Bayesian Analysis Results from JET: Thomson Scattering and Equilibrium

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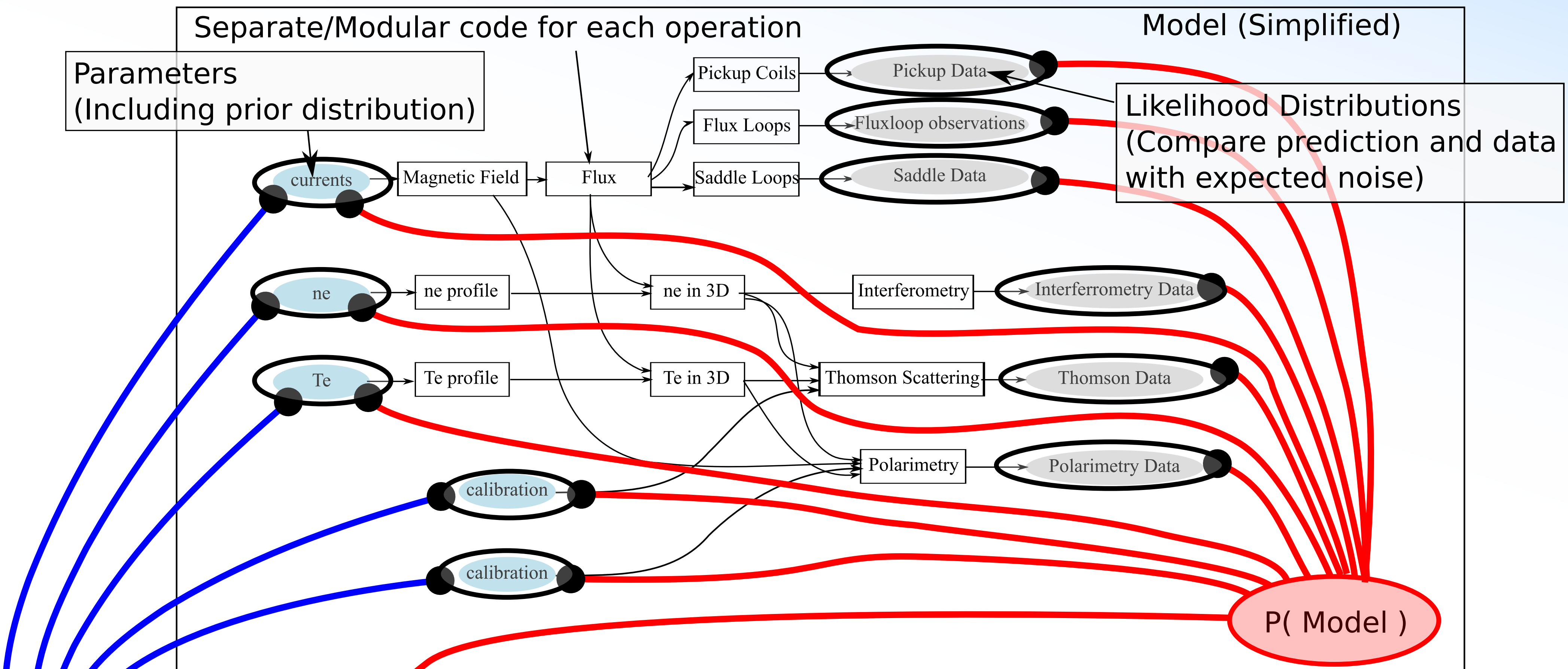
* See the Appendix of F. Romanelli et al., Fusion Energy Conference 2008 (Proc. 22nd Int. FEC Geneva) IAEA

Forward Modelling and Bayesian Inference

The basic idea:



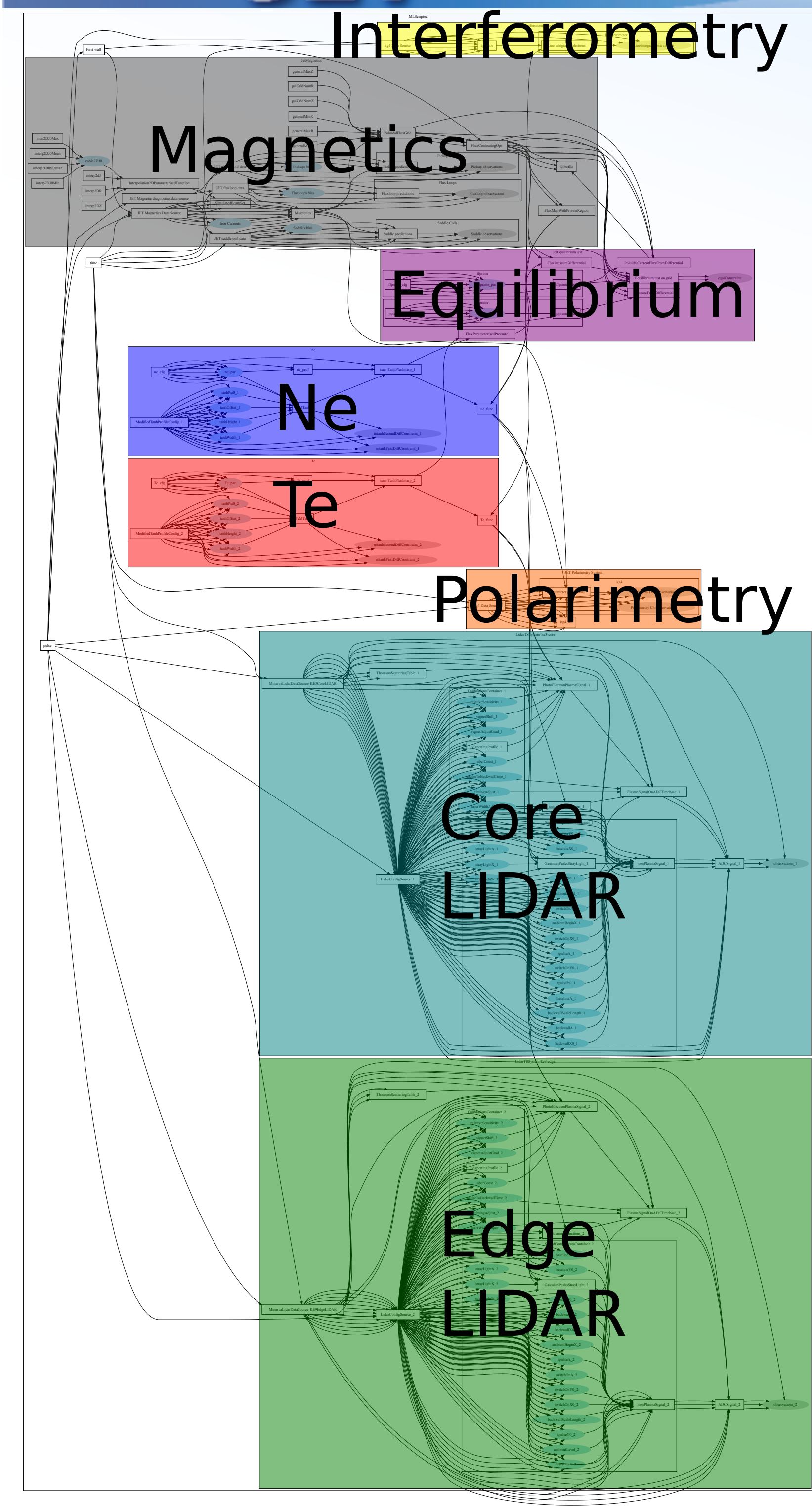
Bayes Theorem: $P(\text{Te, Ne, J} \mid \text{Data}) \sim P(\text{D} \mid \text{Ne, Te, J})\, P(\text{Te, Ne, J})$

Practically: Solve and explore using external algorithms:

Linear Gaussian Solver
(Best fit and PDF
covariance)

Genetic Algorithms
(Non-linear best fit)

Metropolis Hastings
MCMC Non-linear Exploration:
--> Uncertainty



Software and Models

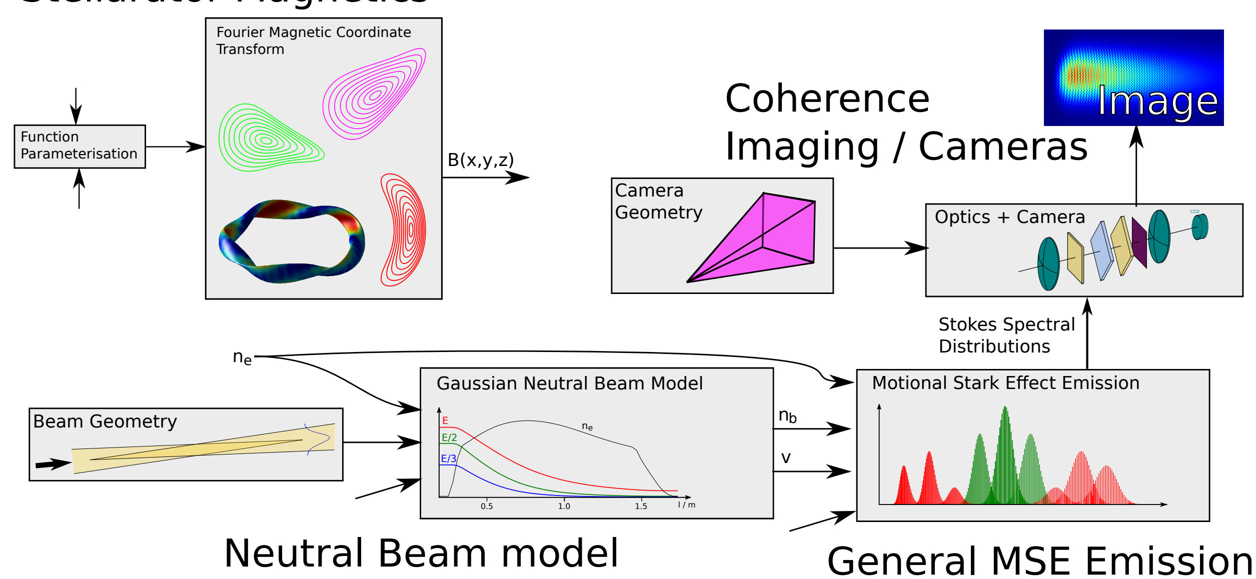
Write nodes and wire them together.
Software framework handles the rest.
Even automatically generates the graphical representation.
We can re-wire the graph and redefine/modify the problem
at will, even during a run.

