

# Bayesian Analysis Results from JET: Thomson Scattering and Equilibrium

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D. C. McDonald,<sup>3</sup> A. Meakins,<sup>3</sup> E. Solano,<sup>3</sup> JET-EFDA Collaborators\*

1: Blackett Laboratory, Imperial College, London SW7 2BZ, UK

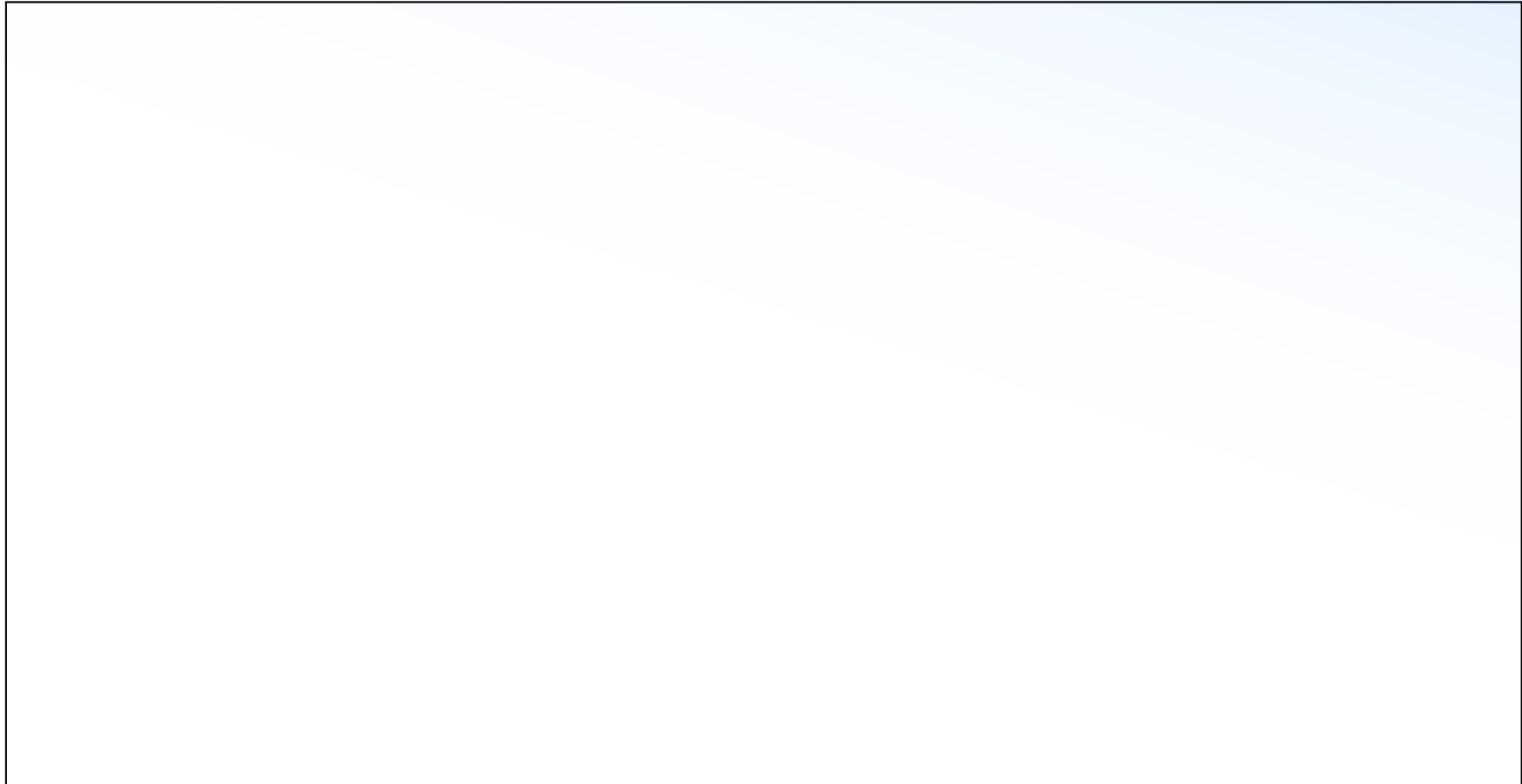
2: Max Planck Institute, Teilinstitut Greifswald, Germany

3: UKAEA Fusion Association, Culham Science Centre, OX14 3DB, UK

\* See the Appendix of F. Romanelli et al., Fusion Energy Conference 2008 (Proc. 22nd Int. FEC Geneva) IAEA

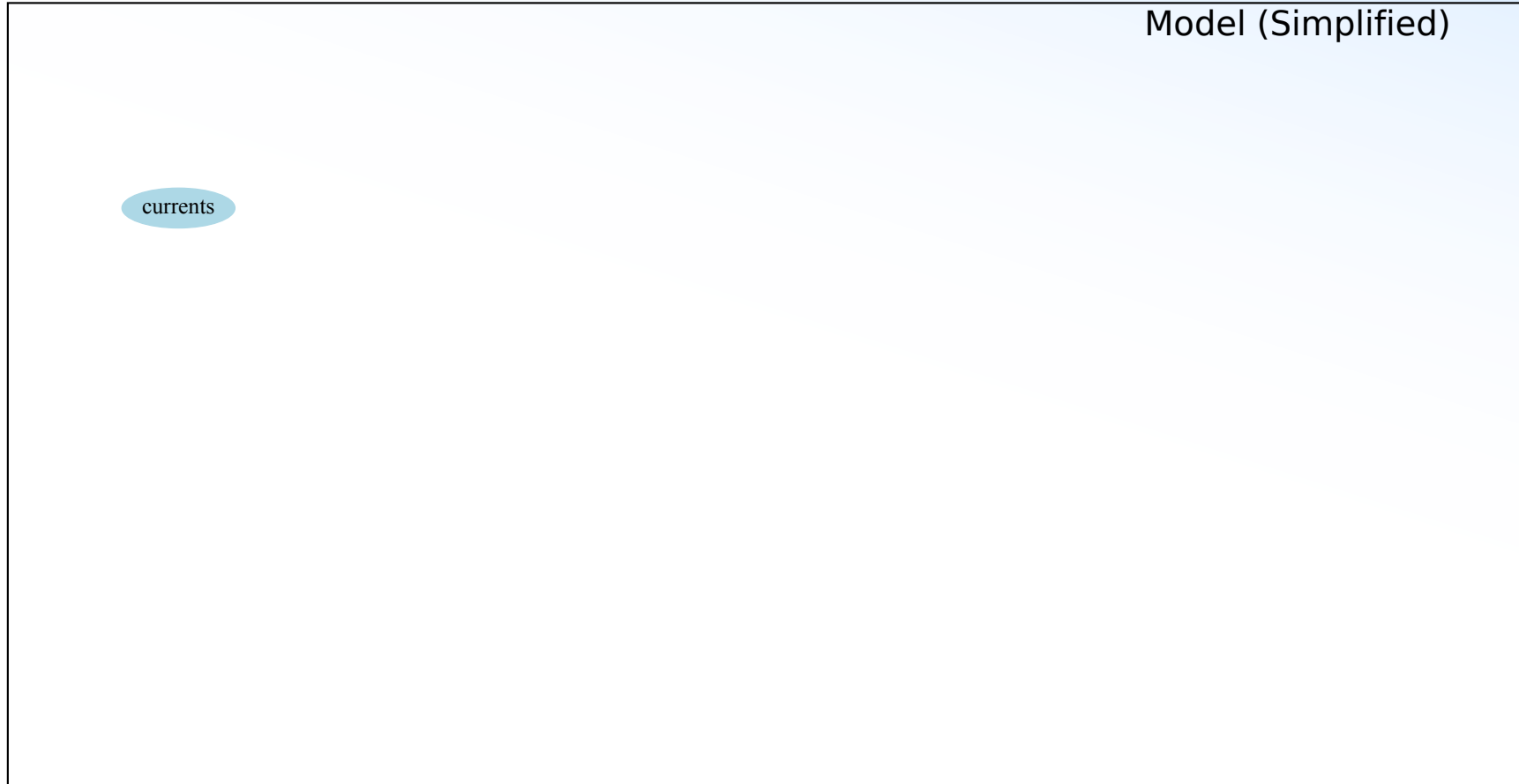
## Forward Modelling and Bayesian Inference

The basic idea:



