approximation is sufficient:
- Correction from cold model of 24% for ITER.

b) V.V. Mirnov (2007): Argues mass increase of electron is important and derives a weakly relativistic approximation. - Gives a correction of 9% for ITER.

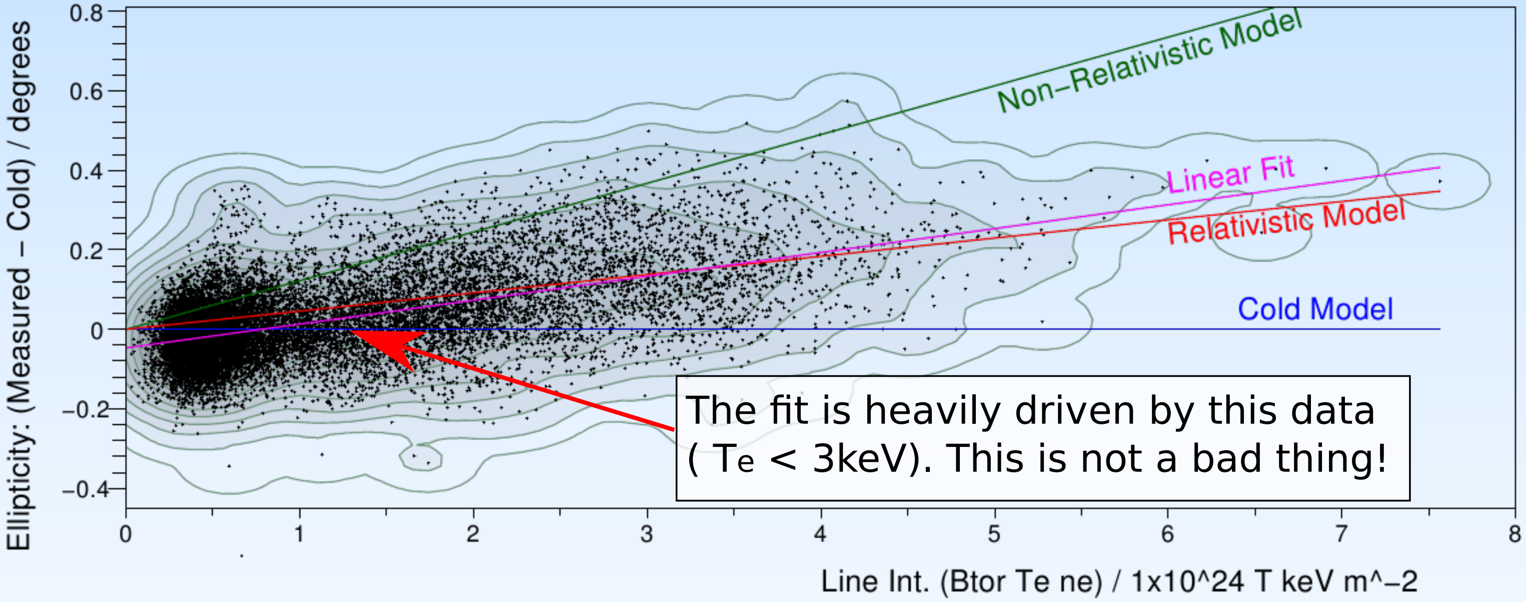
Compare predictions from each model based on Bayesian inferred n_e profiles from interferometry diagnostic.
But... Measurement and prediction for cold plasmas differ unexpectedly and systematically over entire pulses and campaigns due to calibration variation which is not fully understood.



Calibration uncertainty larger than model differences and systematic for pulse:
Can easily find pulses that agree with any model.

Polarimetry - Verification of finite T_e effects and relativistic polarisation theory.

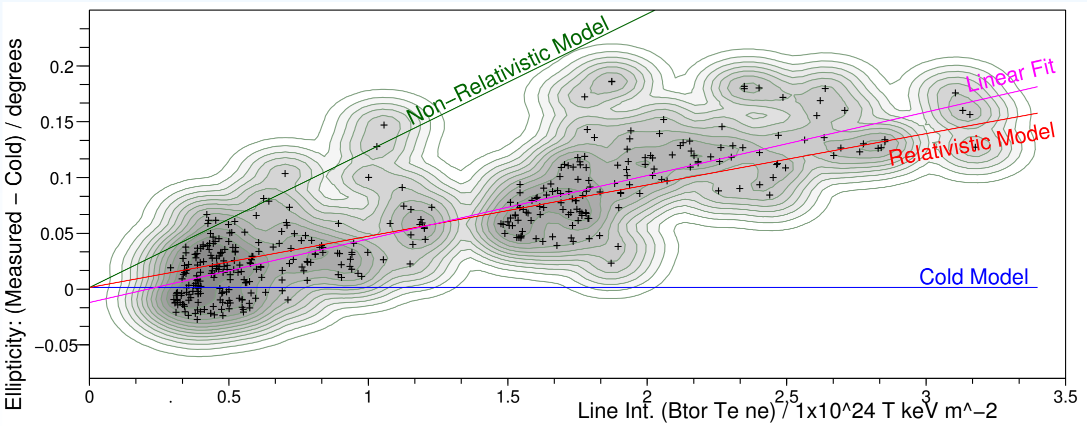
Diagnostic uncertainty is due to calibration and cannot depend on plasma core T_e.
Polarimeter has been on JET for a long time so we have a lot of data...



There were suggestions to run an experimental campaign at very high T_e to check these theories but the information was already in the data.

Independently:
1) Adjust calibration parameters to make cold plasma model agree for cold plasmas

2) Compare data in high T_e period: →



"*Forward modeling of JET polarimetry diagnostic*" - Rev. Sci. Instrum **79** 10F324 (2008)

"*Experimental verification of relativistic finite temperature polarimetry effects at JET*"
Plasma Phys. Control. Fusion **51** 065004 (2009)   (Included in IOP select and PPCF highlights 2009.)