Parts previously written: JET Magnetics, Interferometry.

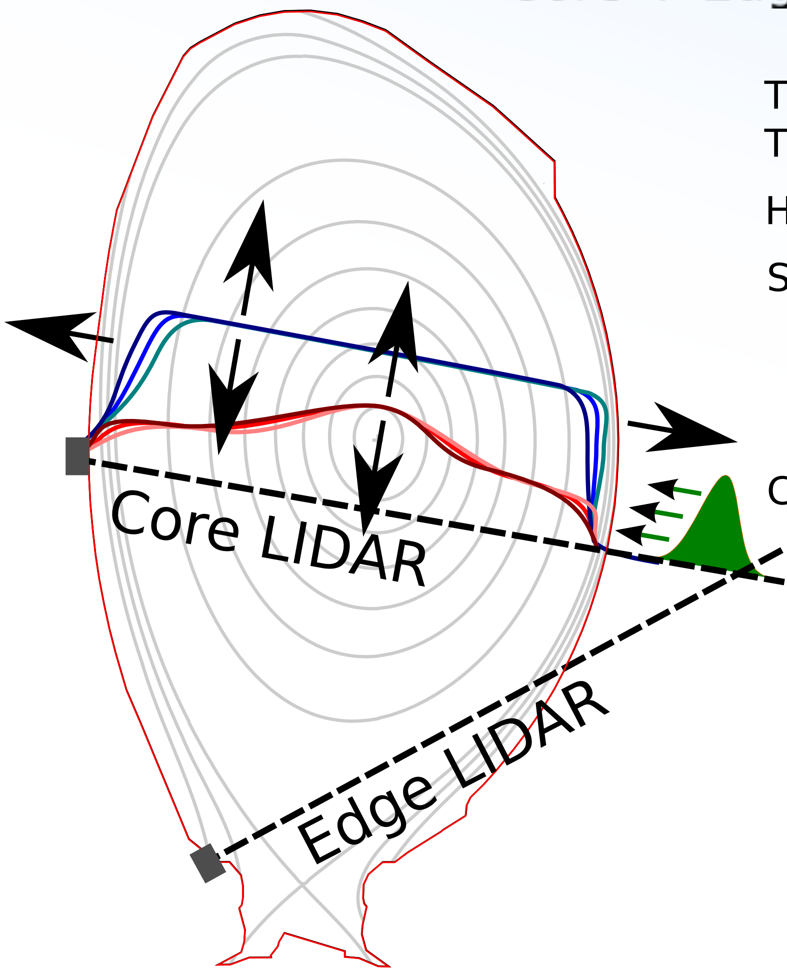
Parts I wrote as part of my PhD:
- Polarimetry
- Core + EDGE LIDAR Thomson Scattering
- Equilibrium (Grad-Shafranov Test)

Parts written by others:
- JET: MSE, Reflectometry, Infrared strikepoint camera
- MAST: Magnetics, MSE, Thomson Scattering

Stellarator Magnetics



Core + Edge LIDAR: The systems and the problem



Thomson Scattering diagnostics with single spectrometer.
Time of flight for positioning.

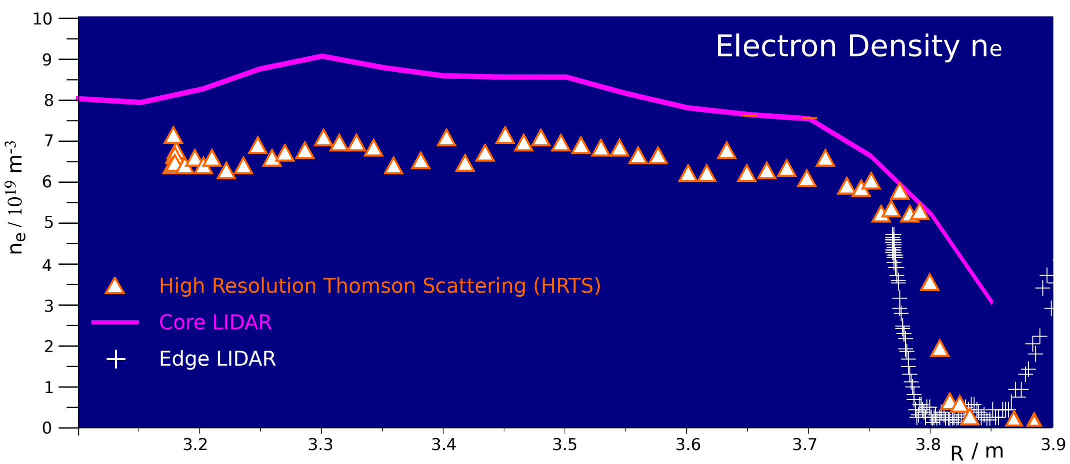
Hardware system very complex.

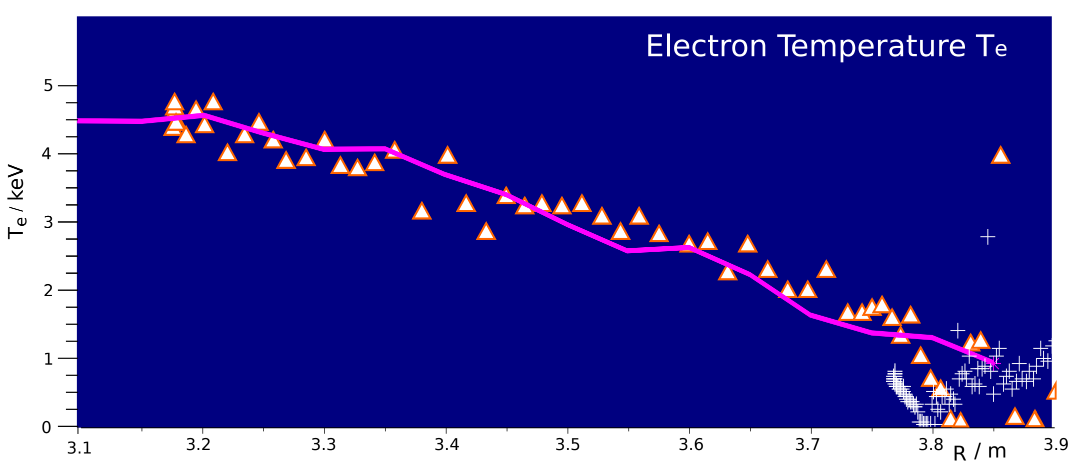
Spatial Resolution:

- Effective convolution of light signal.
- If ignored: Convolves n_e but complex effect on T_e.
- No problem for forward modelling: we just convolve the signal.

Calibrations:

- Beam dump position + timing --> Uncertain position.
- Optical transmission + laser energy --> n_e magnitude.
- Spectrometer Relative Sensitivities --> T_e magnitude.
- Relative Channel timing --> T_e + n_e shape!





Imperial College London

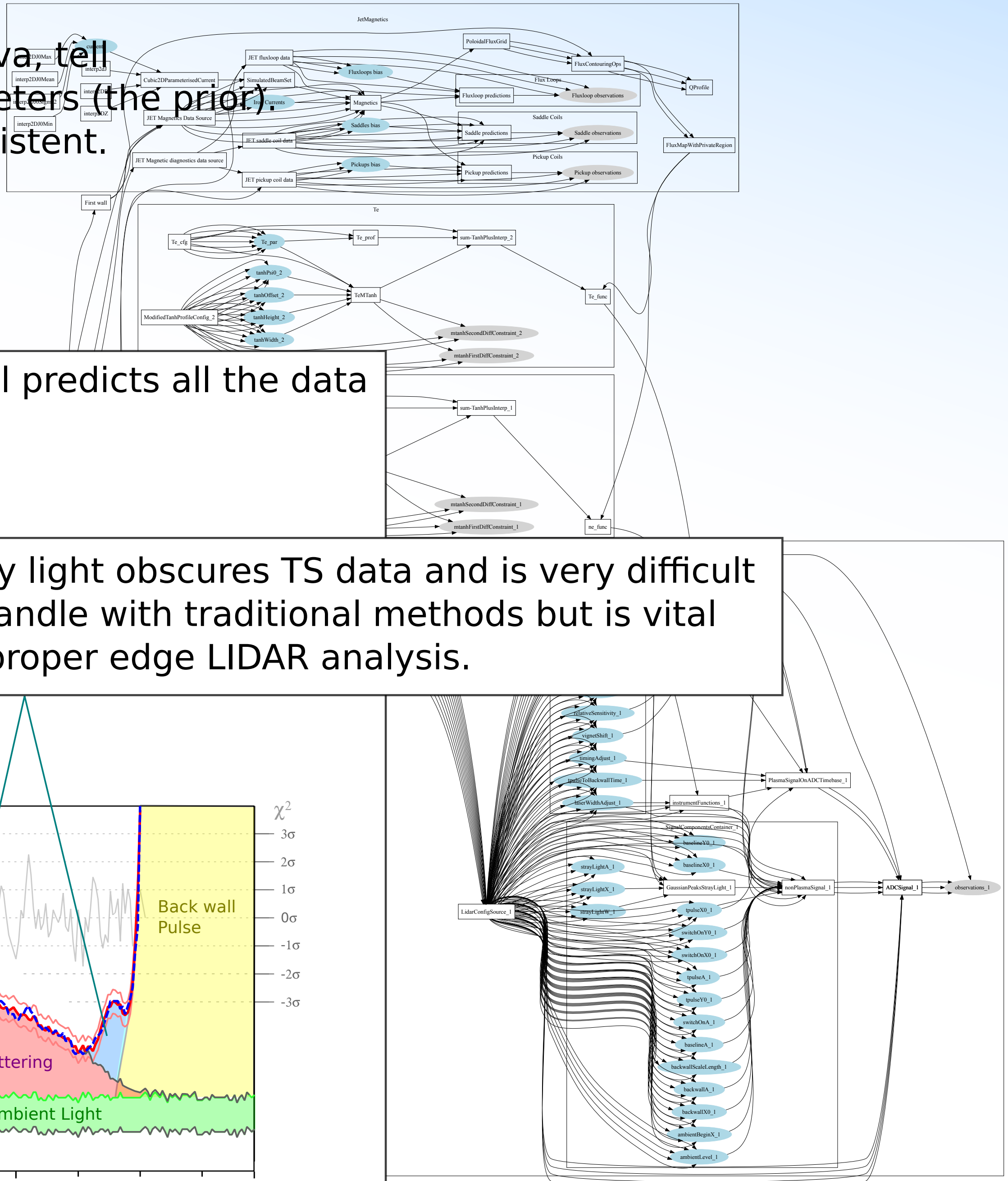
Core + Edge LIDAR: The model

So how do we deal with disagreement with other diagnostics?