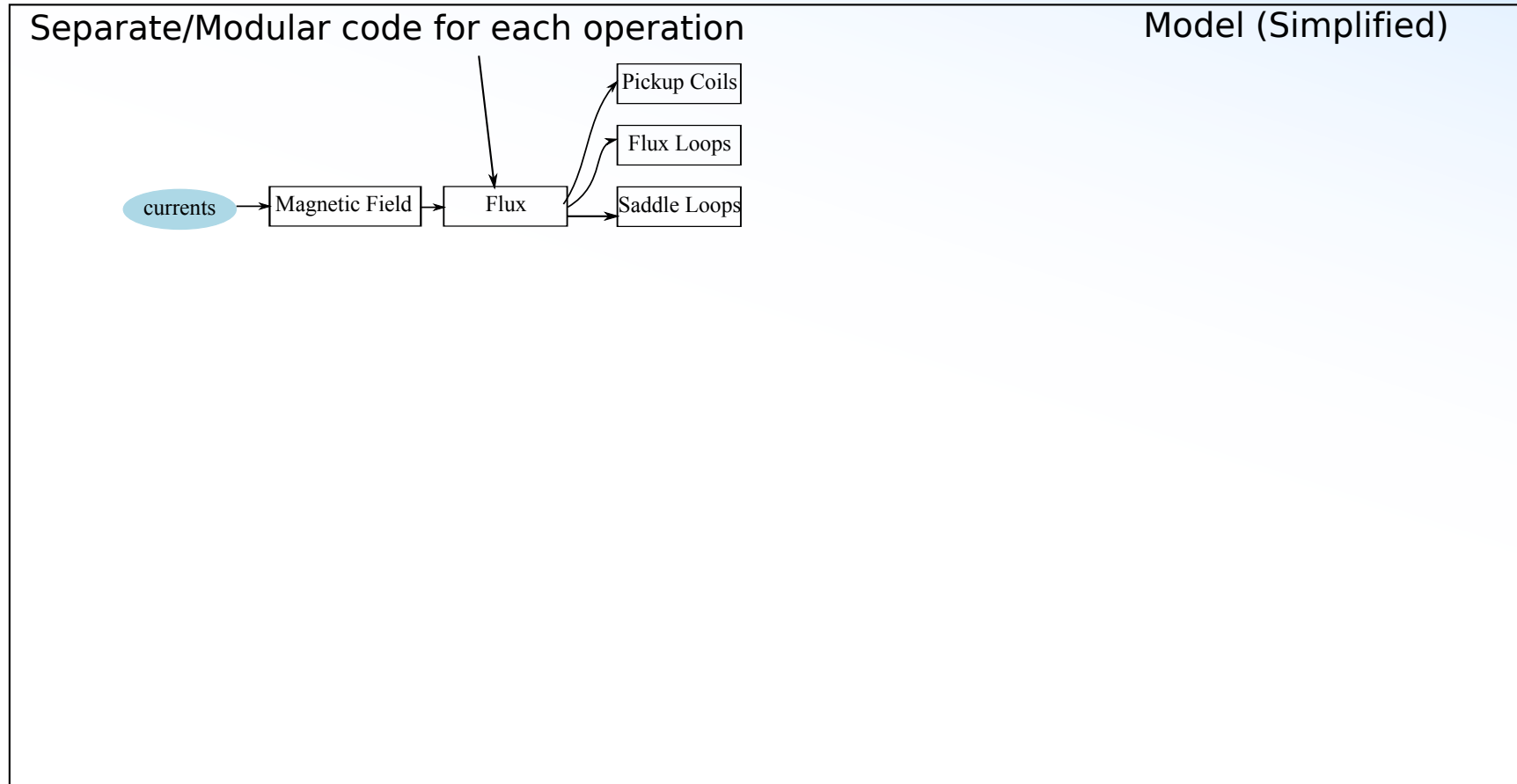
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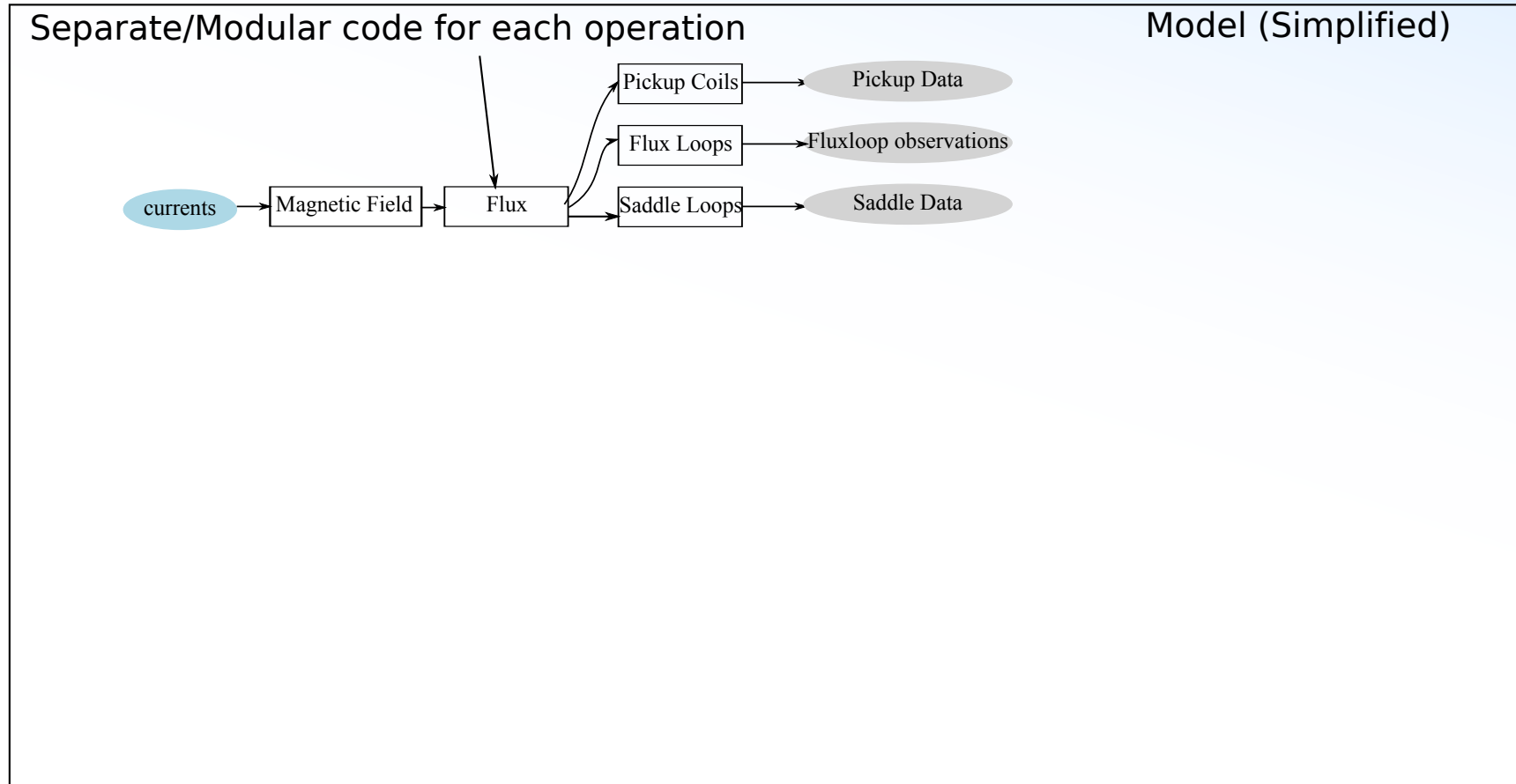
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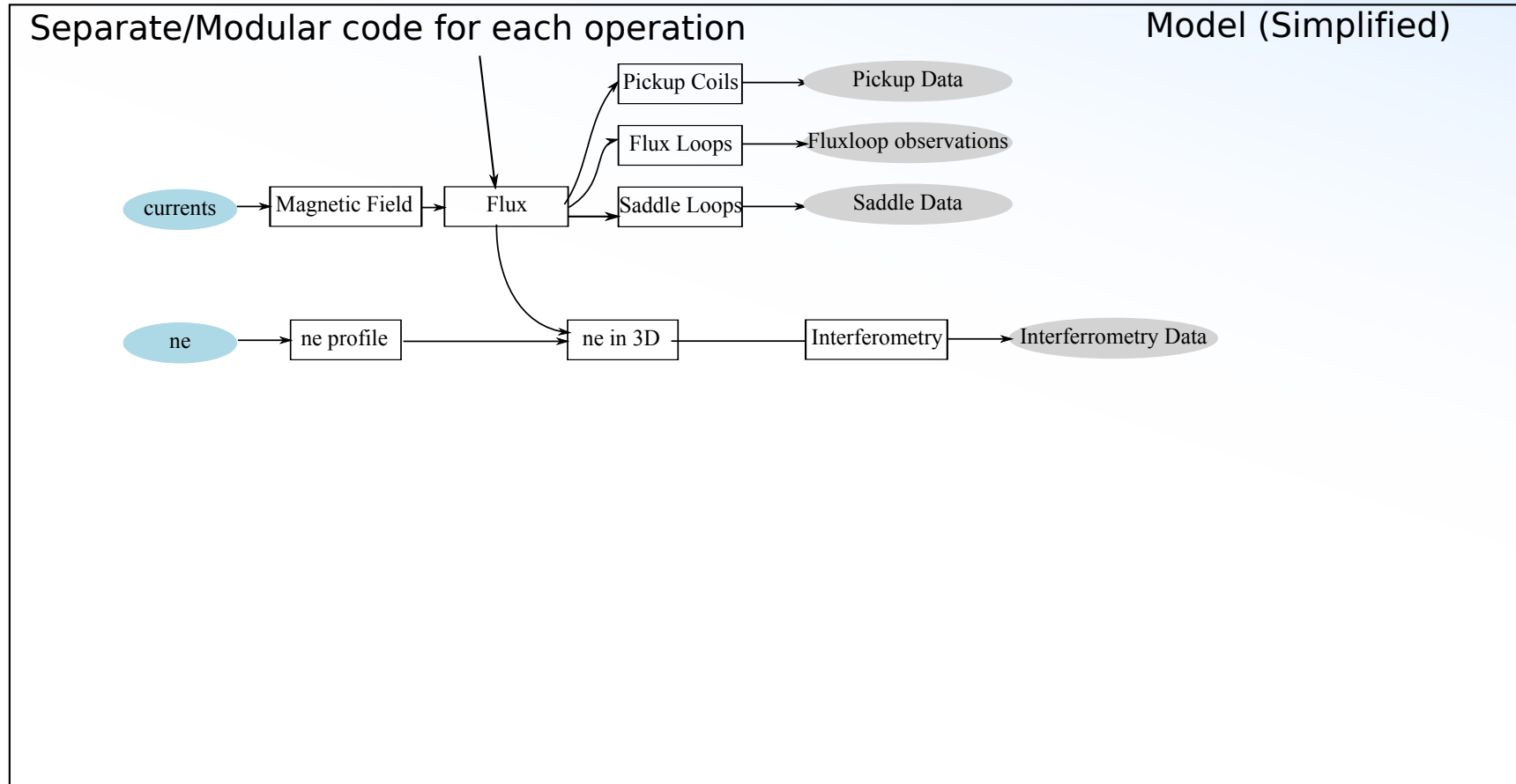
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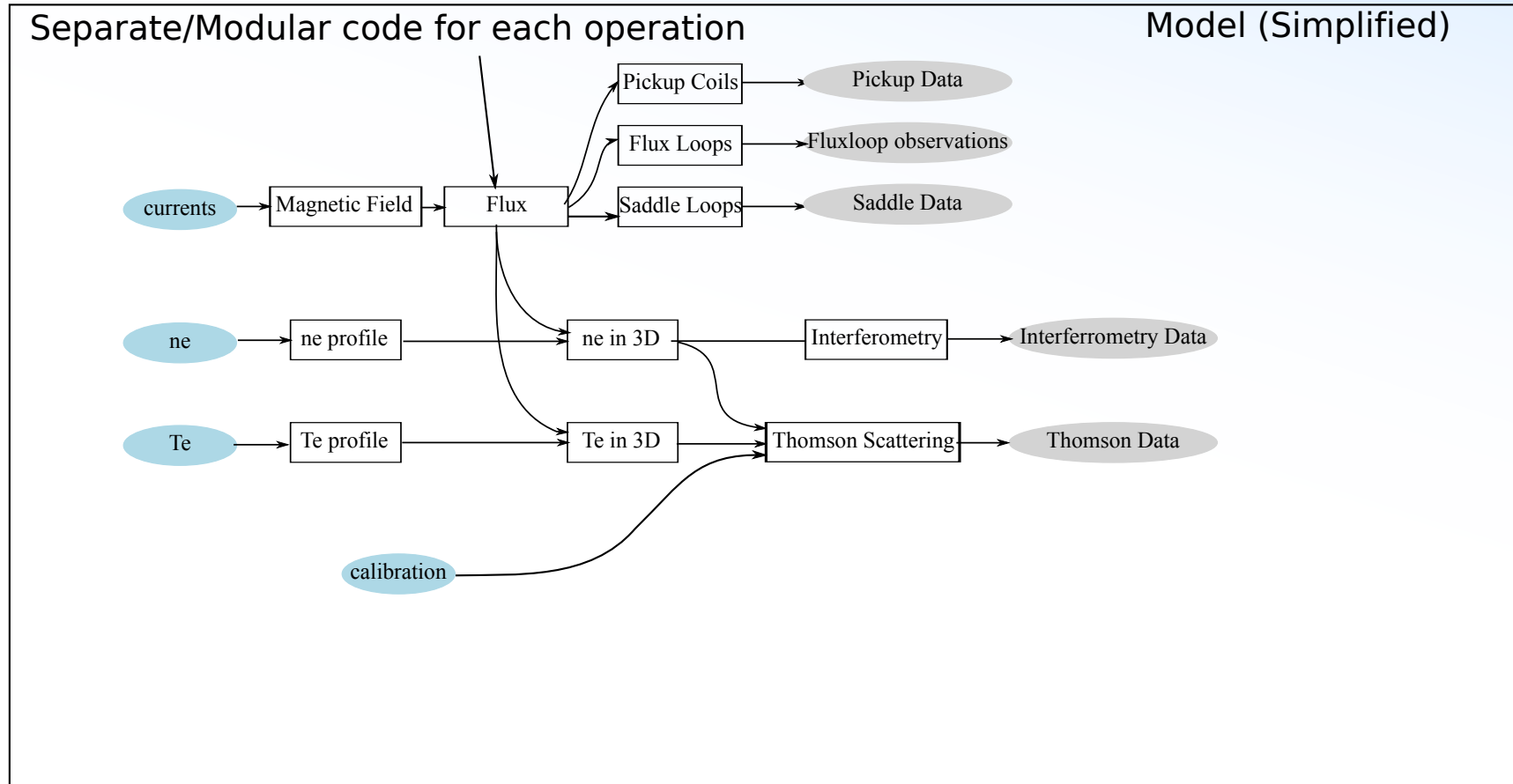
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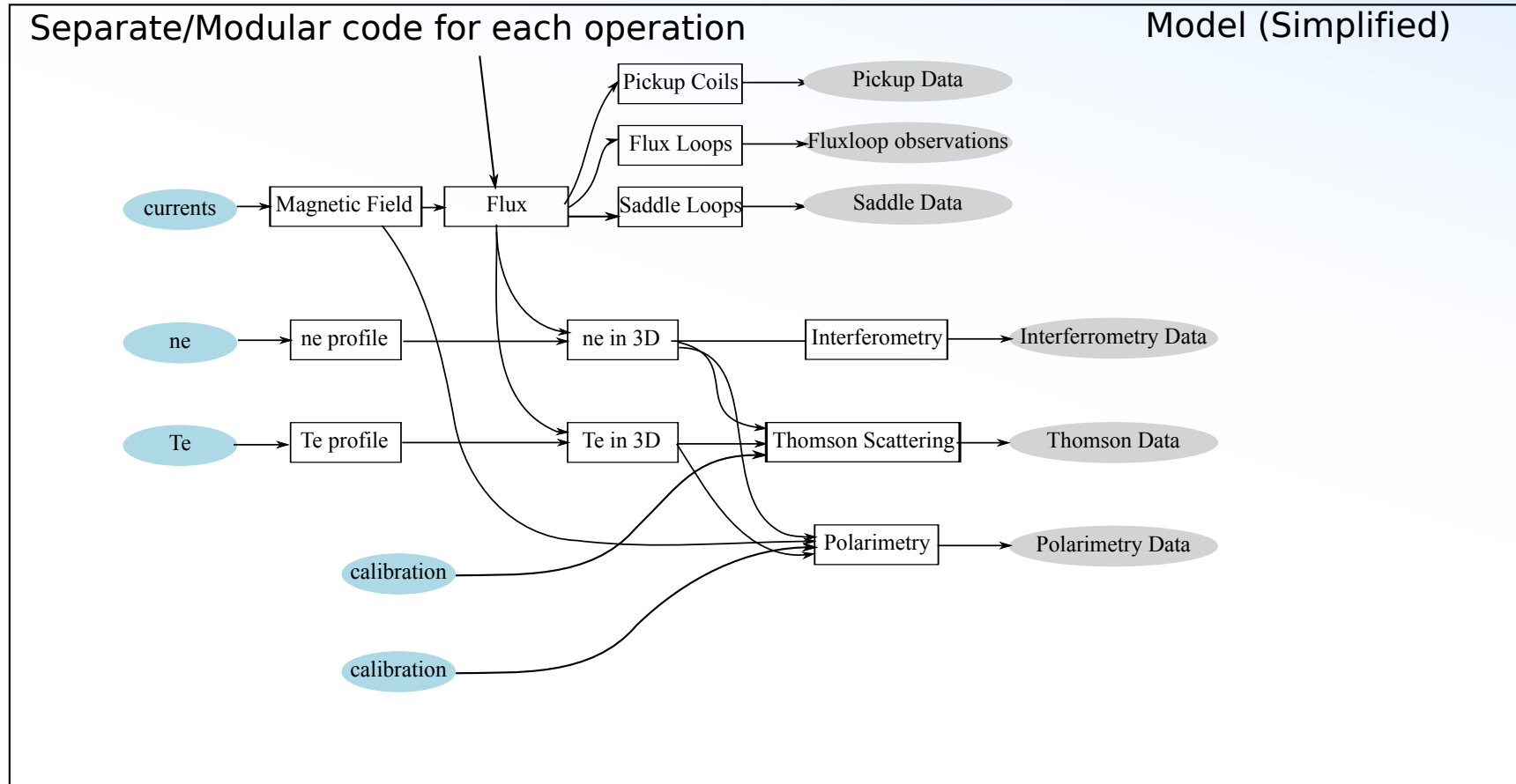
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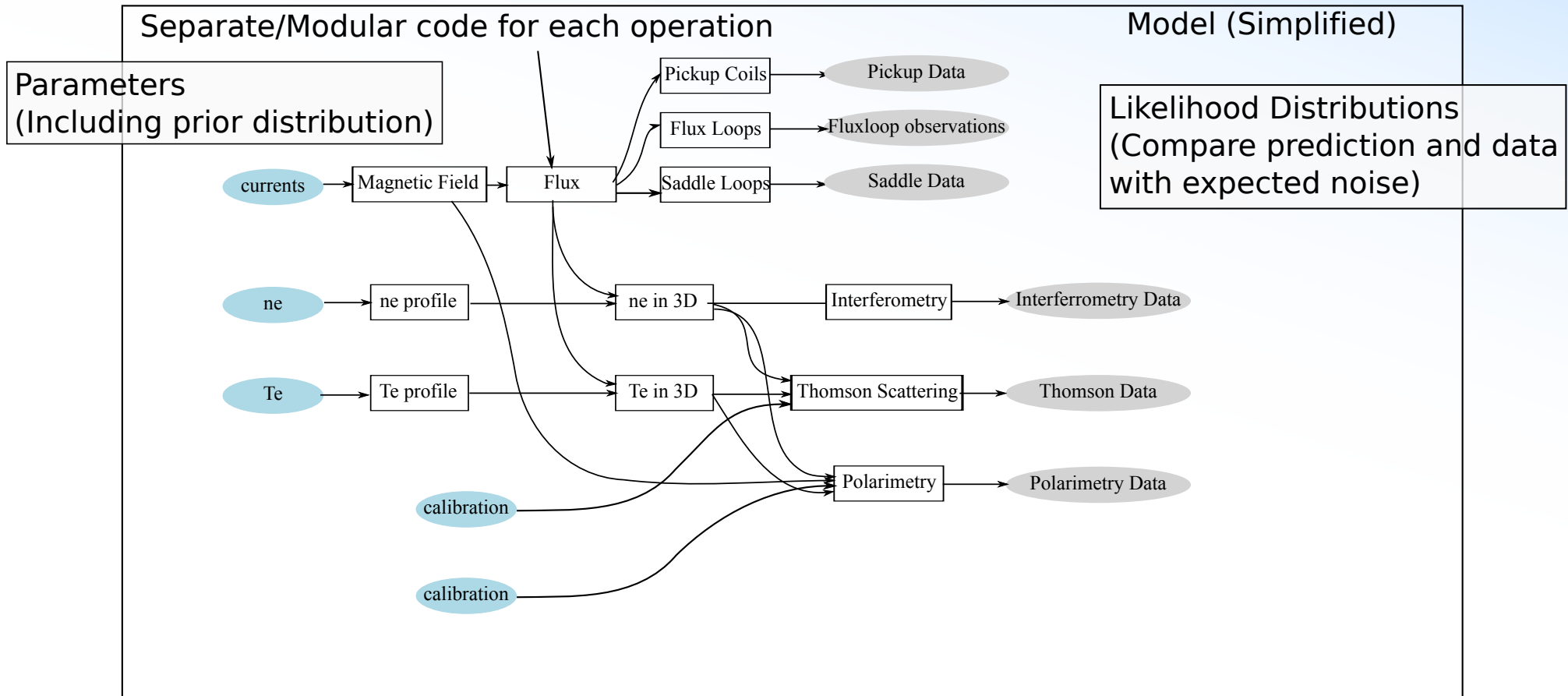
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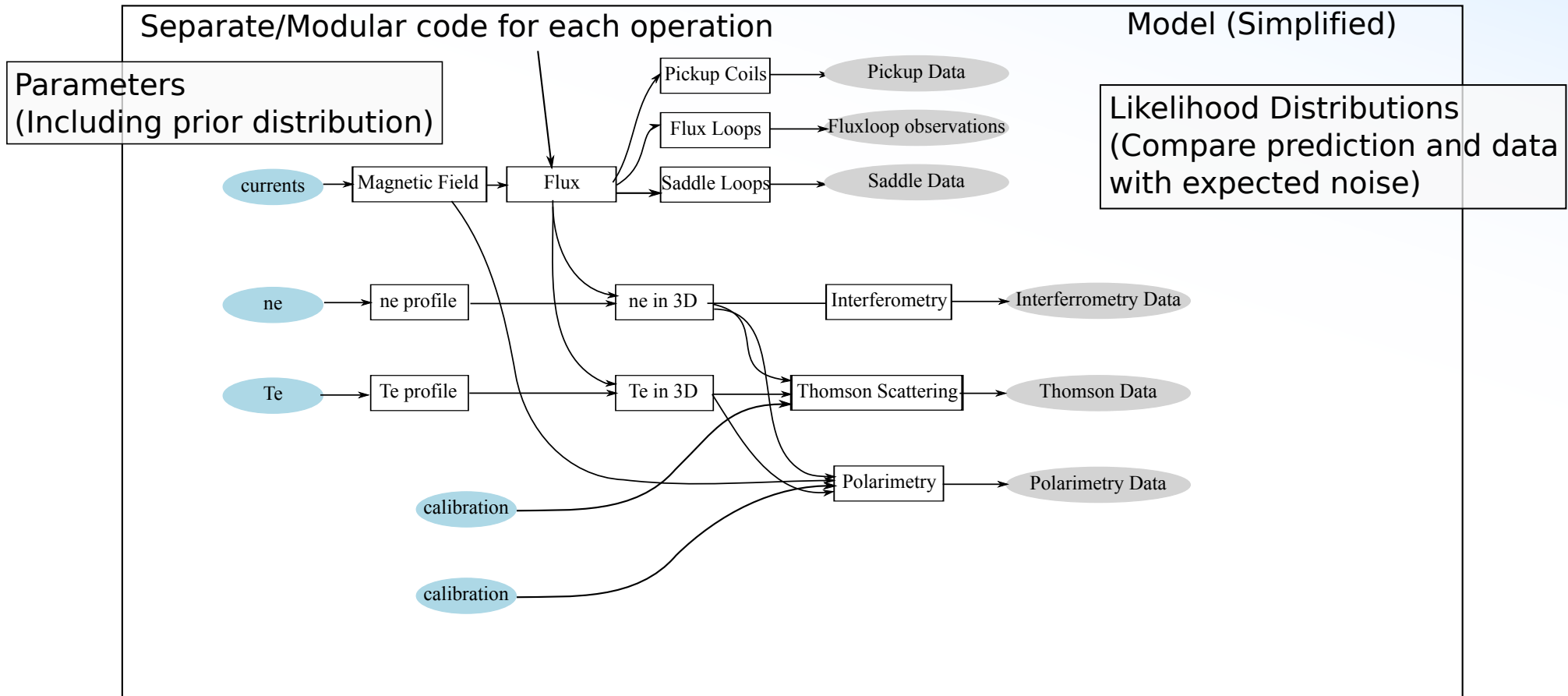
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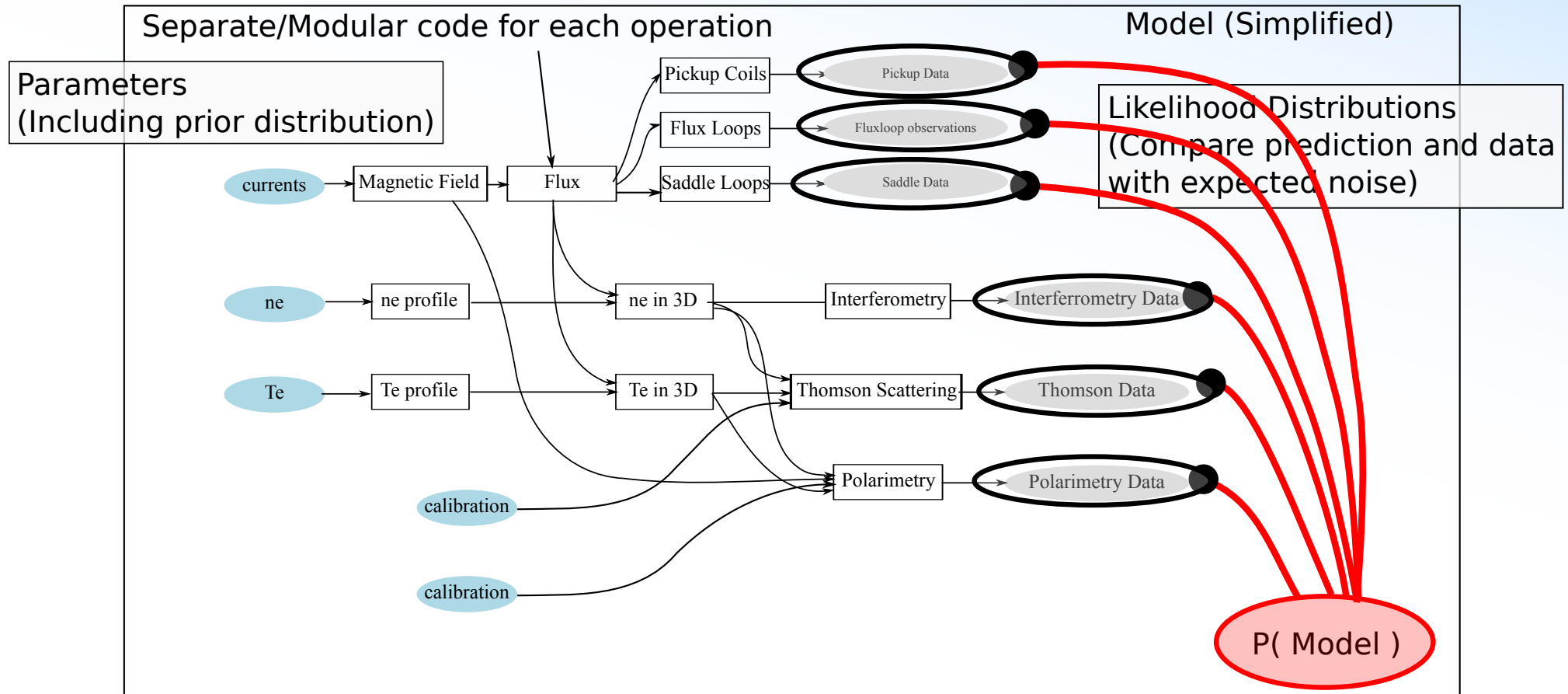