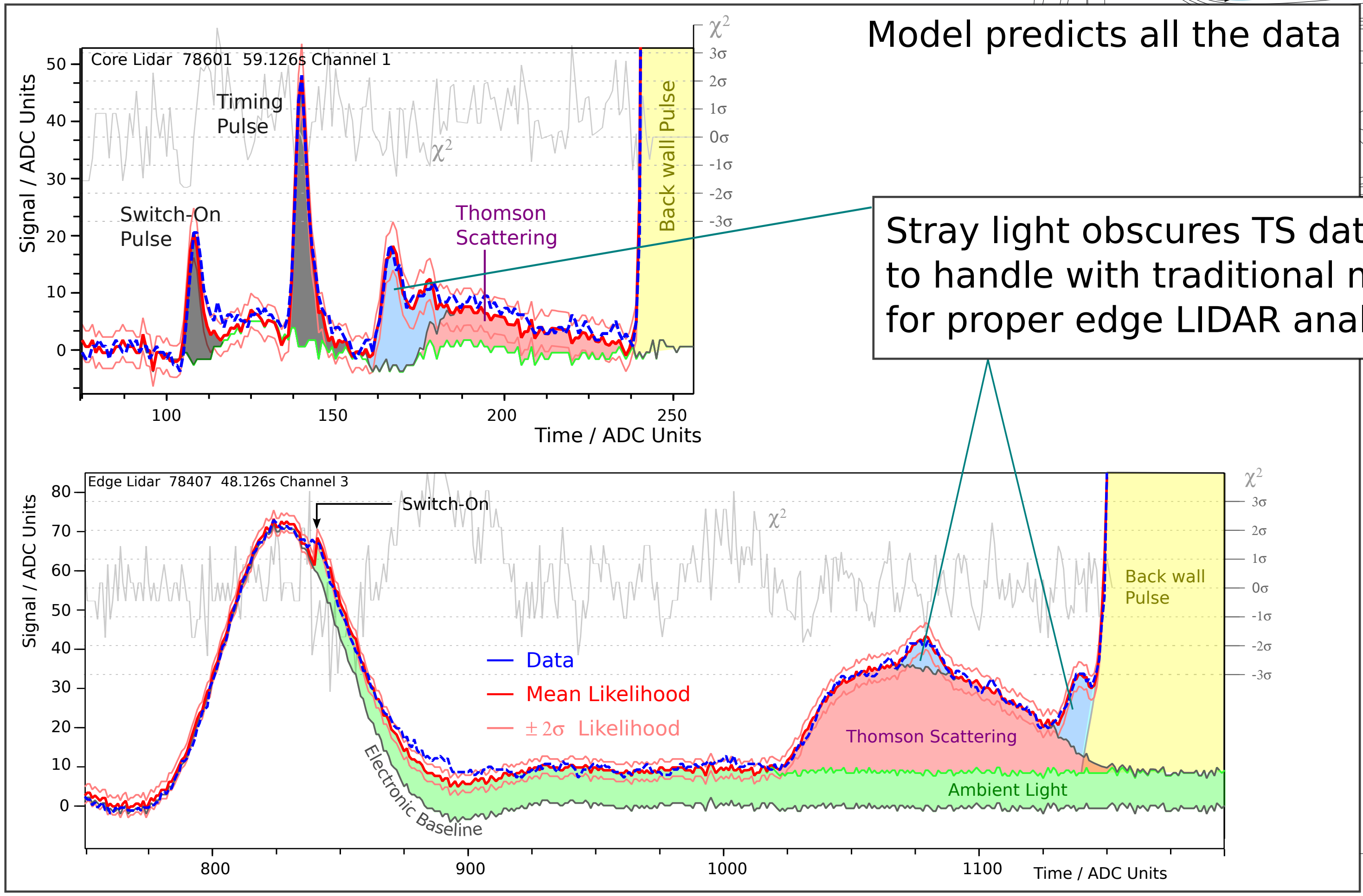
Shift and scale output profiles to match?

No - Build the model for each and wire up to Minerva, tell it what we do know about the calibration parameters (the prior) and let it work out how to make everything consistent.

We must really understand each part of the system:
Laser Pulse, TS physics, Optics, Filters, Photomultipliers, Counting Noise (PDFs), ADCs.



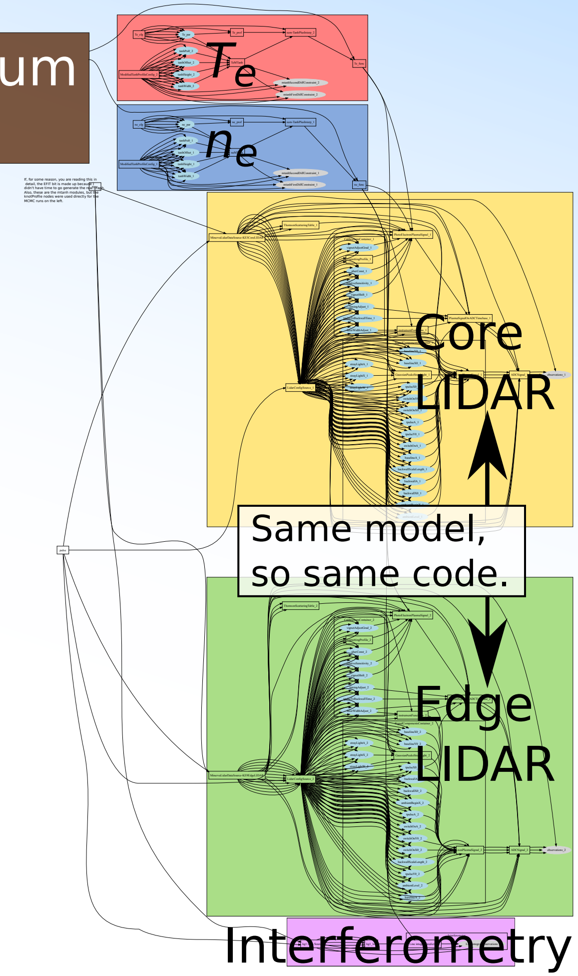
Model predicts all the data

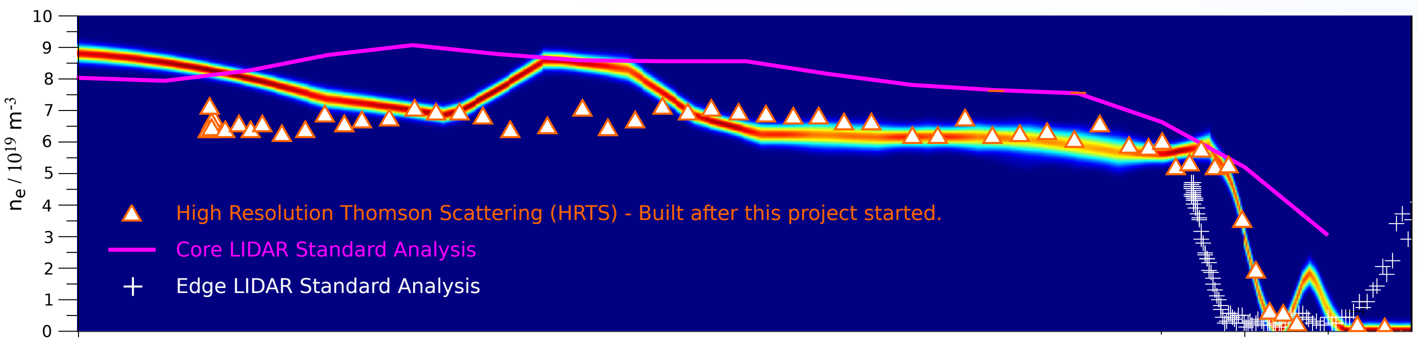


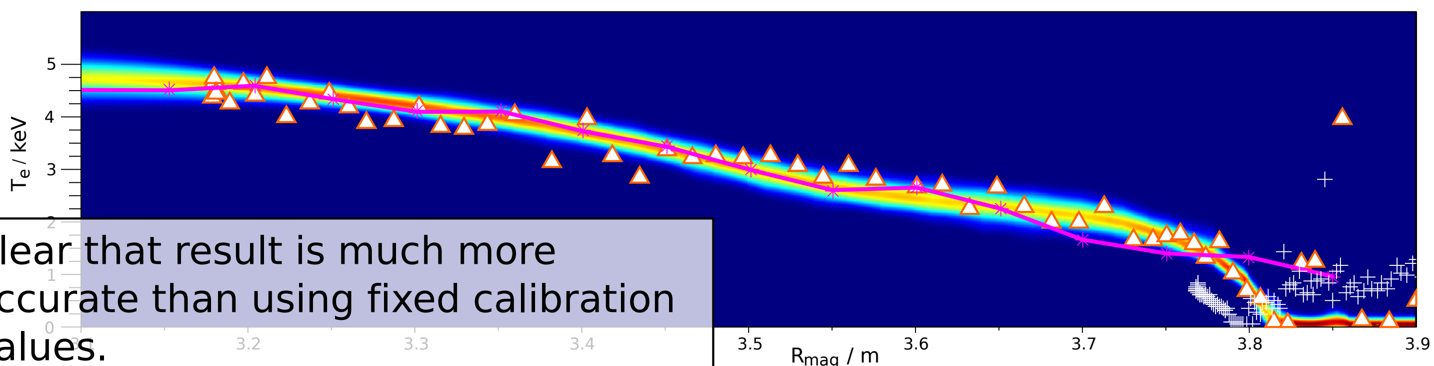
Stray light obscures TS data and is very difficult to handle with traditional methods but is vital for proper edge LIDAR analysis.