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$$P( Te, Ne, J | Data ) \sim P( D | Ne, Te, J ) P( Te, Ne, J )$$

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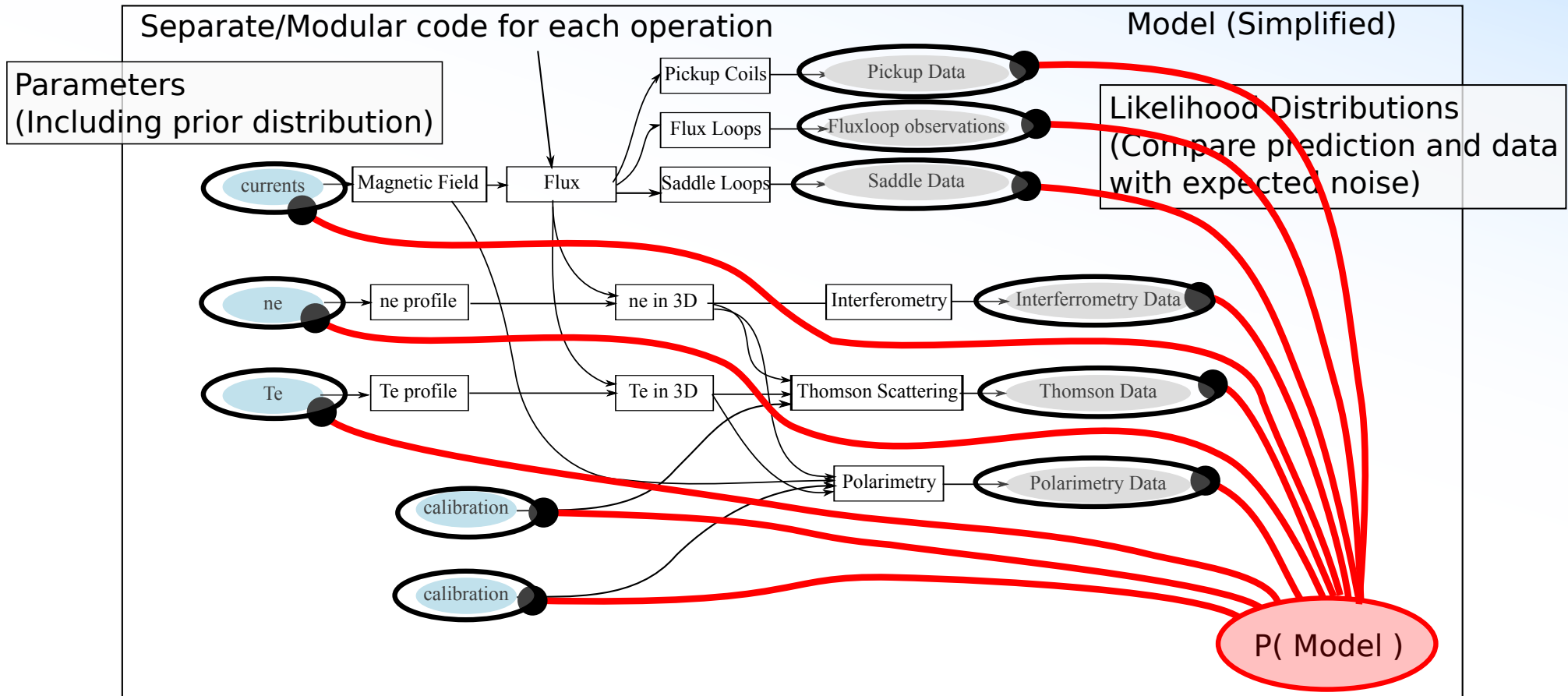


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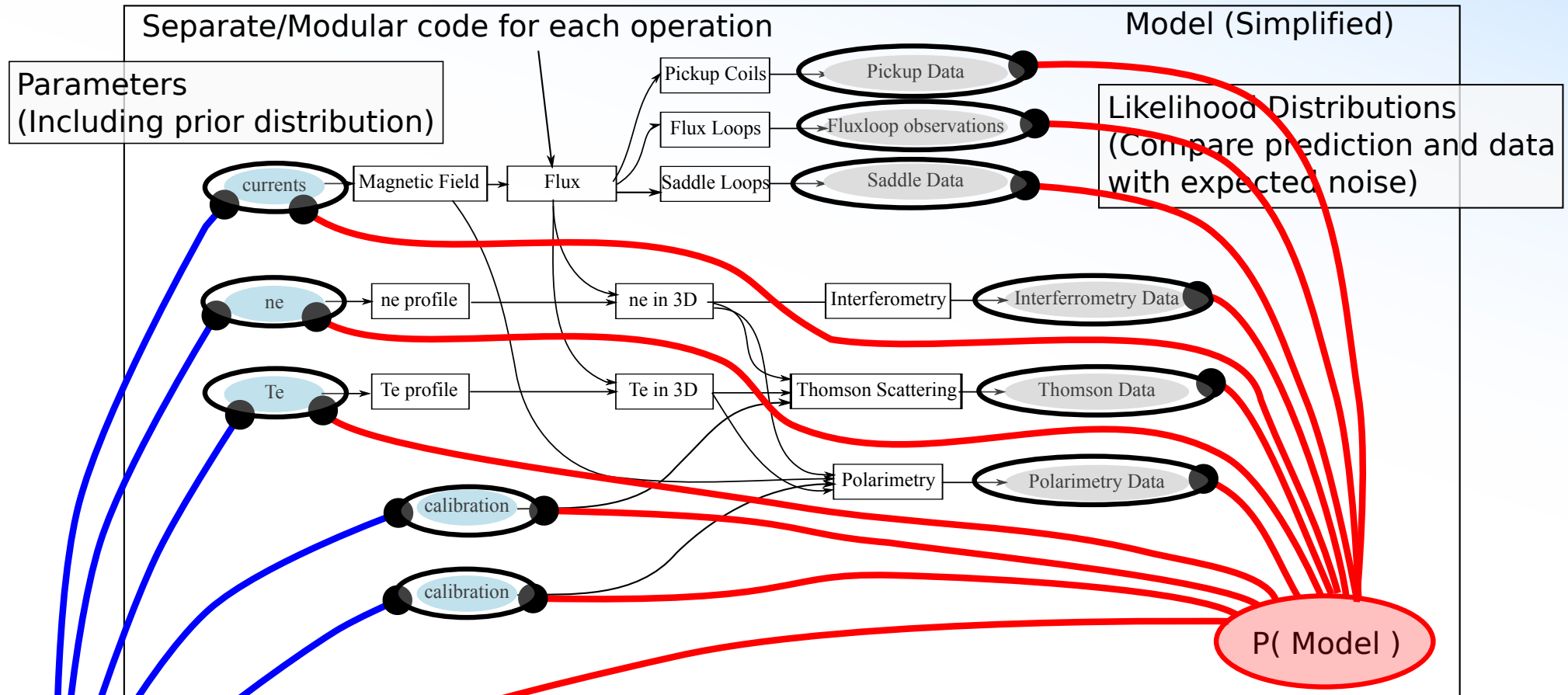


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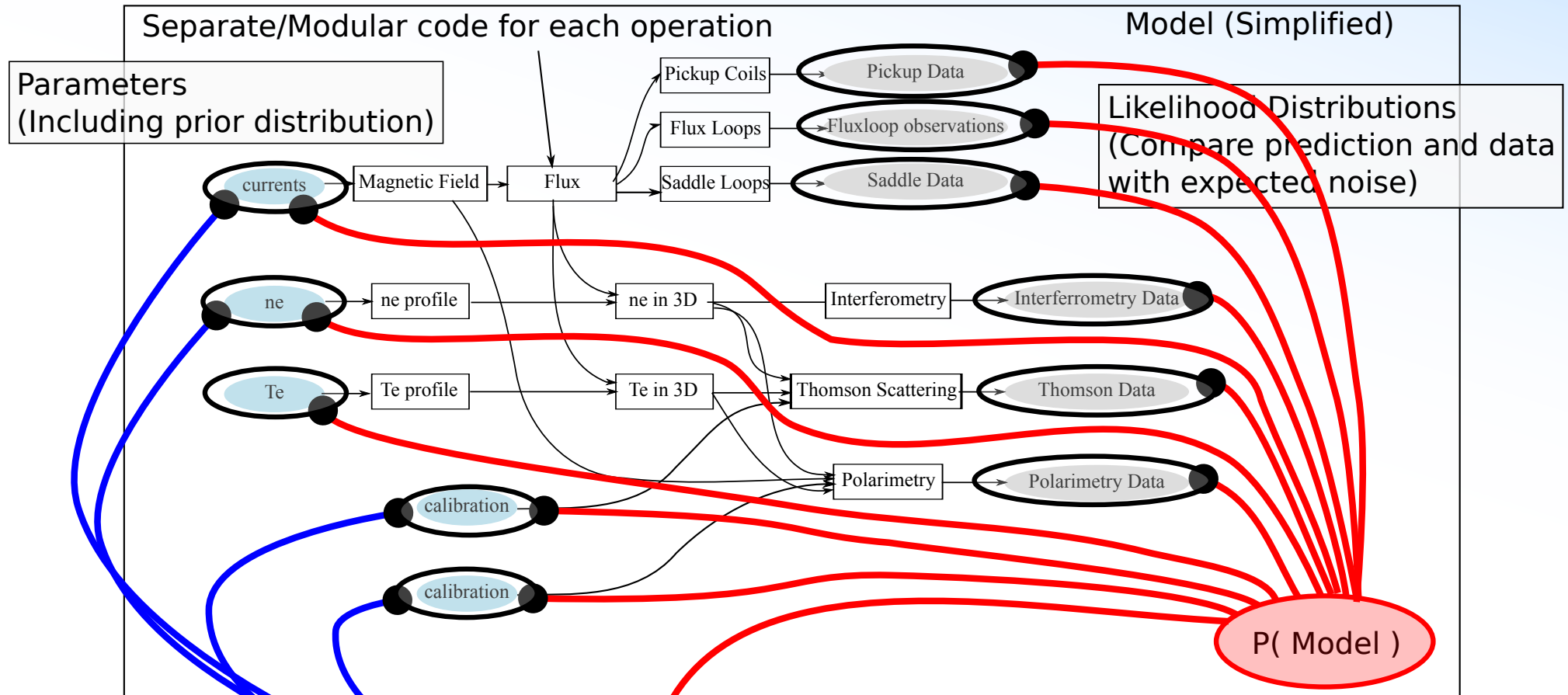
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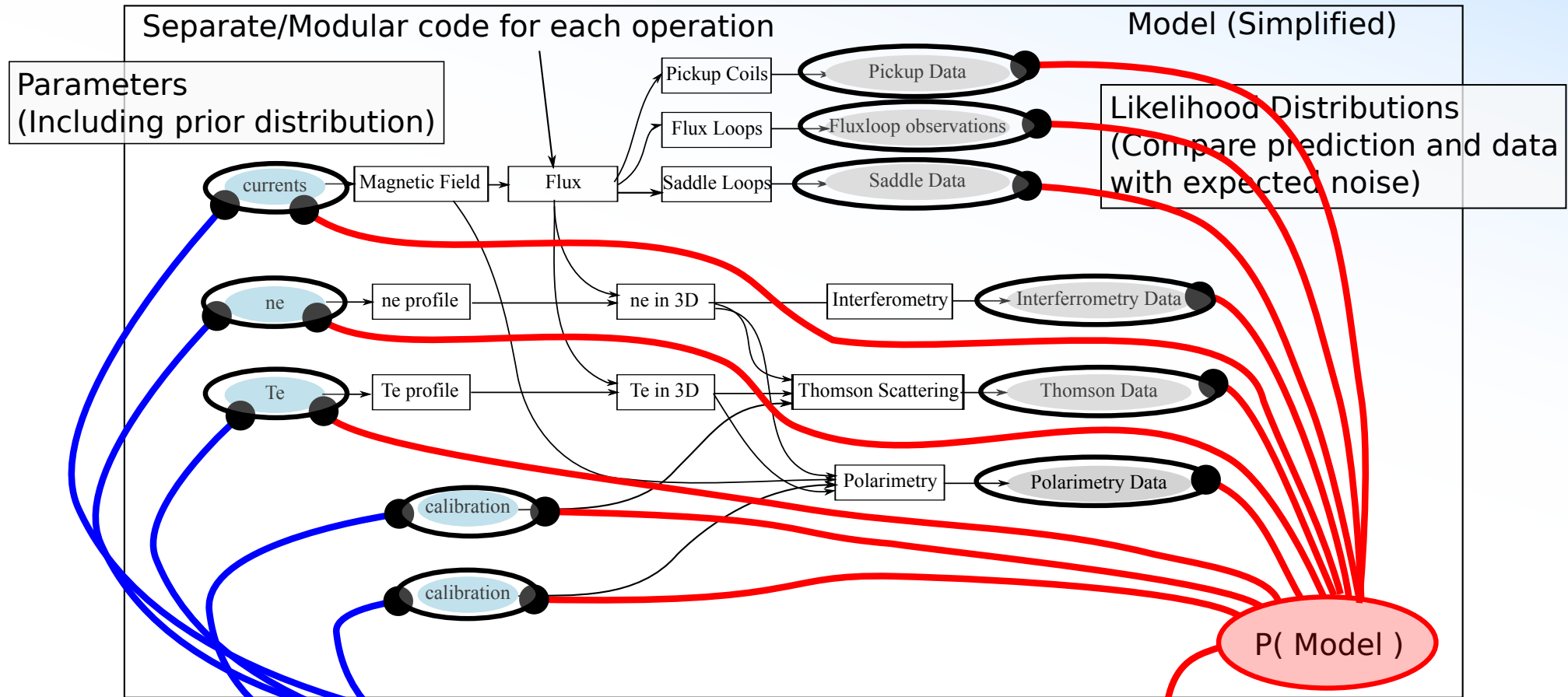
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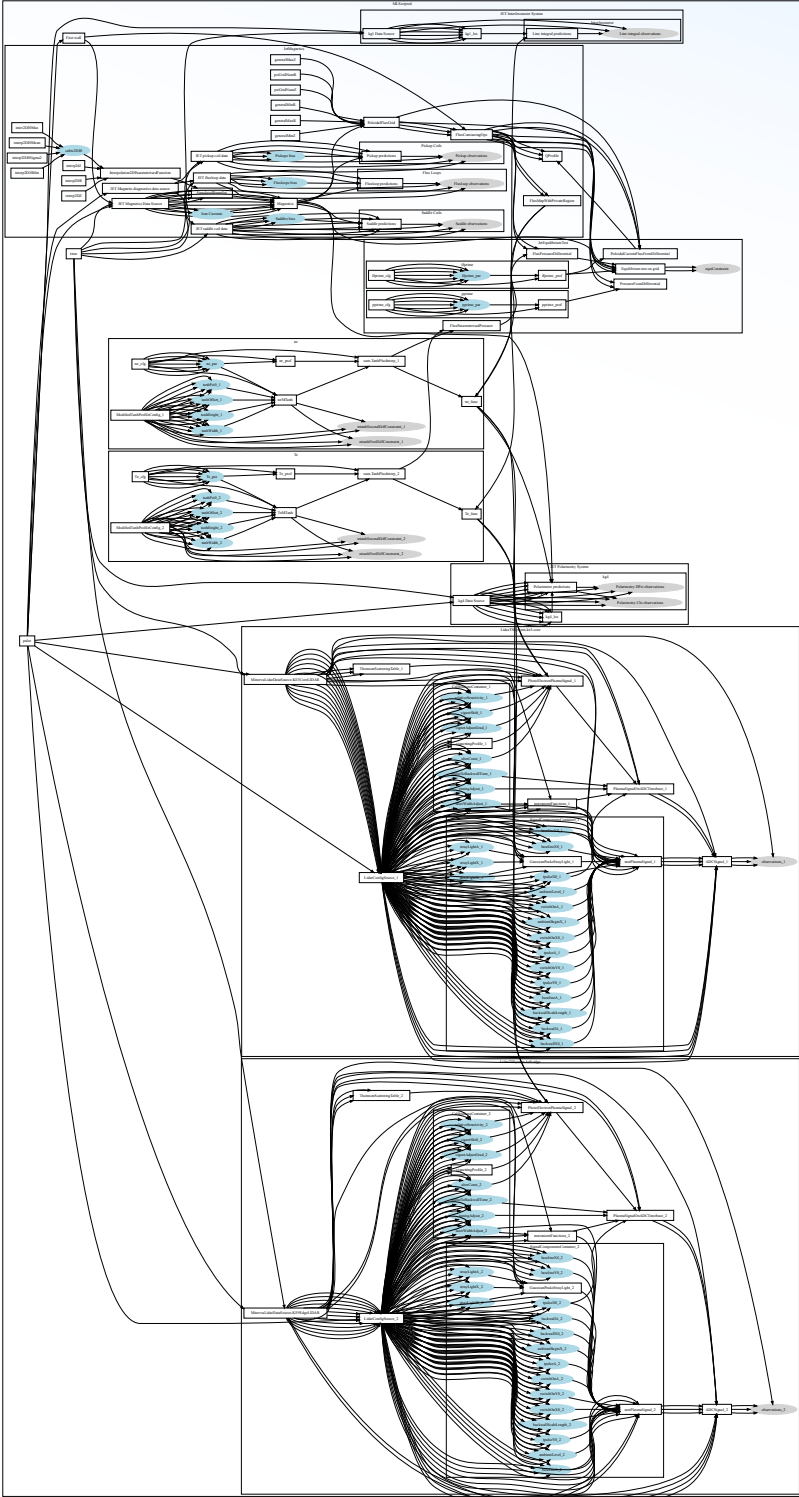
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Metropolis Hastings  
MCMC Non-linear Exploration:  
--> Uncertainty