Core LIDAR + Edge LIDAR + Interferometry

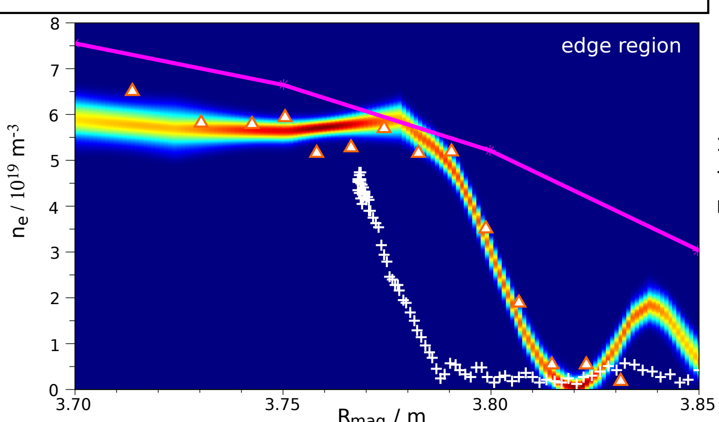
A typical high density H-mode pulse:
- Connect up the model.
- Give all calibrations some uncertainty (what we believe).
- Give some less trusted calibrations almost complete freedom (uniform prior).
- Throw the complete problem at the distributed GA for MAP (best fit) and then at the distributed MCMC for the PDF (uncertaint...

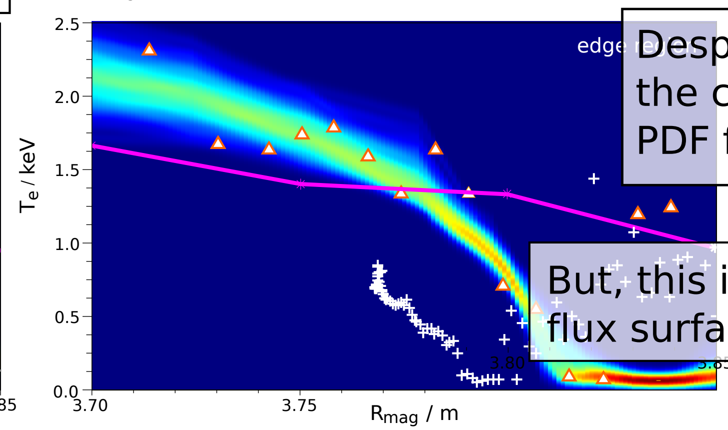






Clear that result is much more accurate than using fixed calibration values.





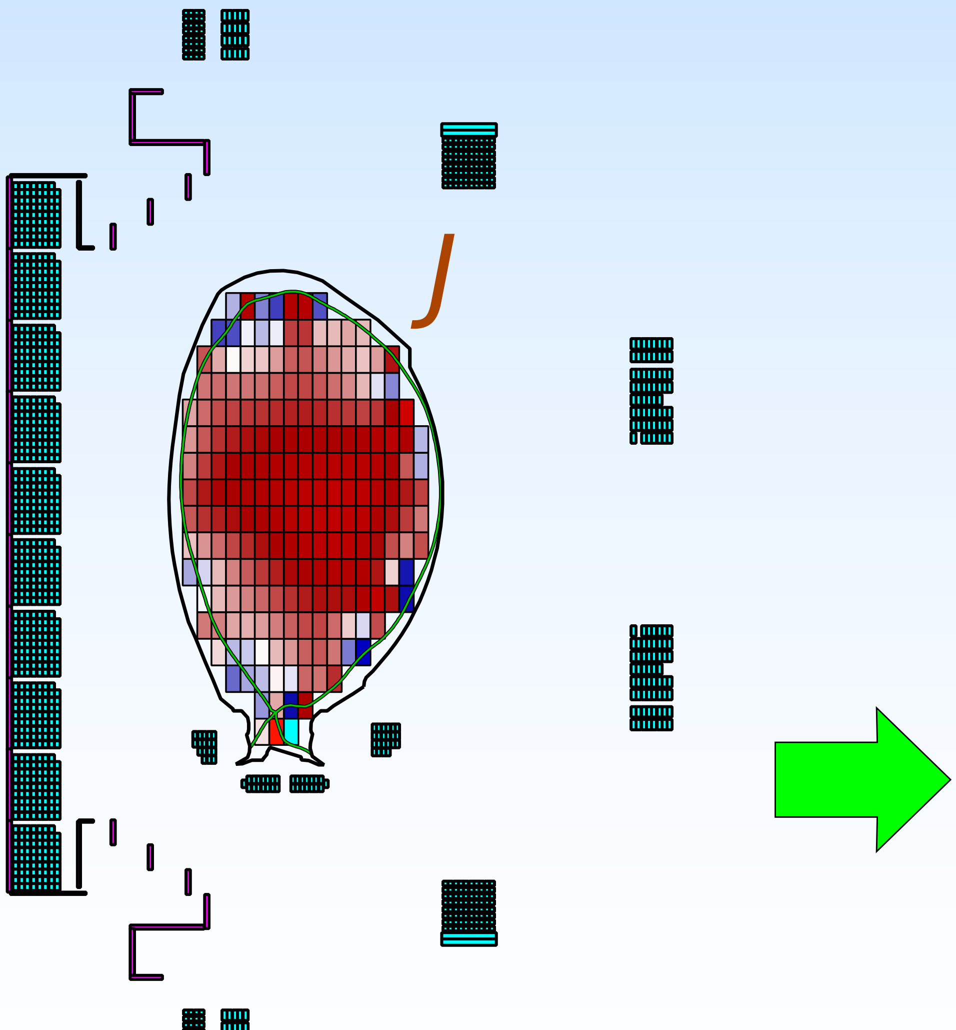
Despite completely free T_e calibration, the combination can fix Te and gives a PDF for the calibration values.

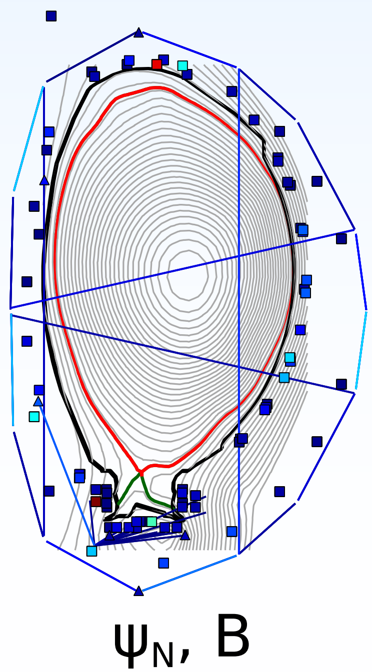
But, this isn't complete - we are still using flux surfaces fixed to the equilibrium code.

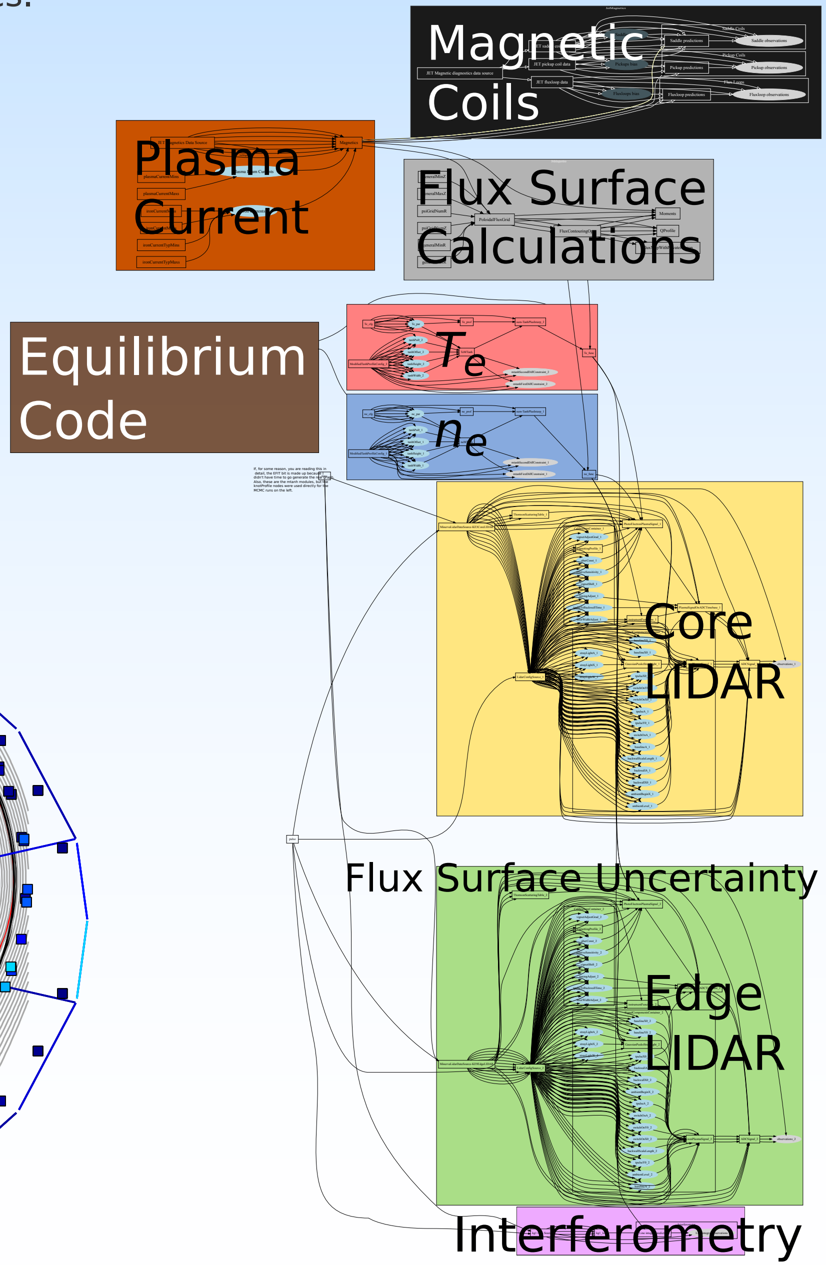
Interferometry + Current Tomography I

Instead, calculate ψ_N from toroidal currents J, include magnetics diagnostics and invert to full posterior:

i.e: Find combinations of J, T_e, n_e, that are consistent with all the diagnostics.