## Software and Models

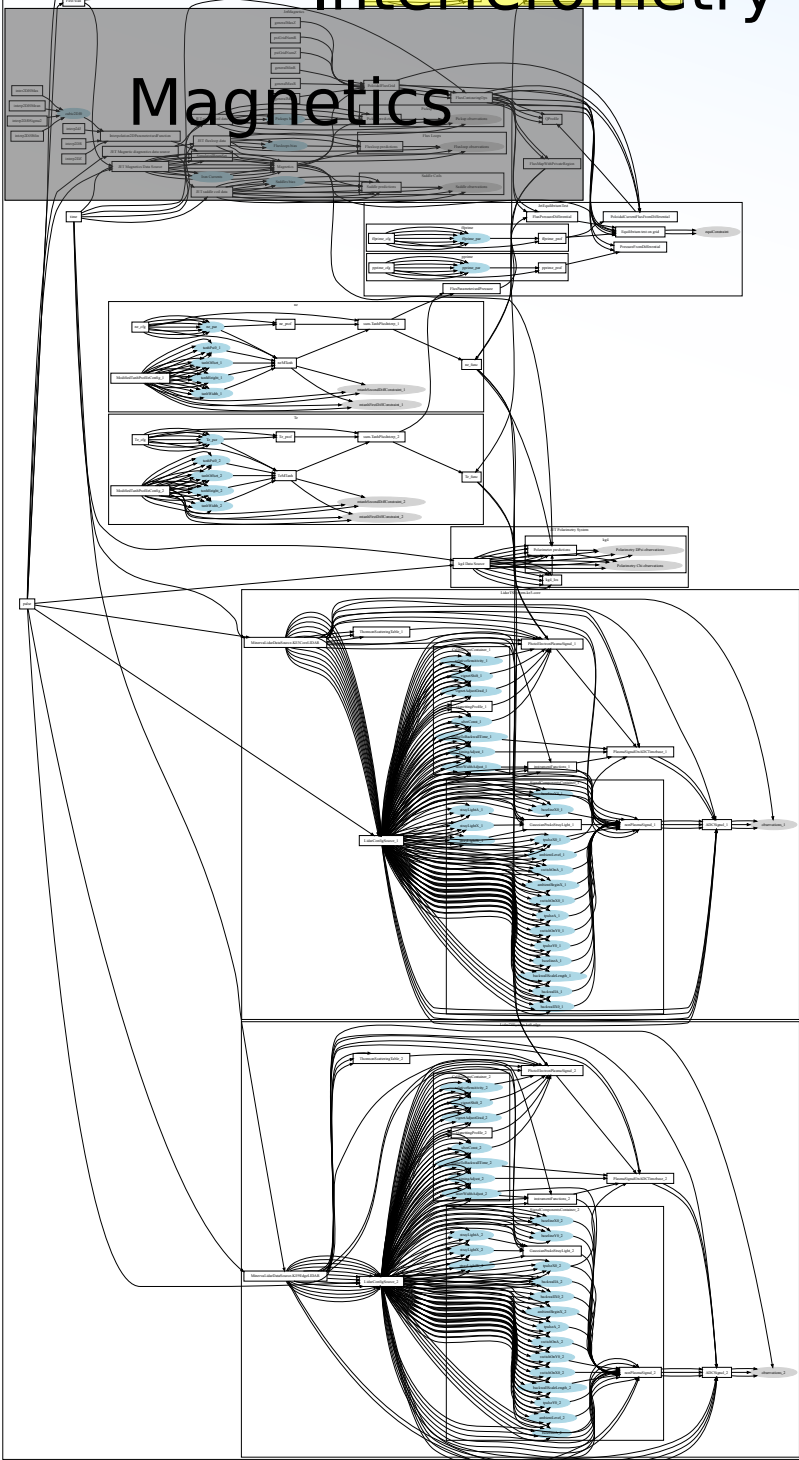
Write nodes and wire them together.  
 Software framework handles the rest.  
 Even automatically generates the graphical representation.  
 We can re-wire the graph and redefine/modify the problem  
 at will, even during a run.





# Interferometry

## Magnetics



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 LIDAR

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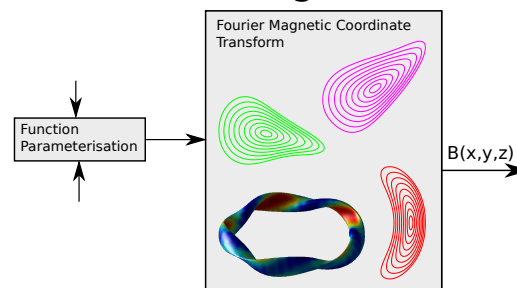
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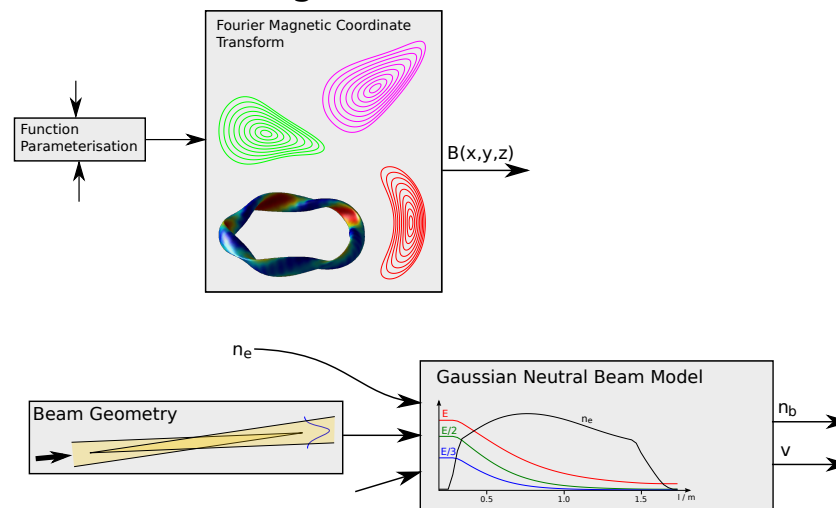
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Neutral Beam model

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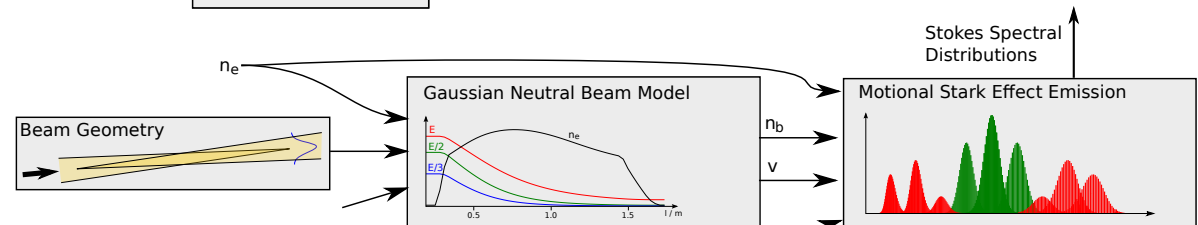
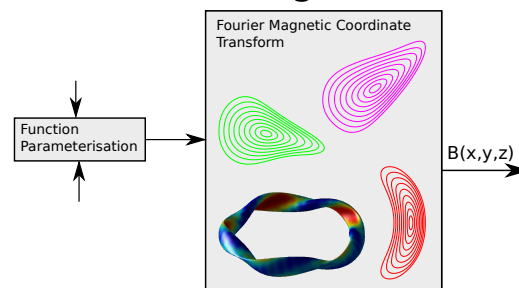
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Neutral Beam model

General MSE Emission



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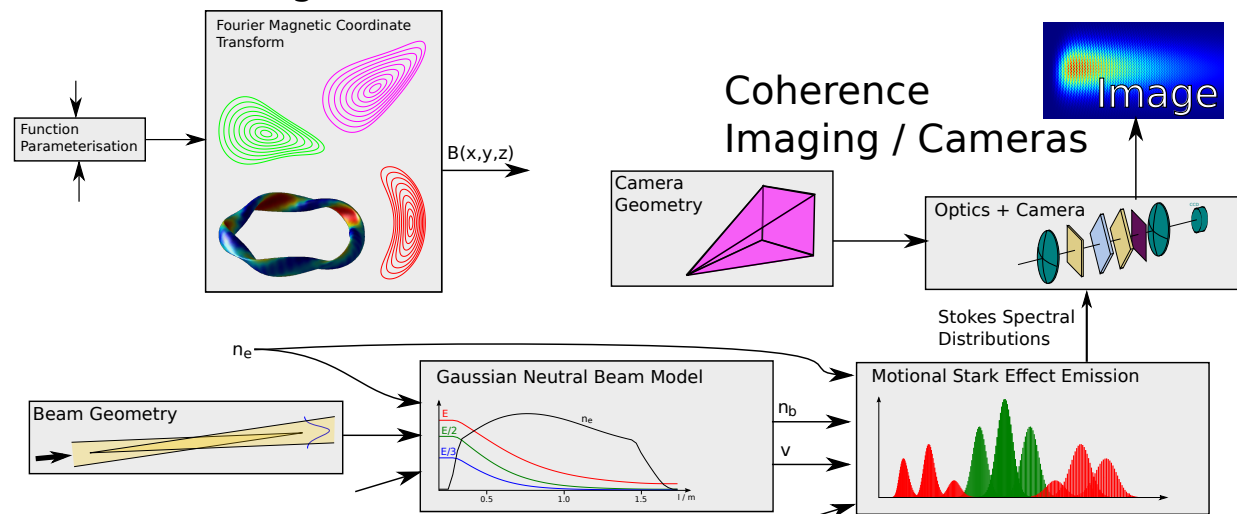
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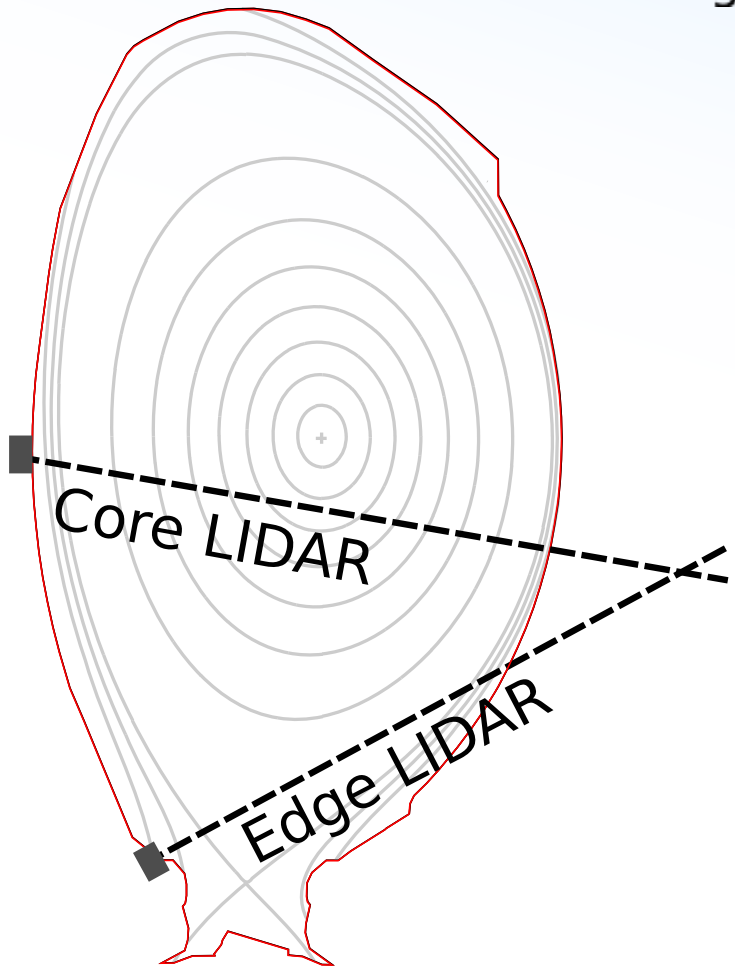
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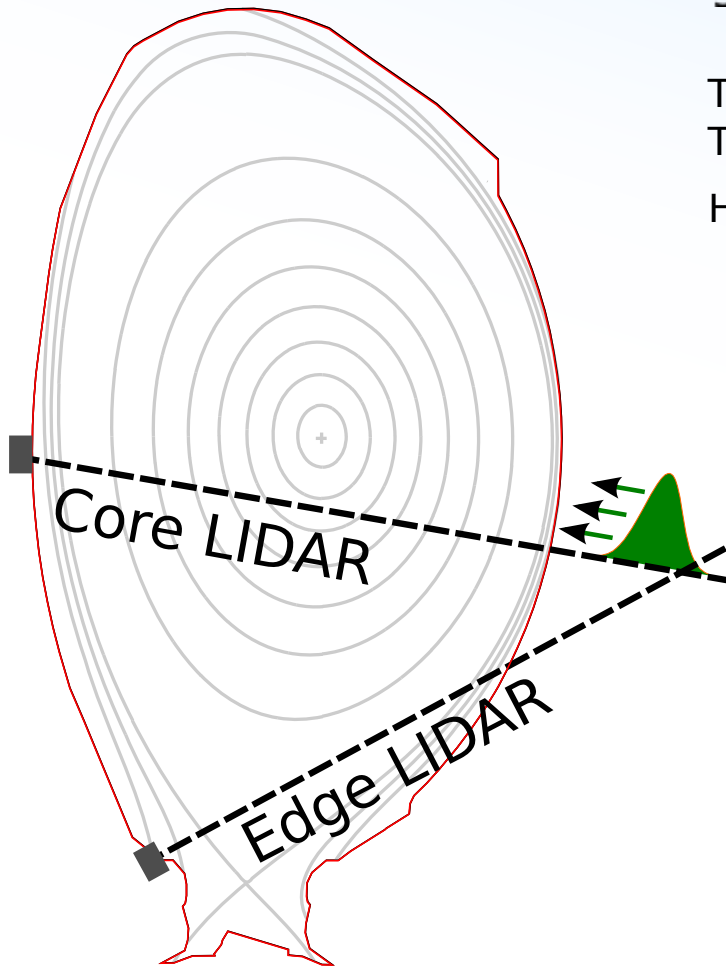
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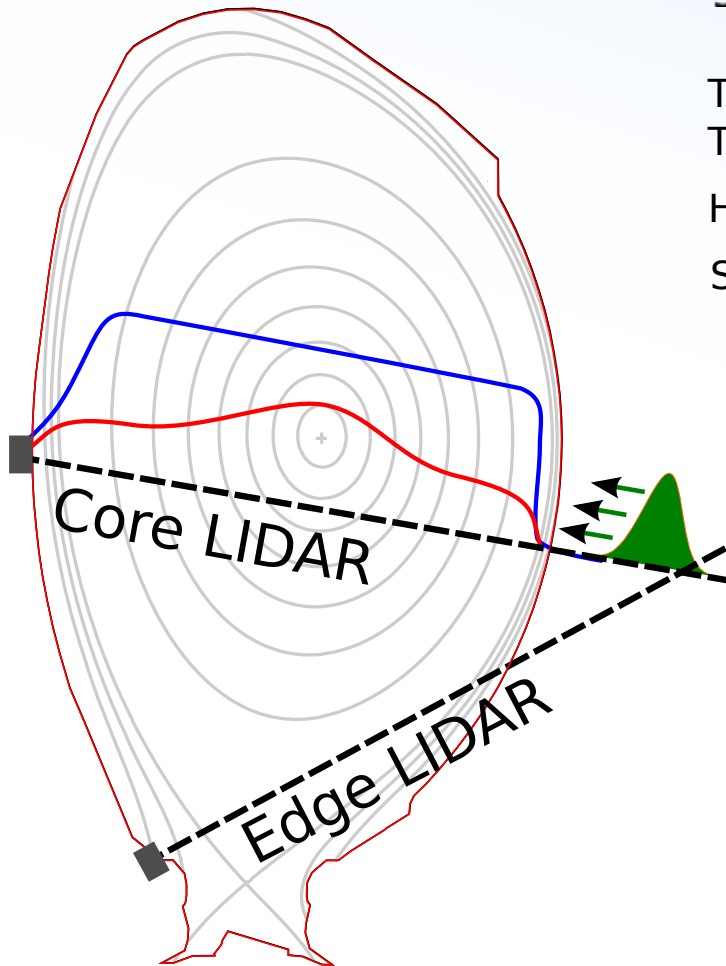


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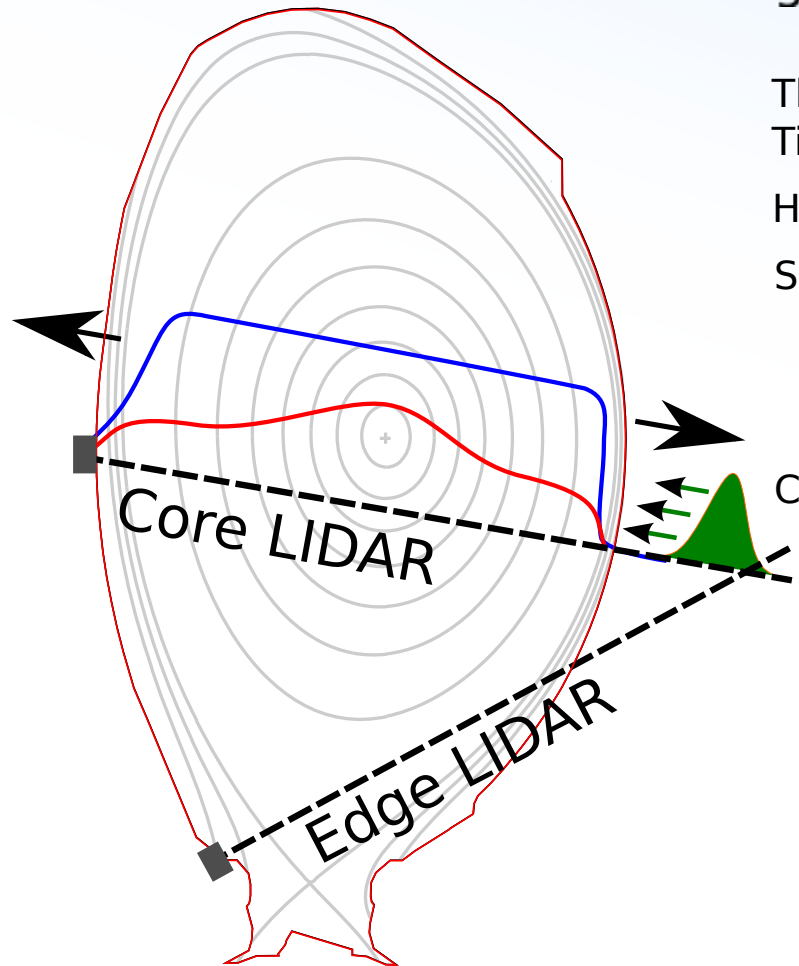
Spatial Resolution:

Effective convolution of light signal.