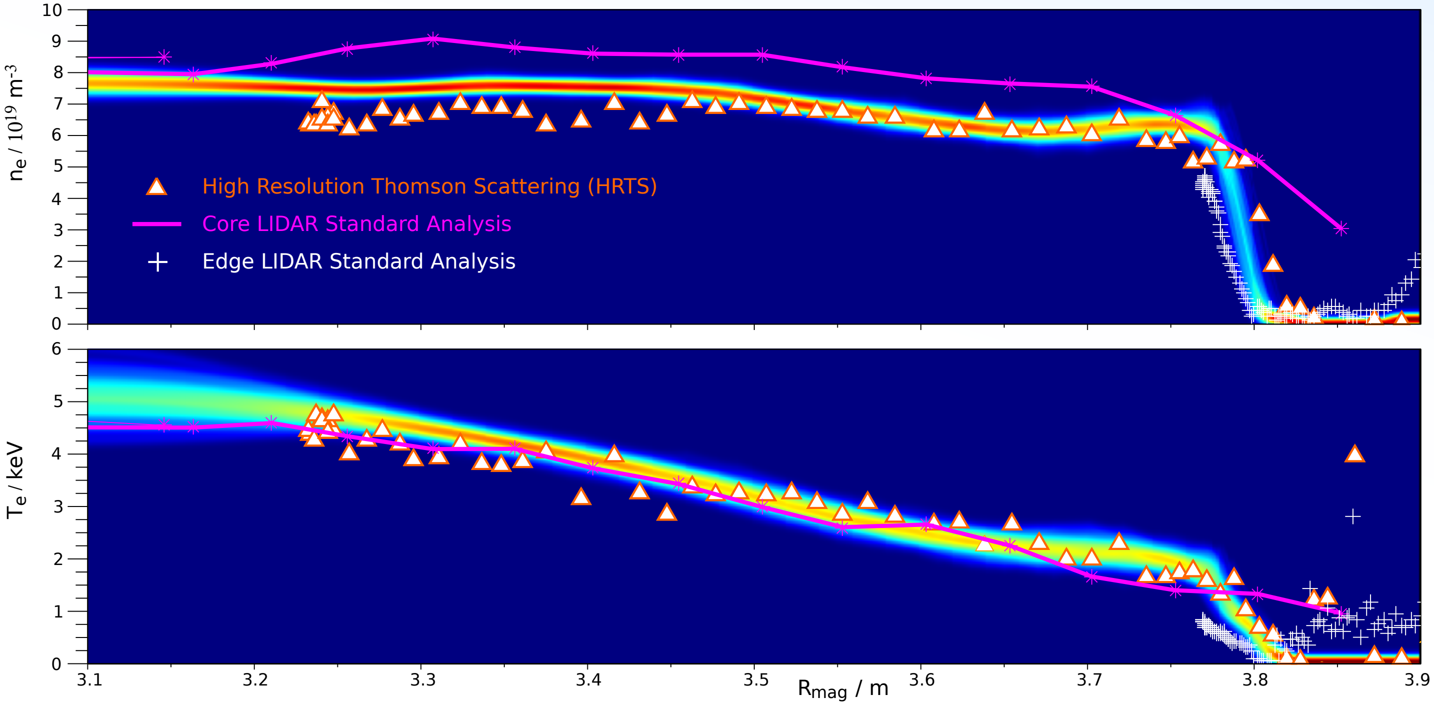


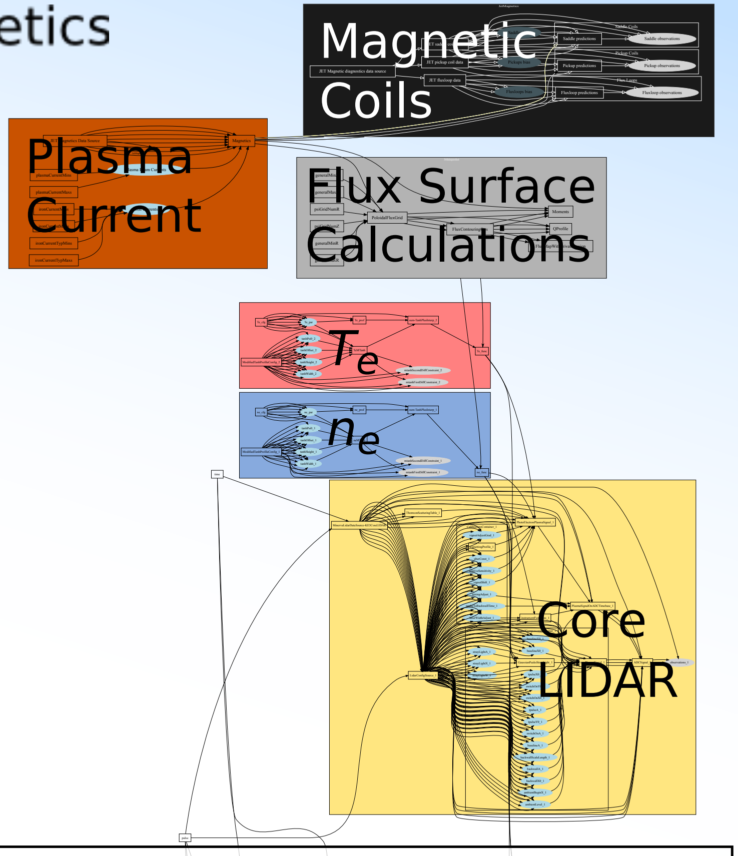


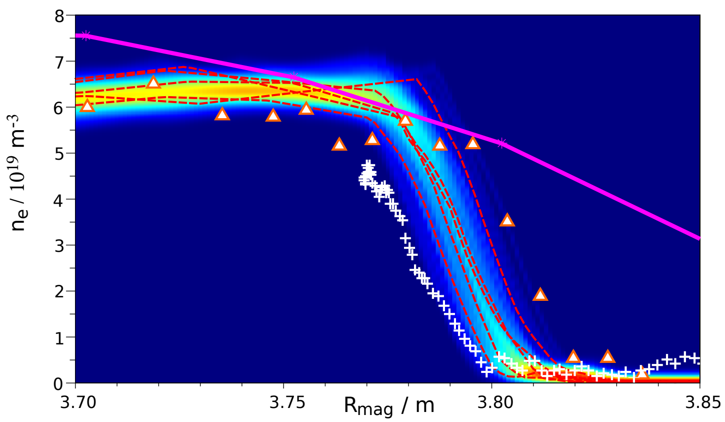


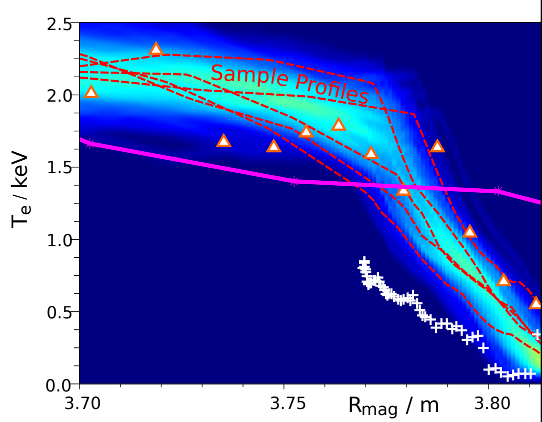
Core LIDAR + Edge LIDAR + Interferometry + Magnetics

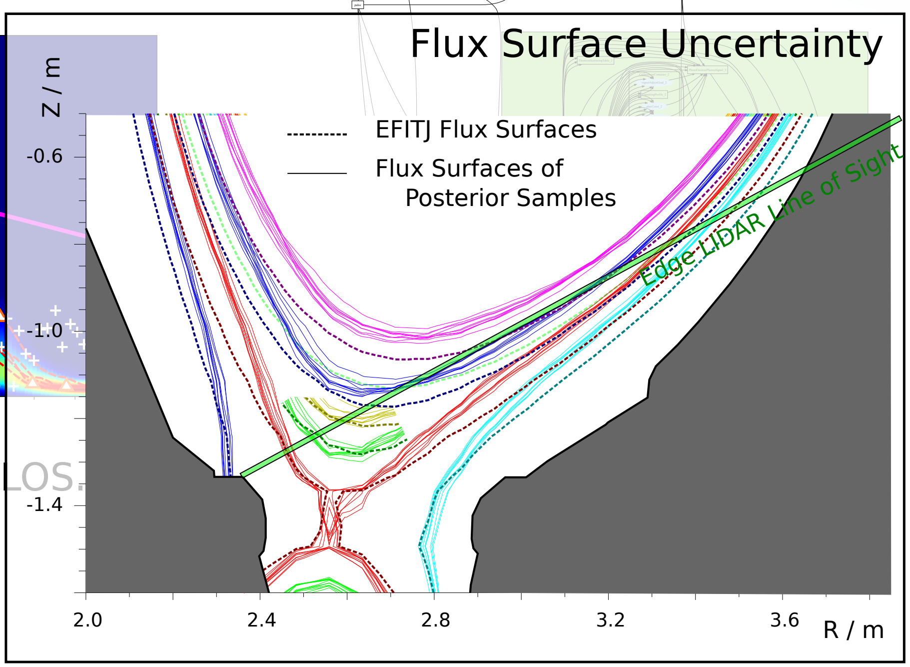
Connect magnetics model and run inversion.









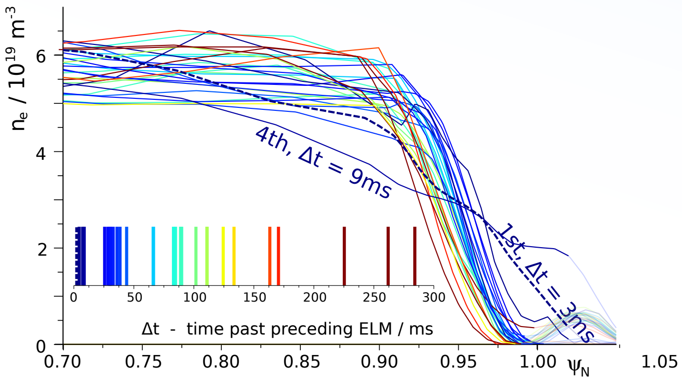


The TS diagnostics provide information on plasma current near LOS.

Plasma current one of the most important and least diagnosed parameters in Tokamaks.

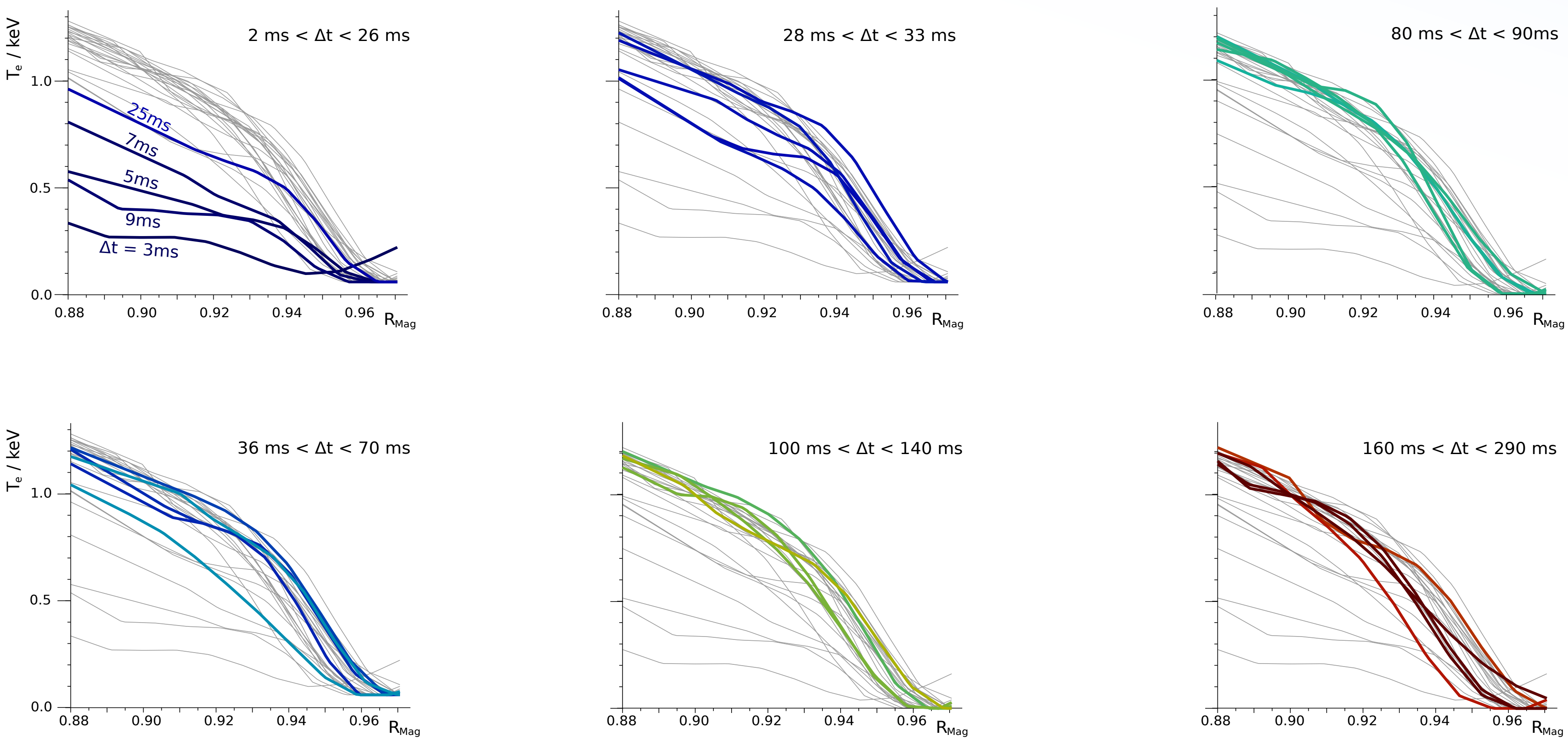
Core LIDAR + Edge LIDAR + Interferometry: Pedestal Evolution Study

Looked in detail at evolution of ne/Te pedestals through the ELM cycle. 28 time points over 6 almost identicle pulses.



Appears to be two distinct phases for T_e:

1) Rapid rise in height and gradient during first 50ms.
2) Slow rise in height and width at fixed gradient until next ELM.

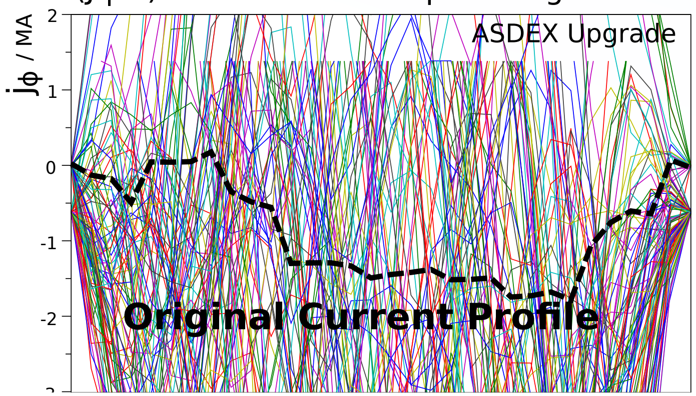


Current and Flux Surfaces

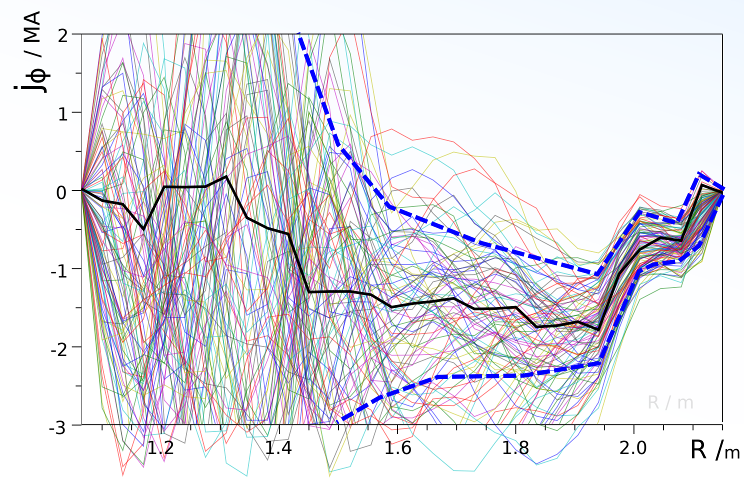
Inference of plasma current and flux surfaces P(ψ_N | ...) is the big problem.

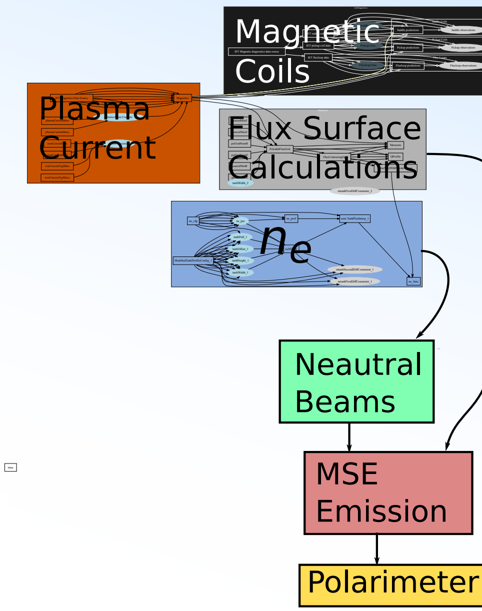
1) Current Tomography and external magnetics:

P(J | D): Possible current profiles given data

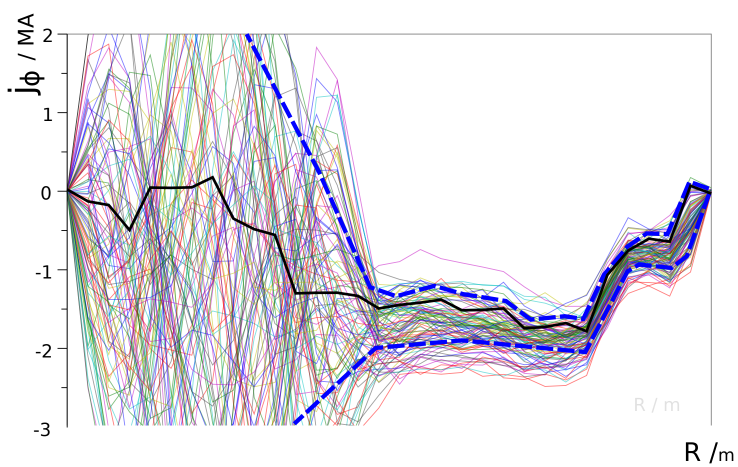


2) Add an MSE diagnostic:
Better





3) Add a 2D MSE System:
Much better!



In the meantime....

Add the 'prior knowledge' that the plasma should be somewhere near, something like an equilibrium....

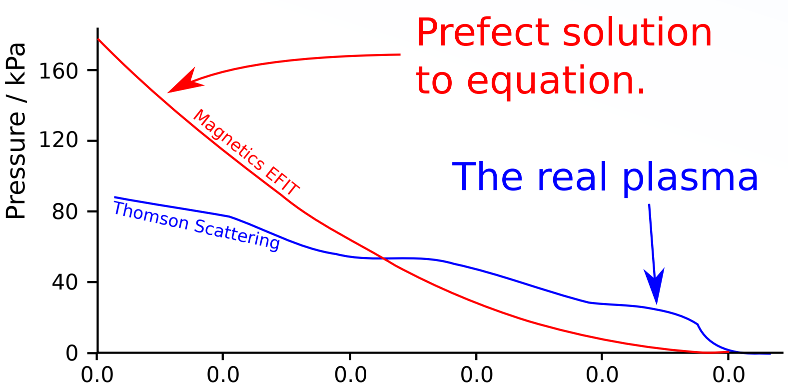
$$J_\phi = Rp' + \frac{\mu_0}{R} f f'$$

(We can also think of this as the observation that the plasma hasn't exploded).