If ignored: Convolves  $n_e$  but complex effect on  $T_e$ .

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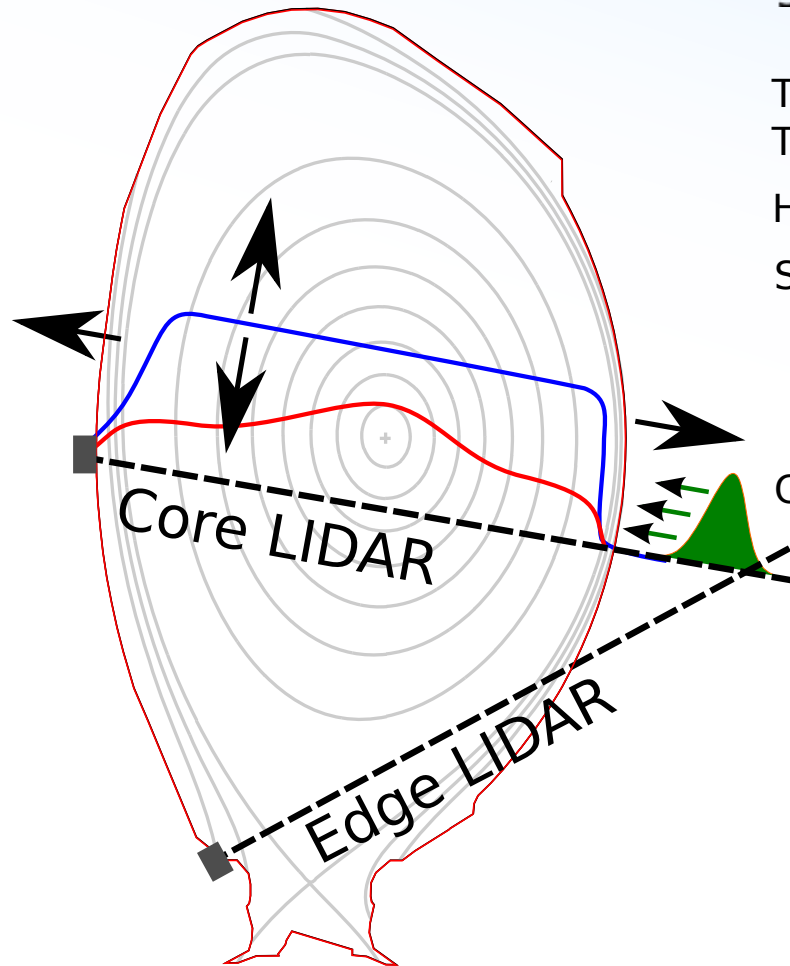
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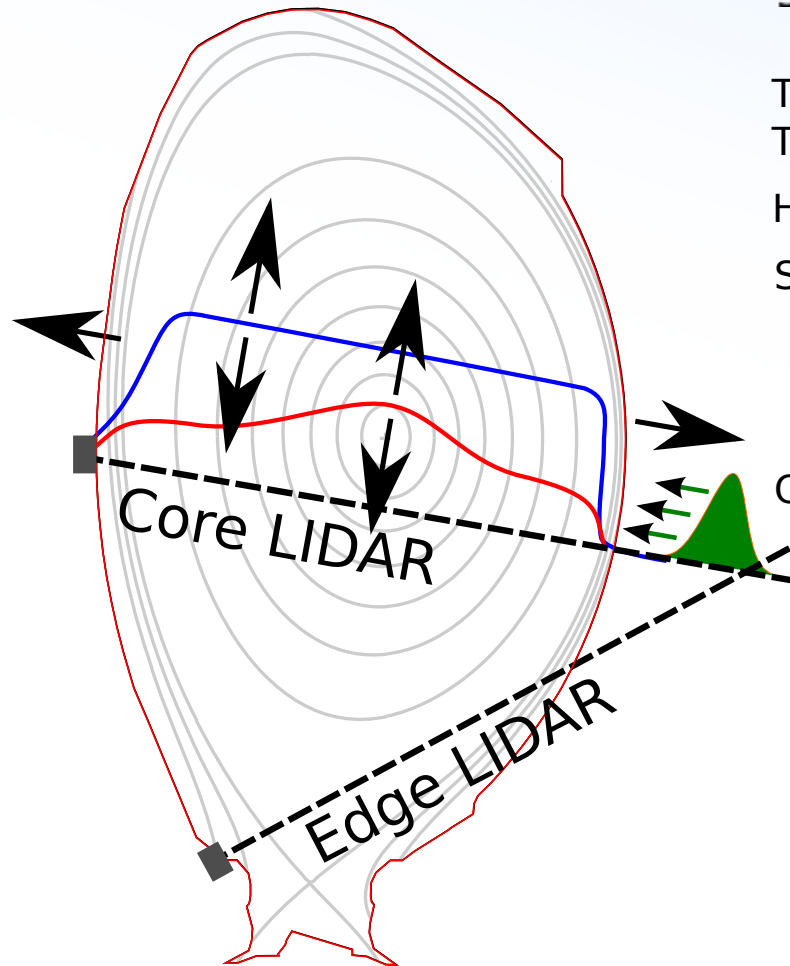
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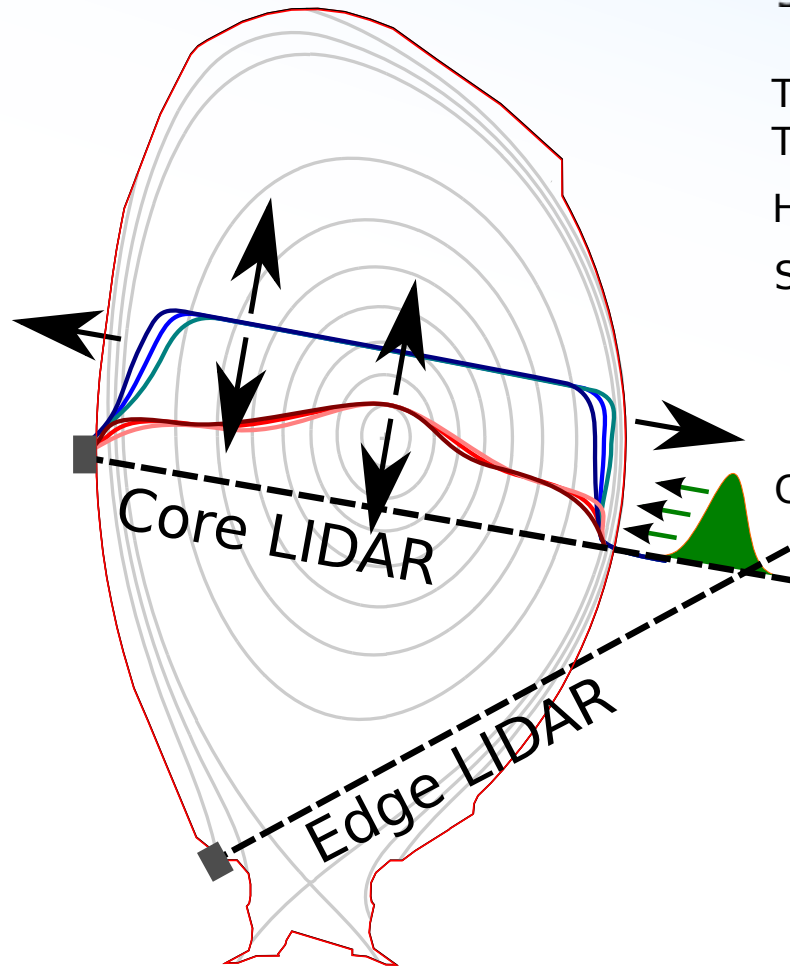
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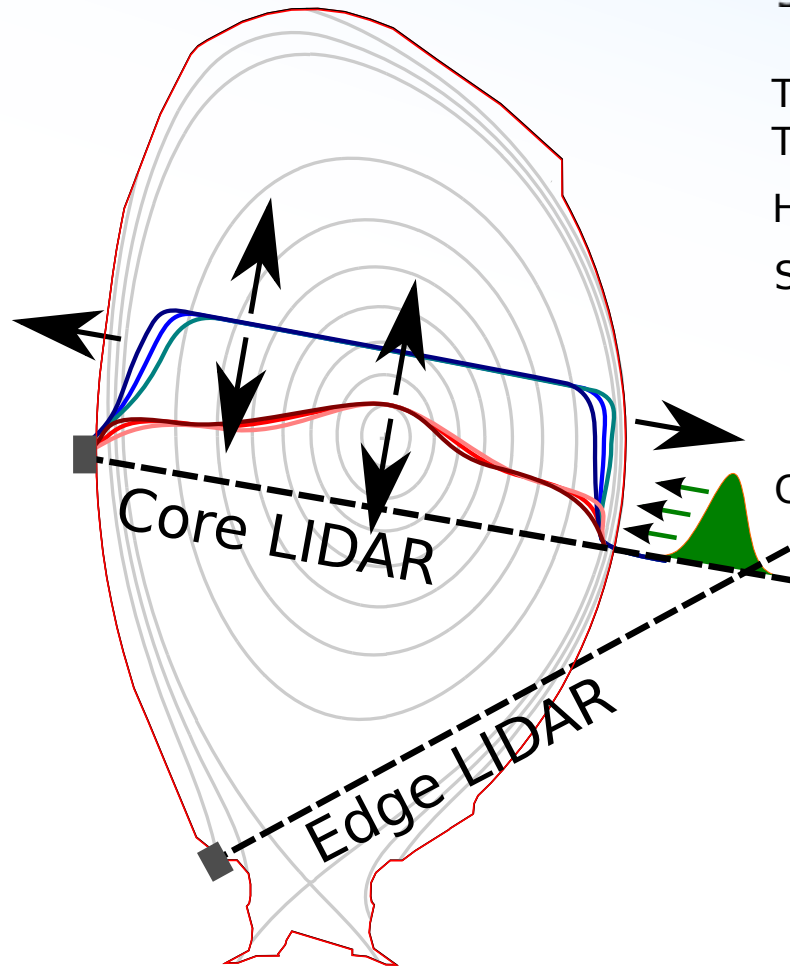
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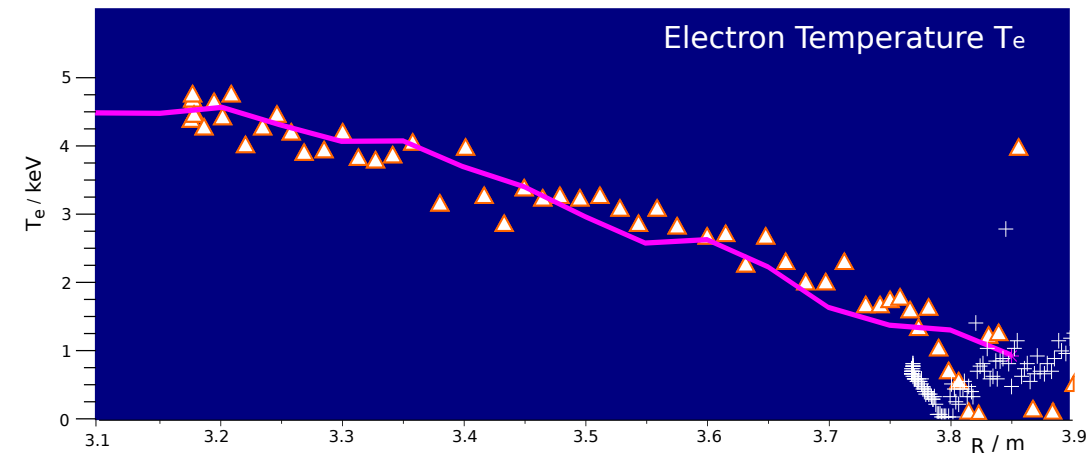
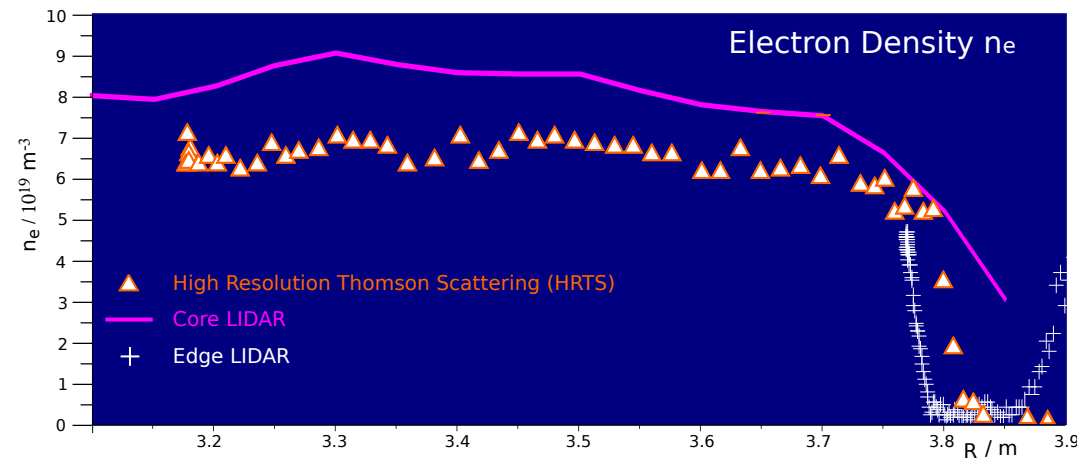
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Shift and scale output profiles to match?

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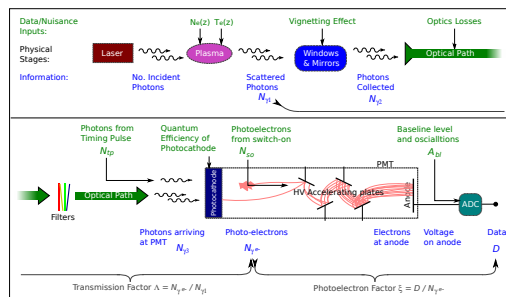
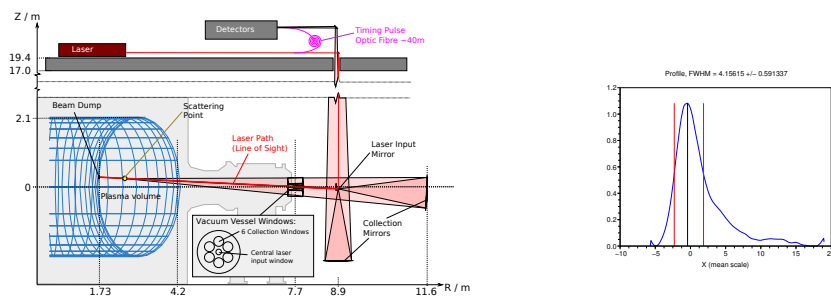
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We must really understand each part of the system:

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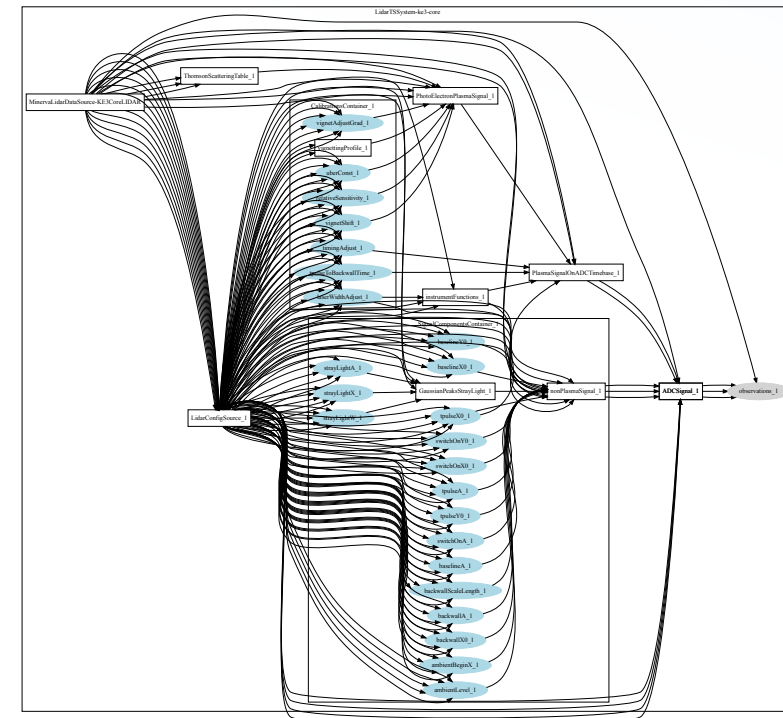
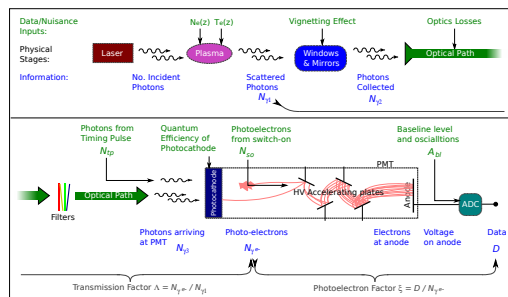
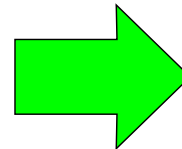
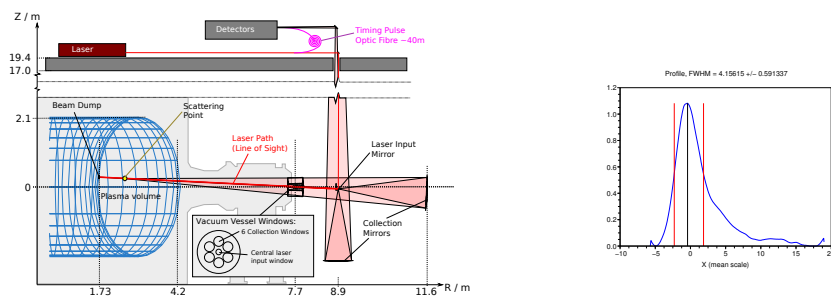
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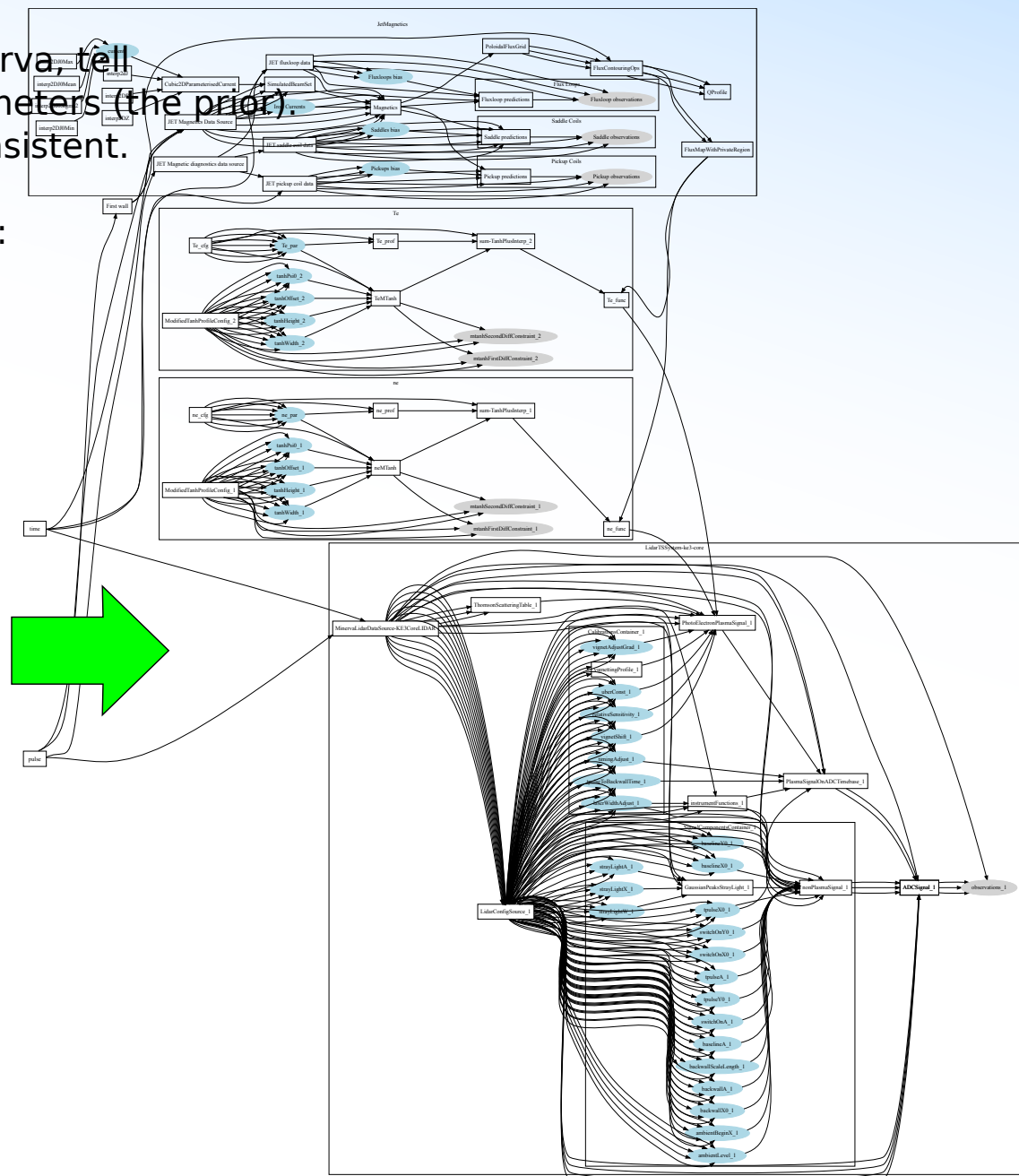
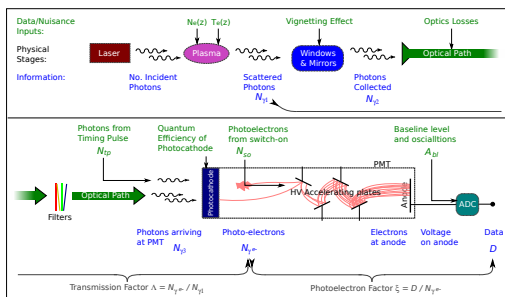
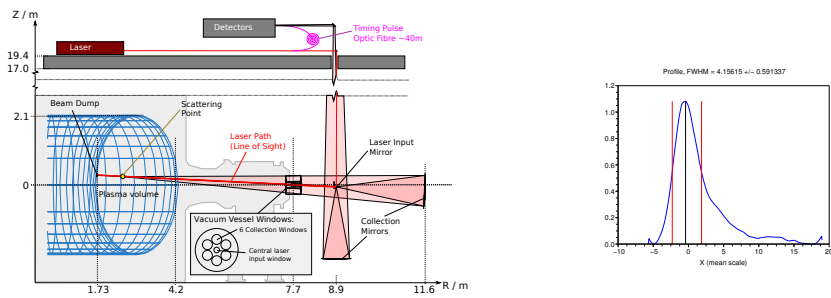
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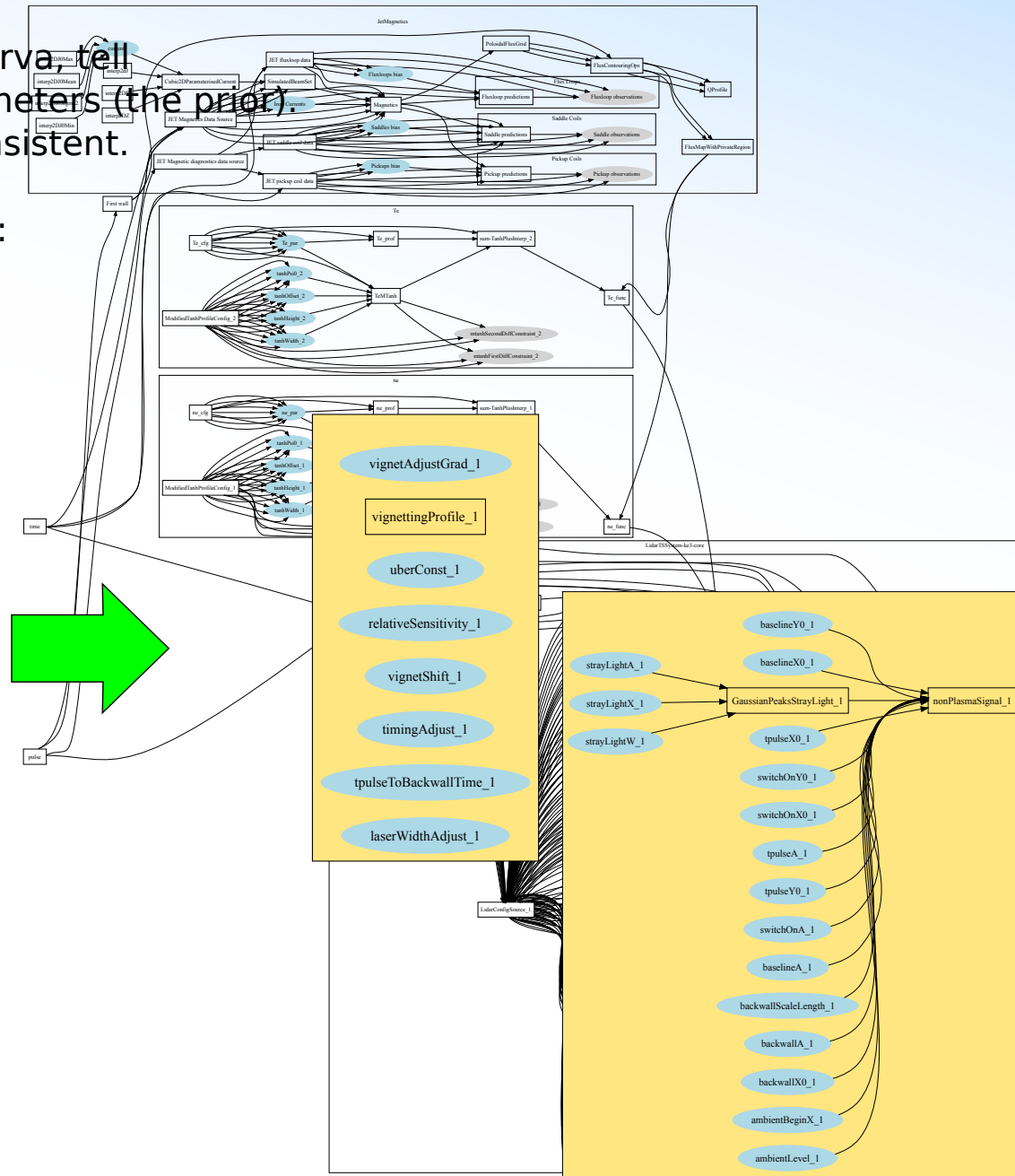
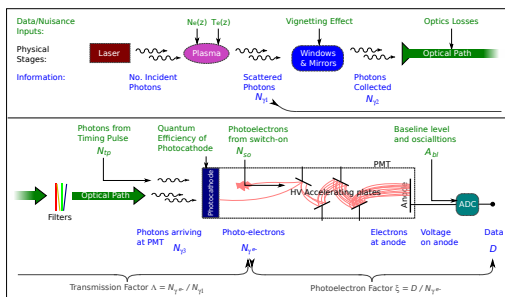
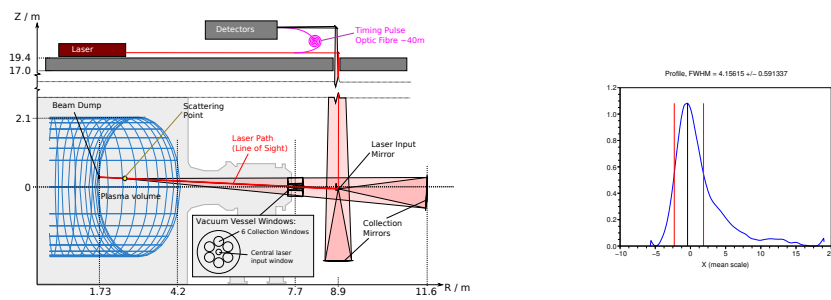
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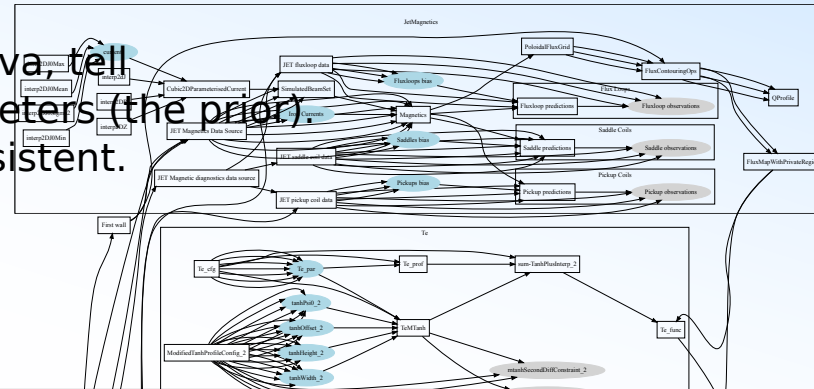


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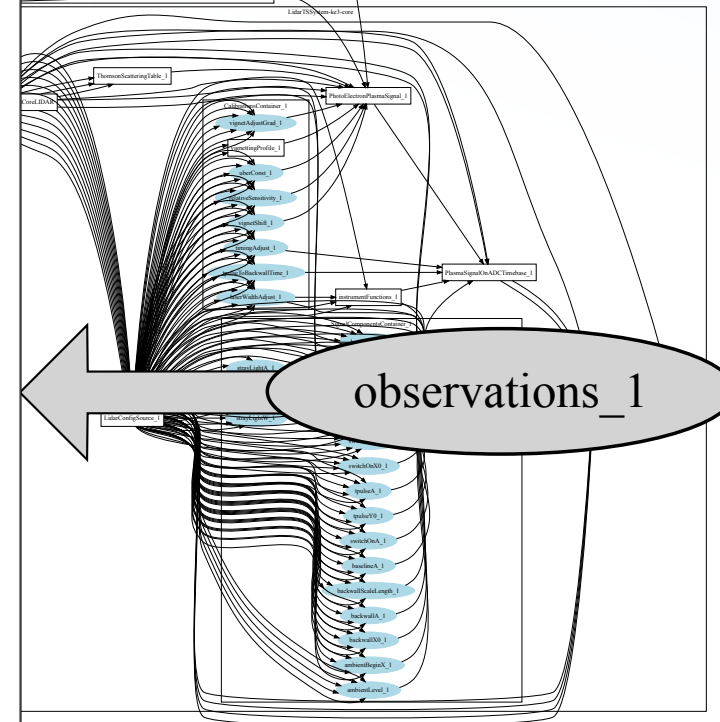
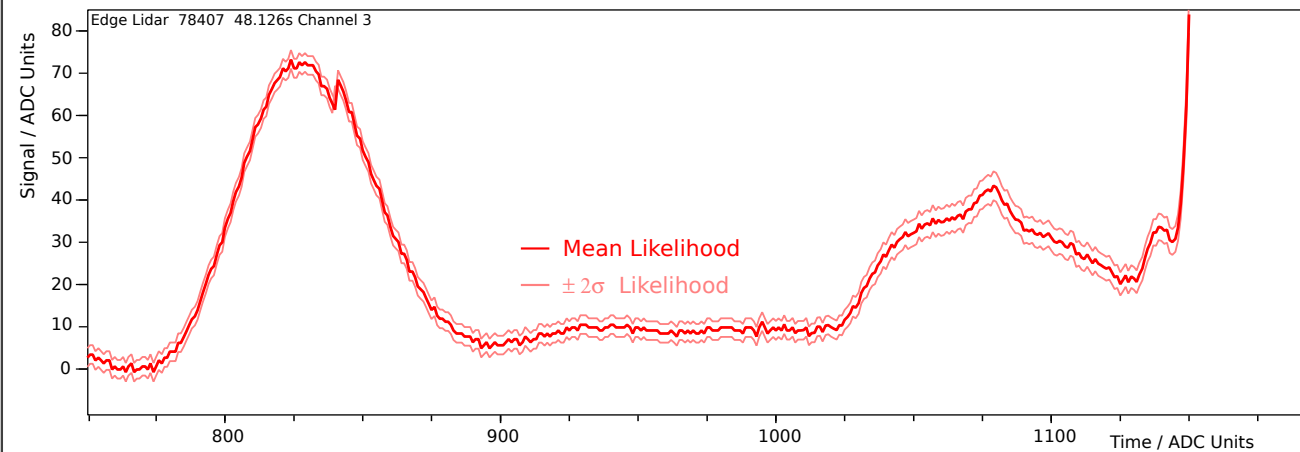
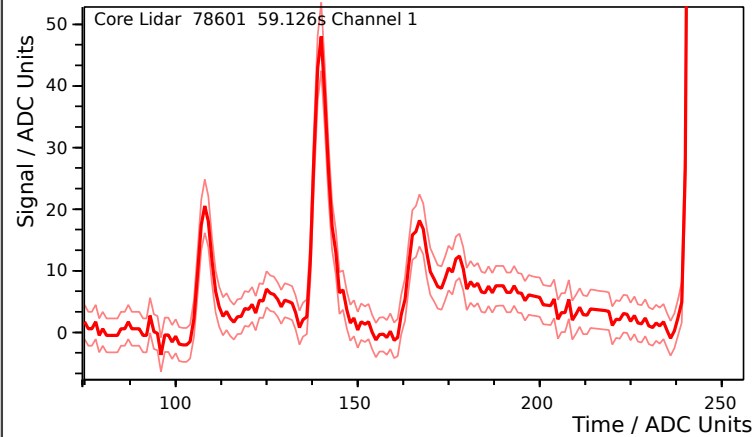
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Model predicts all the data

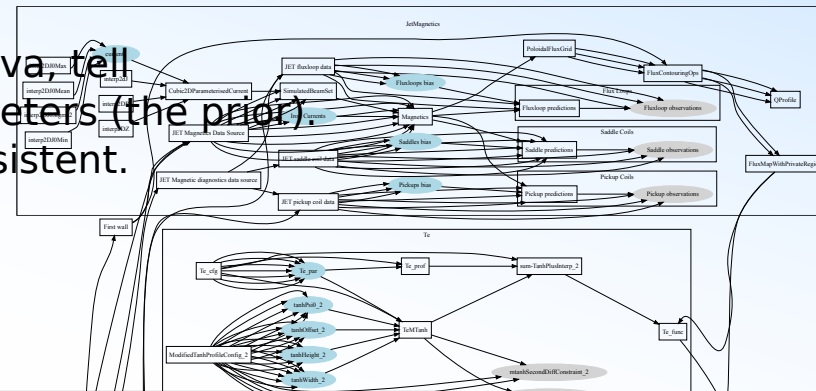


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So how do we deal with disagreement with other diagnostics?

Shift and scale output profiles to match?