What can this and the external magnetics tell us?

Equilbrium I

Isn't this just 'solving' the equilibrium?
1) The *inference* problem cannot be 'solved' - there are a very large range of good solutions that match the magnetic diagnostics.



NB: It is **not** clear how helpful force balance (GS equation) actually is. It is almost always used with strong prior constraints on *p* (equilibirum pressure) and *f* (poloidal current flux).

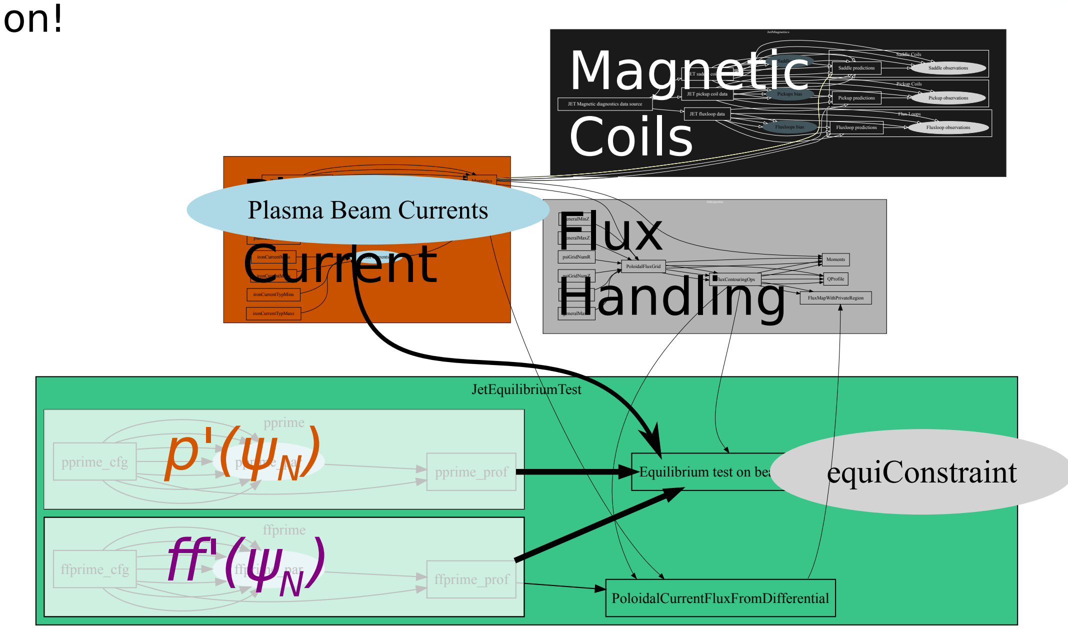
2) The equation is approximate anyway: $J_\phi = Rp' + \frac{\mu_0}{R} f f'$ + Flow + Anisotropy + 3D + ????

- We are not interested in solving an equation. We want to know the configuration of the **real** plasma.
- We should trust the diagnostics more than the equation!

Add as prior constraint:

$$P(j_\phi - Rp' - \frac{\mu_0}{R} f f') = G(0, \sigma_{equi})$$

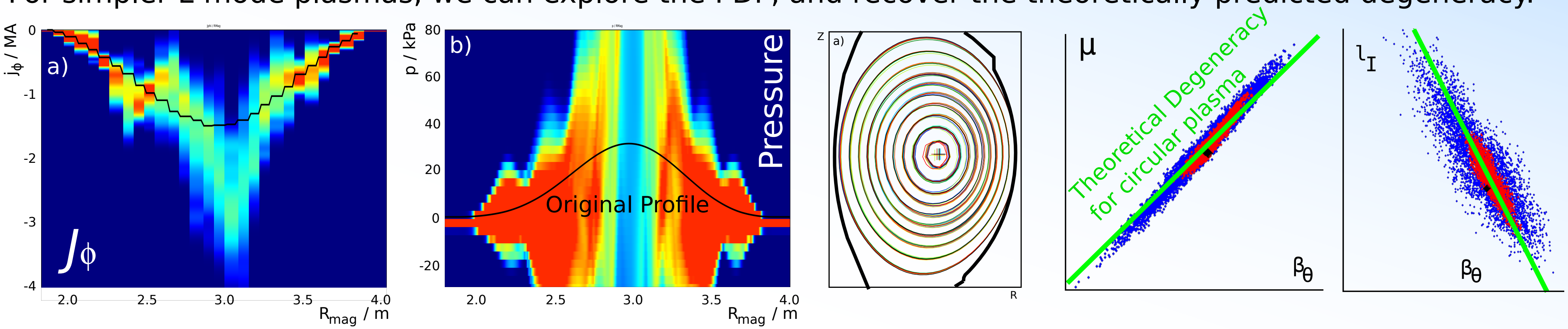
The posterior P(J, p', ff' | D_{diags}, Equilibrium) will include all possible combinations of J, p' and ff' that are consistent with the diagnostics, the priors and describe a plasma very close to equilbrium.



Equilbrium II: Posterior Exploratiand MAP estimates.

78601 High ne
H-Mode (pellets)

For simpler L-mode plasmas, we can explore the PDF, and recover the theoretically predicted degeneracy.

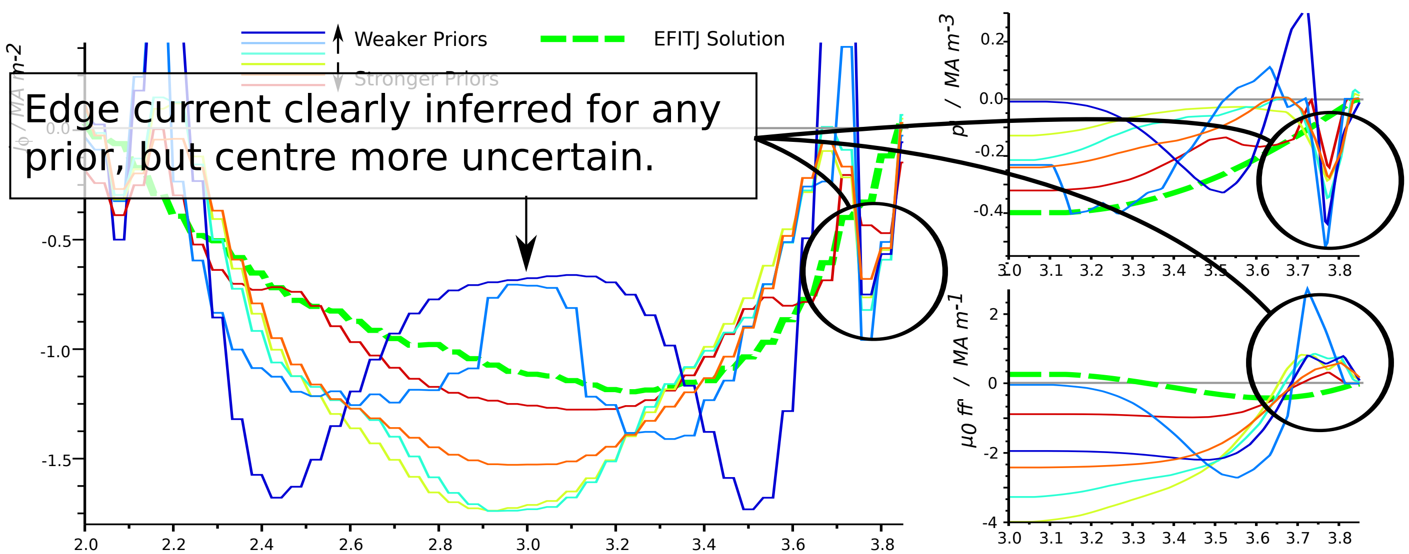


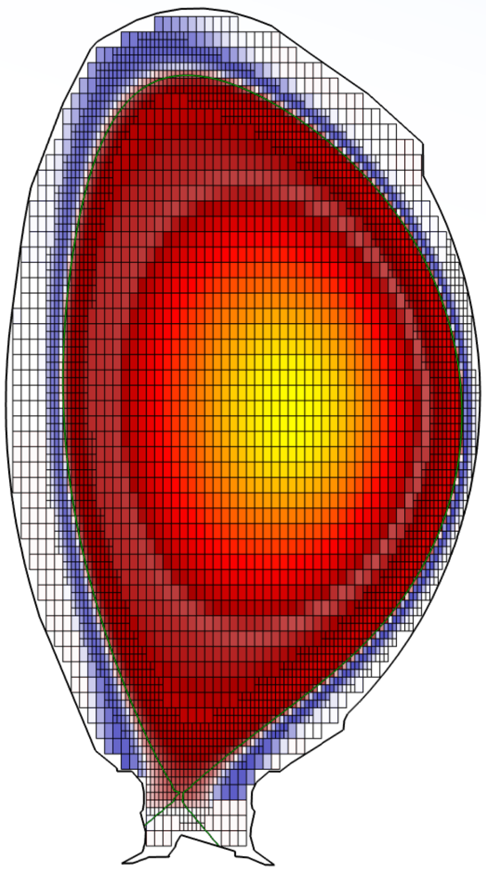
Because of modularity, we can switch parametrisation and priors of J, p' and ff' at will and on-the-fly.
For H-Mode:

J_ϕ: Current beams with higher resolution near edge

$p'(\psi_N)$, $ff'(\psi_N)$: 20 knots, weak smoothing priors.

Too non-linear with too high-dimensionality (4732D) for current MCMC algorithms.
Study MAP with different priors:



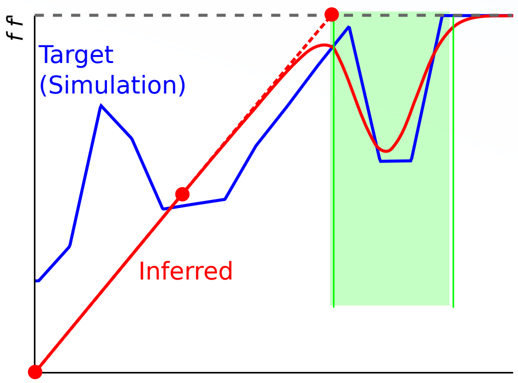


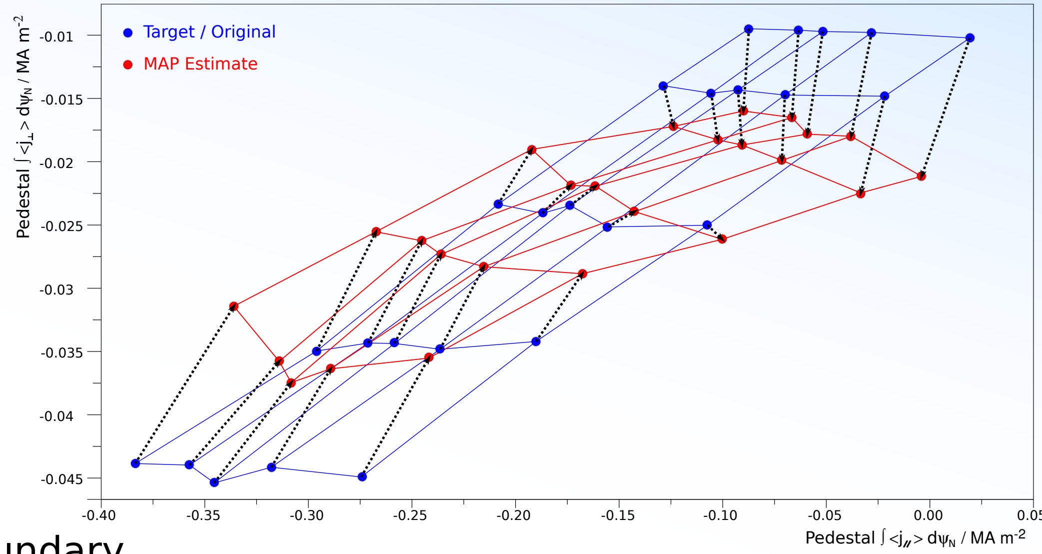
78601 High ne
H-Mode (pellets)

Equilbrium III: Pedestal current evolution

Choose a good prior (e.g. Monotonic pressure), or use stronger parameterisation (e.g. Gaussian at edge):

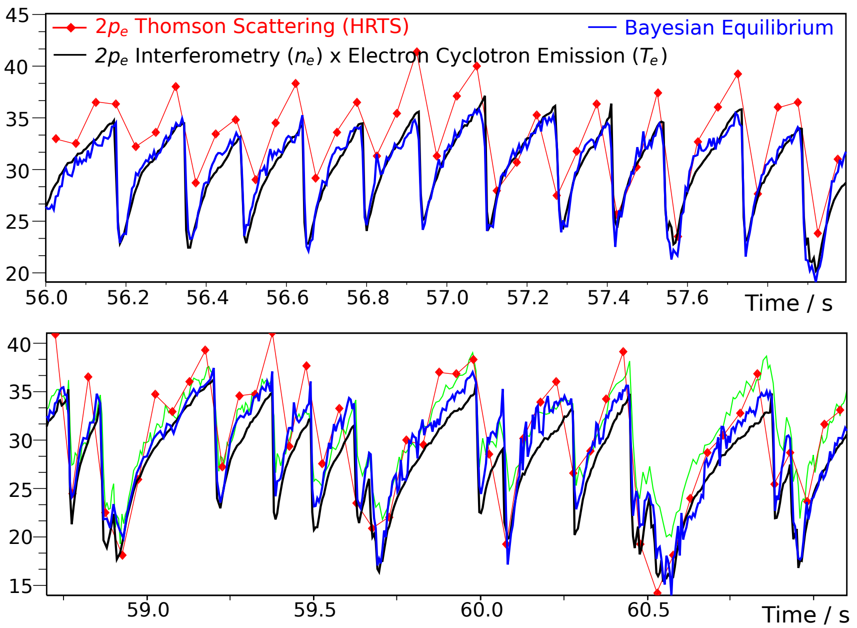
Easy to simulate data and invert to see what can be recovered:

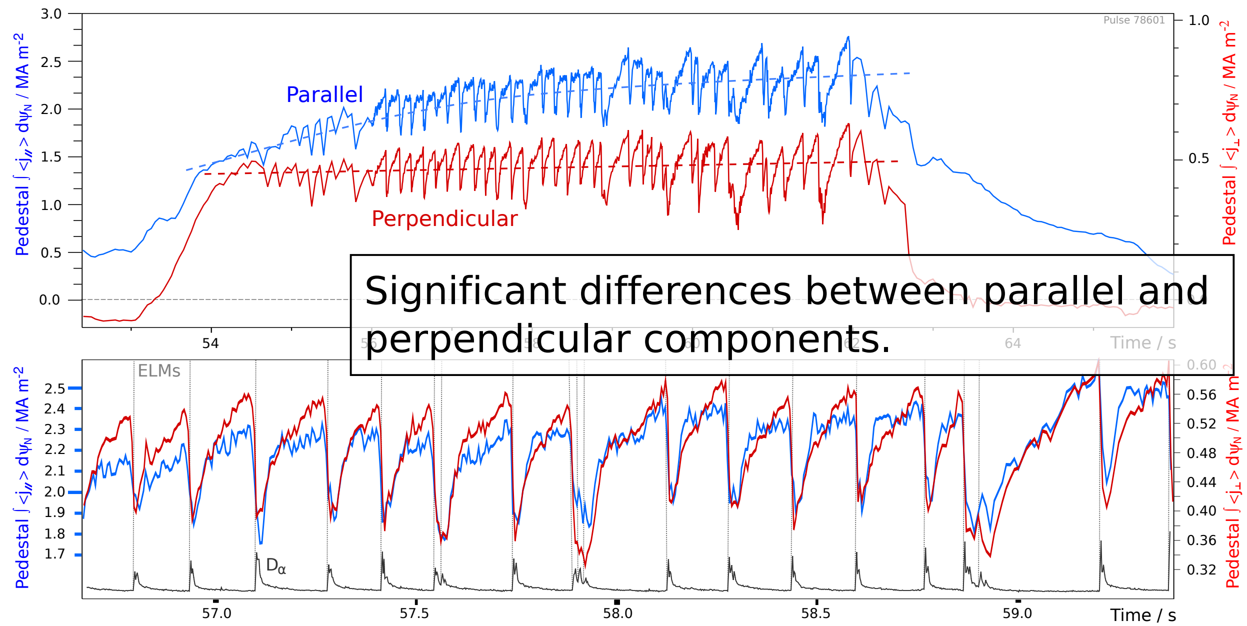




Surprising results:

- We CAN reconstruct information inside boundary.
- Can recover some information about pedestal current both the parallel AND perpendicular (i.e pressure) to the magnetic field.
- Evolution over ELM cycle follows pressure from kinetic measurements incredibly well:

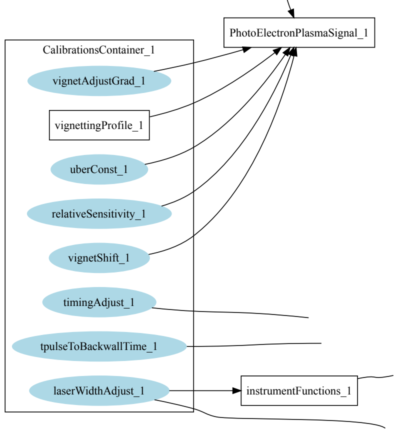




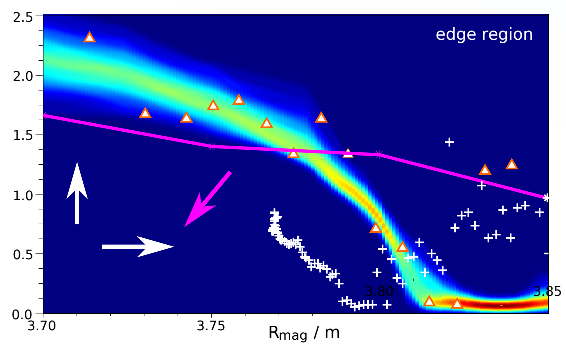
Significant differences between parallel and perpendicular components.

Summary

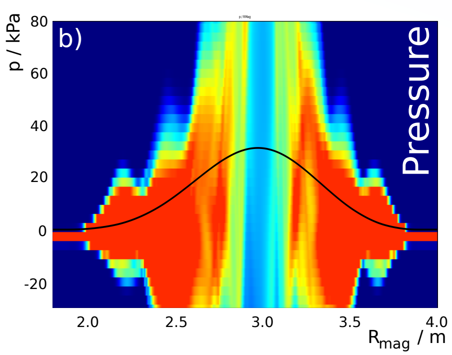
- Develop modular forward models for physics calculations and diagnostics.
- Build up a full description of each problem by connecting modular models.
- Use Bayesian Probability theory to invert data to a distribution over free parameters.



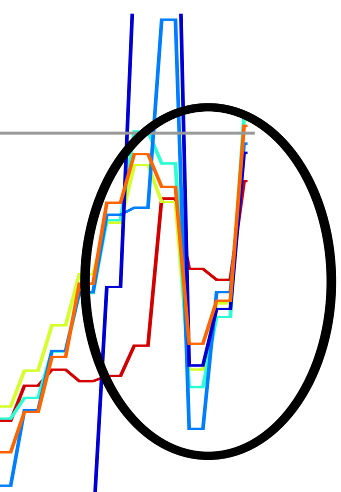
- Forward modelling allows easy handling of many calibration parameters and the complex uncertainties, they result in.



- Combining multiple diagnostics helps infer those calibration parameters from the data:
- Used to examine H-mode pedestal ne/Te evolution at very high spatial resolution.



- Use Bayesian 'posterior PDF' description to examine complex uncertainty in Tokamak equillibria without other strong prior assumptions.



- Surprising amount of detail recoverable from magnetics alone (no internal measurement) when these strong assumptions are not included.