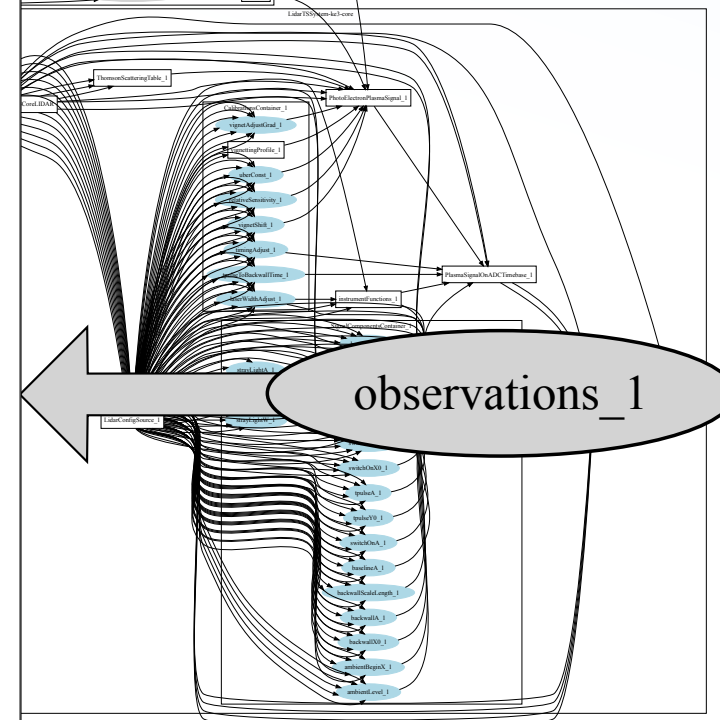
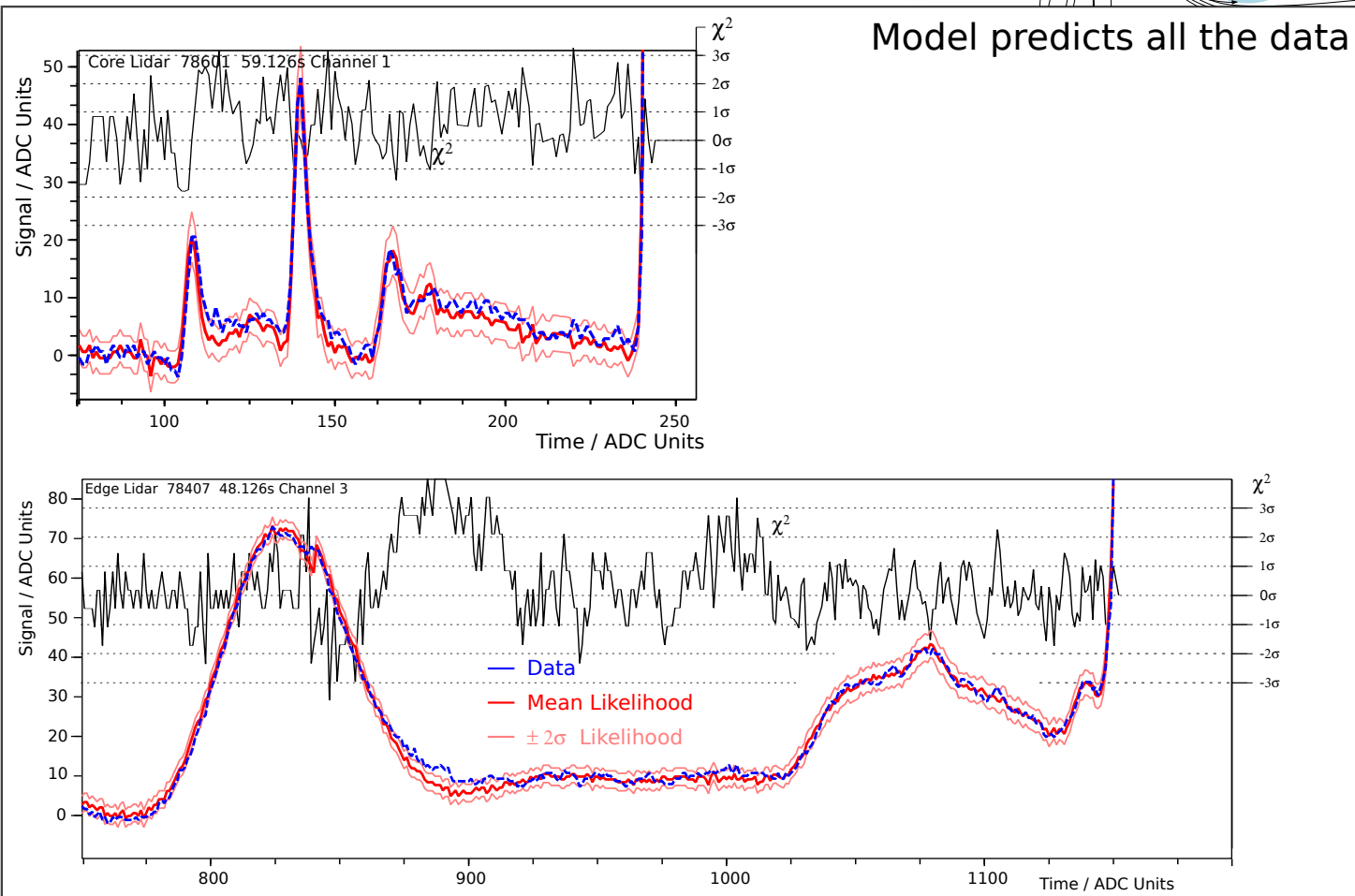
No - Build the model for each and wire up to Minerva, tell it what we do know about the calibration parameters (the prior) and let it work out how to make everything consistent.



We must really understand each part of the system:

Laser Pulse, TS physics, Optics, Filters, Photomultipliers, Counting Noise (PDFs), ADCs.

Model predicts all the data



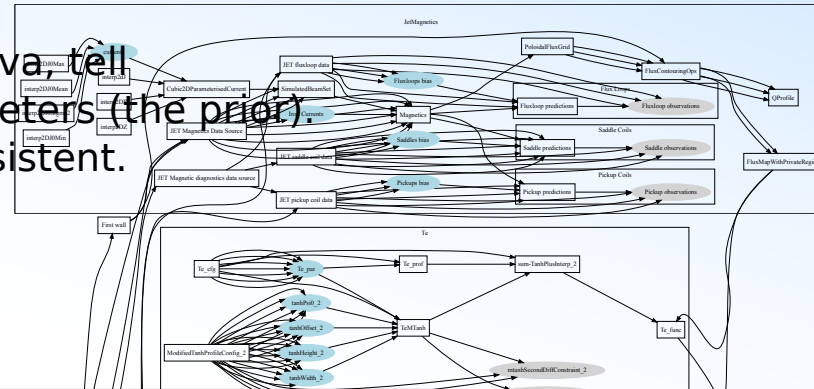


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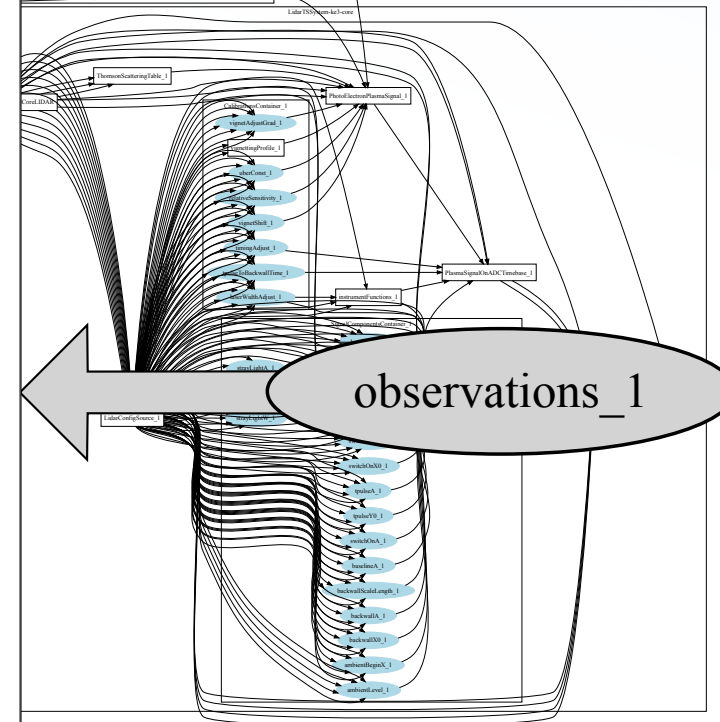
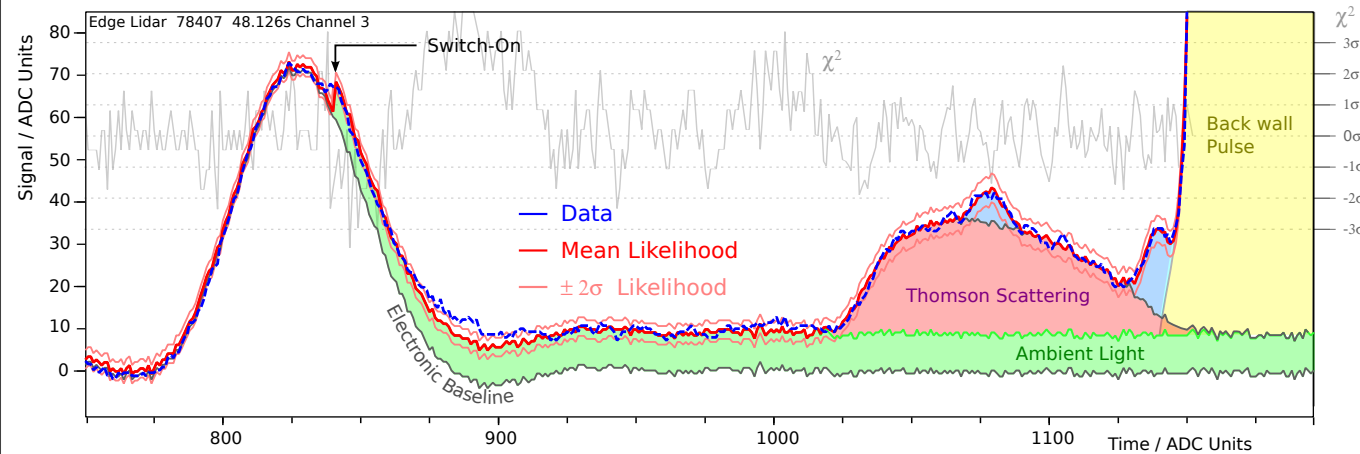
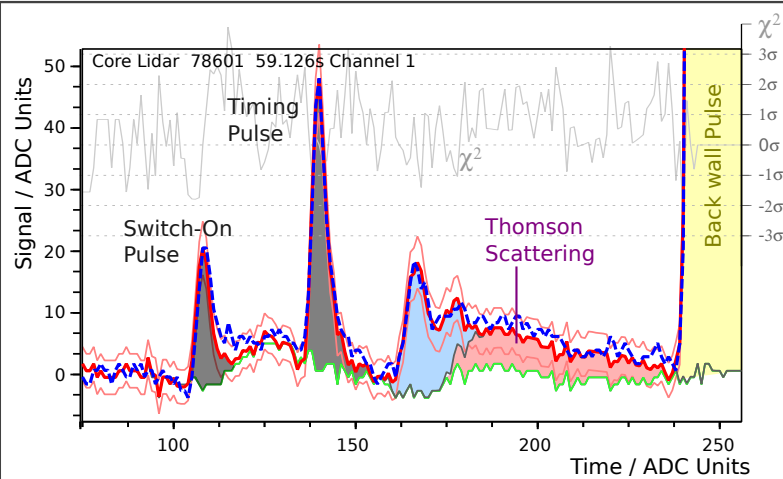
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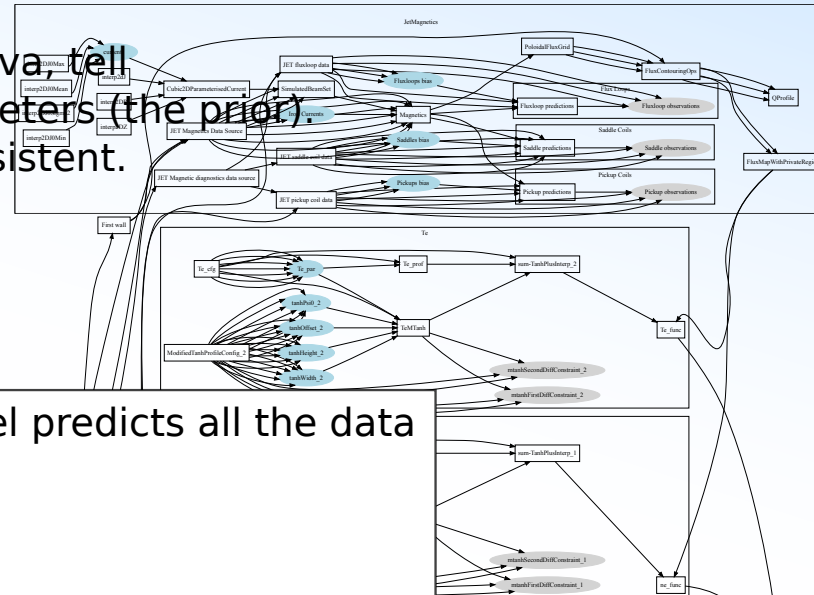


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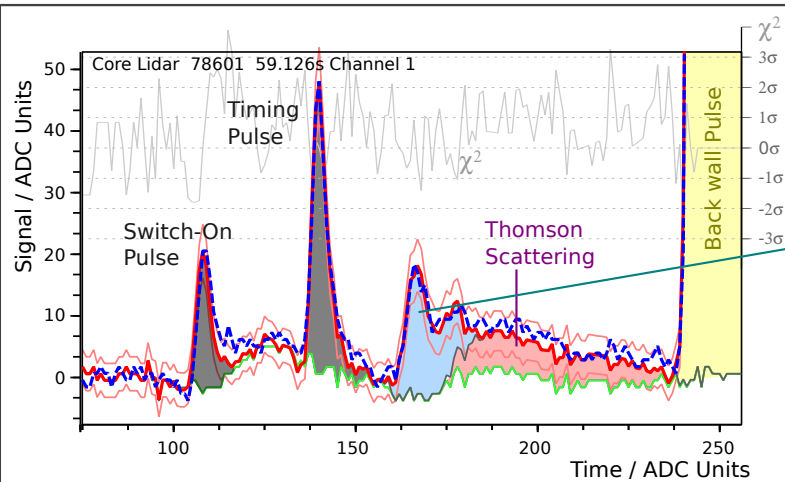
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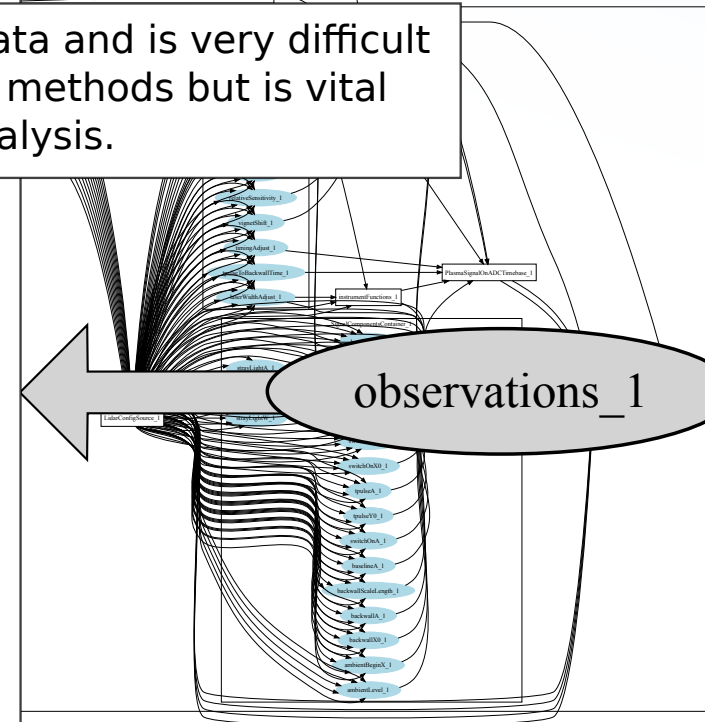
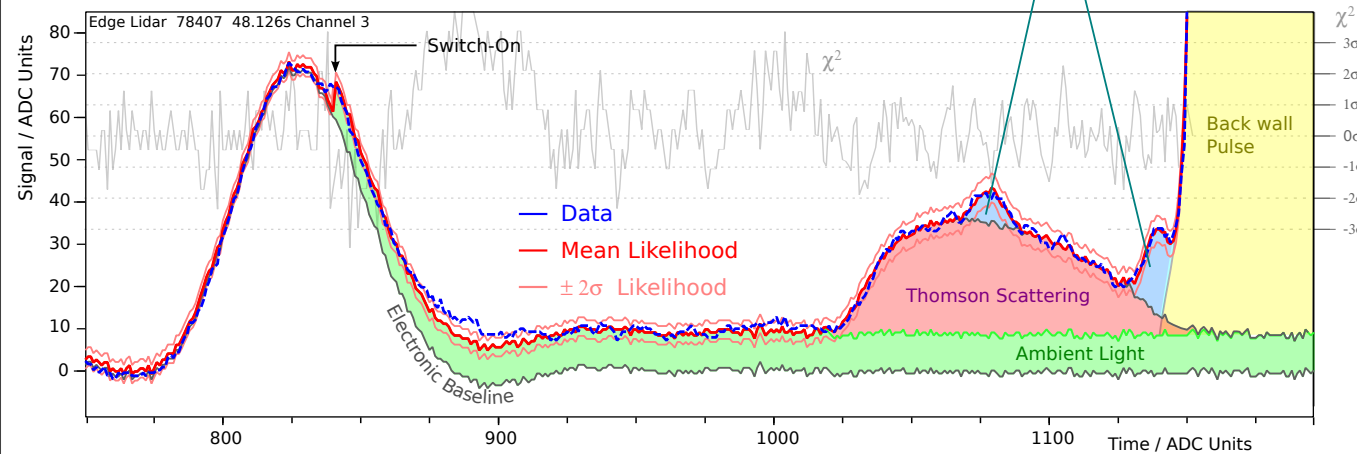
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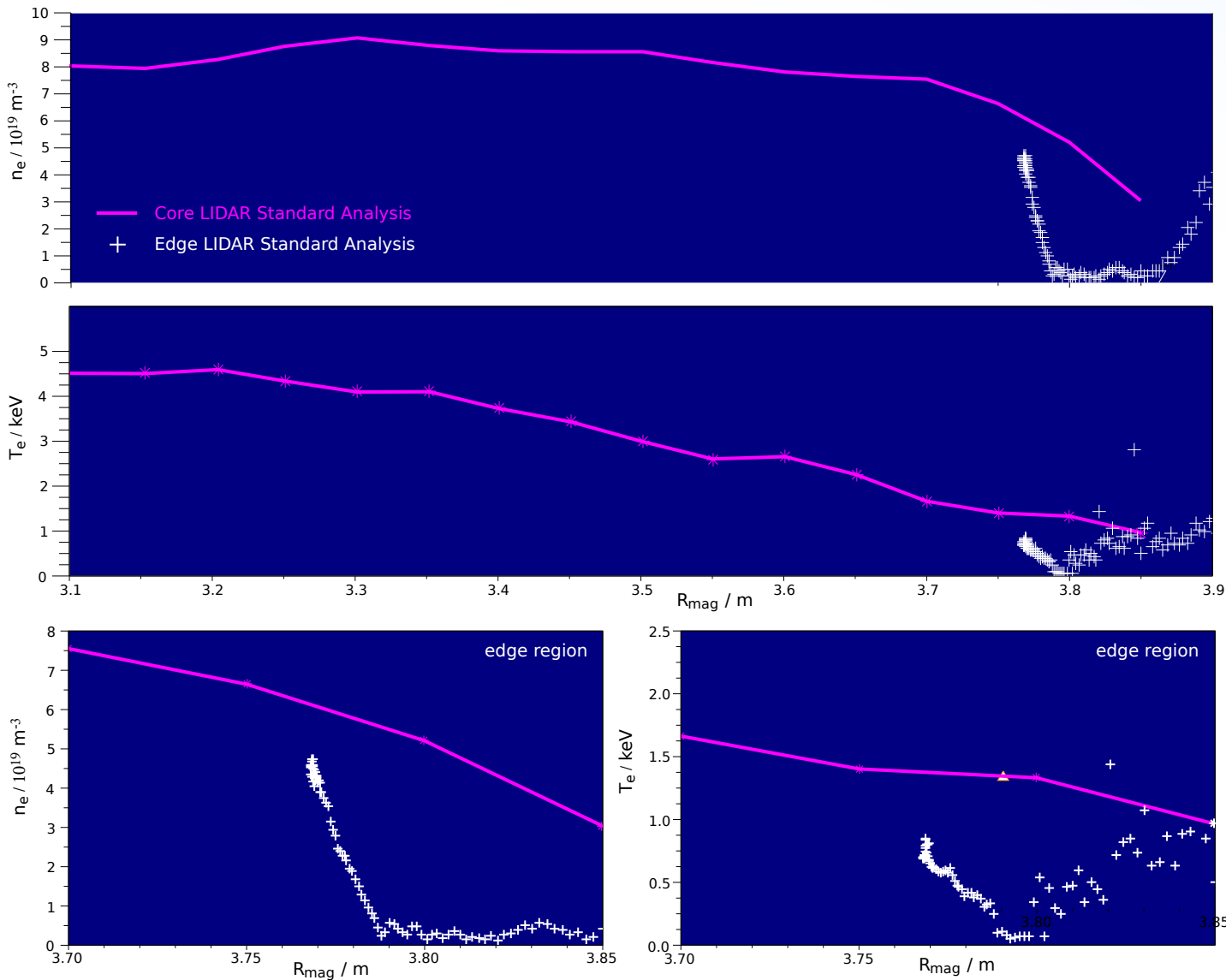
Model predicts all the data

Stray light obscures TS data and is very difficult to handle with traditional methods but is vital for proper edge LIDAR analysis.



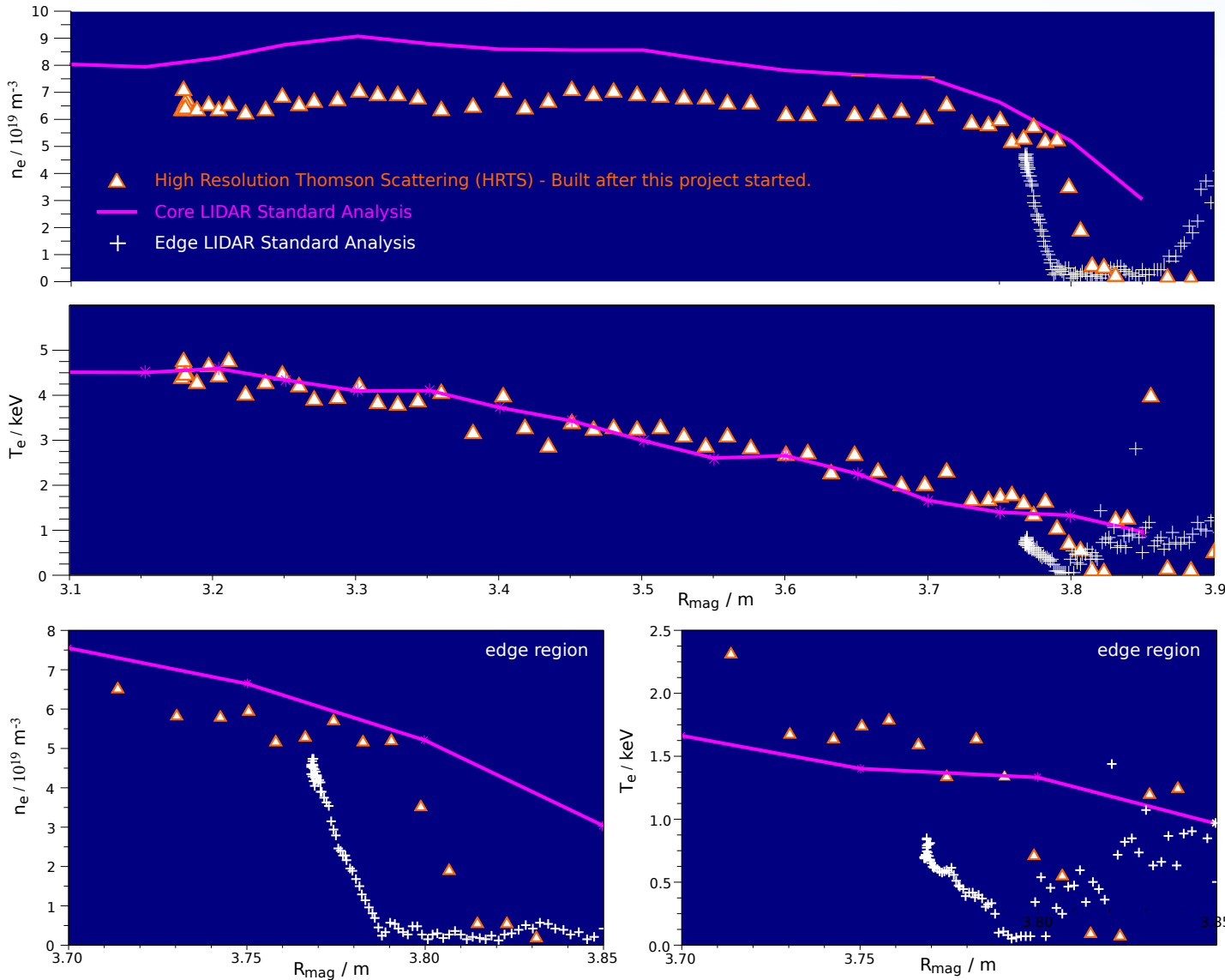
## Core LIDAR + Edge LIDAR + Interferometry

A typical high density H-mode pulse:



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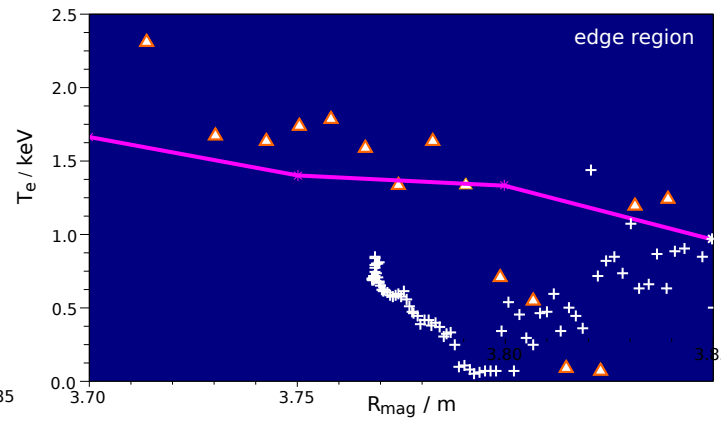
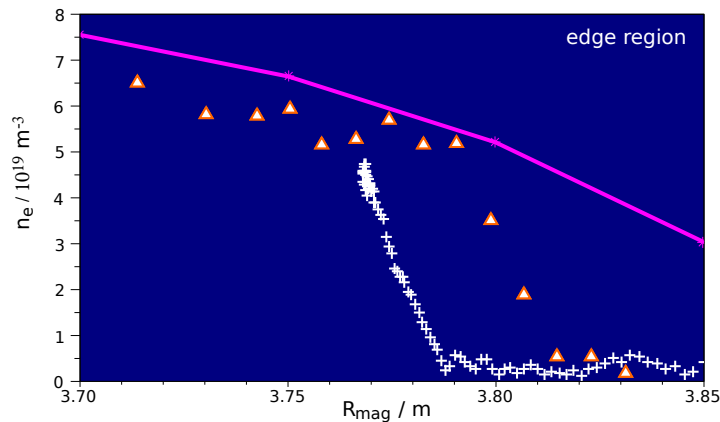
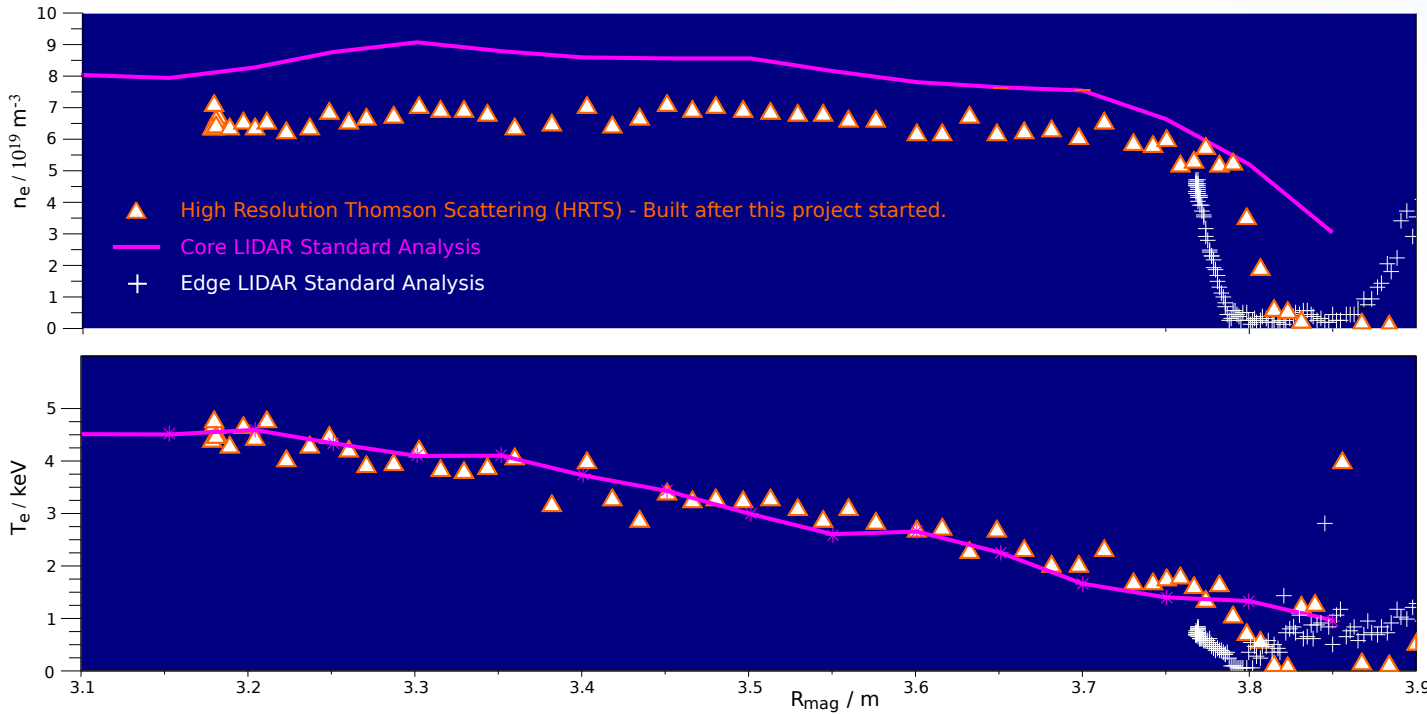
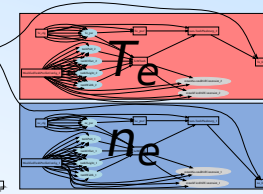
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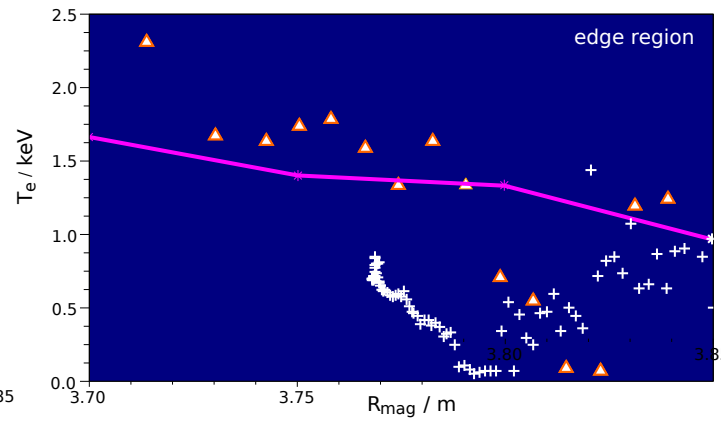
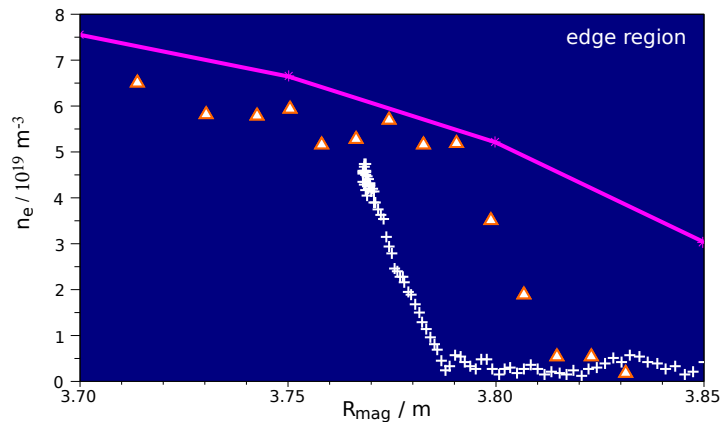
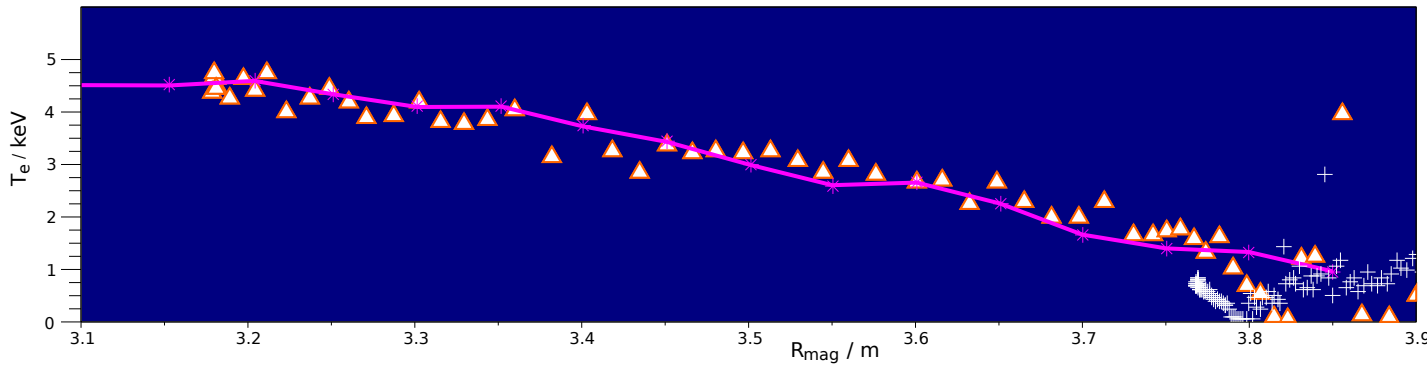
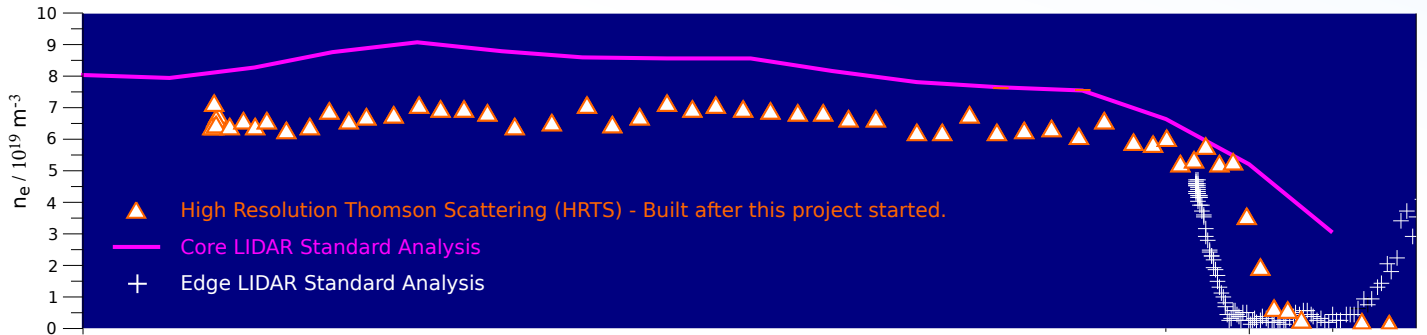
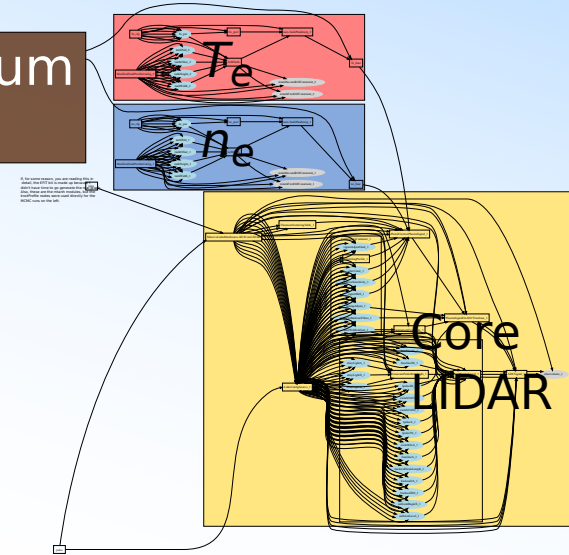
Equilibrium  
Code



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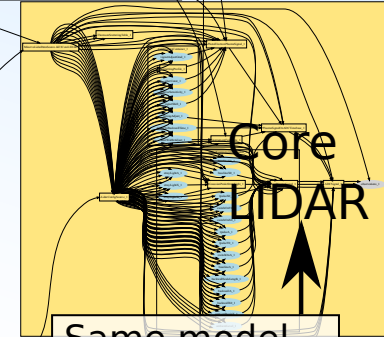
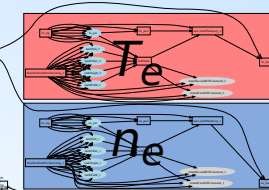
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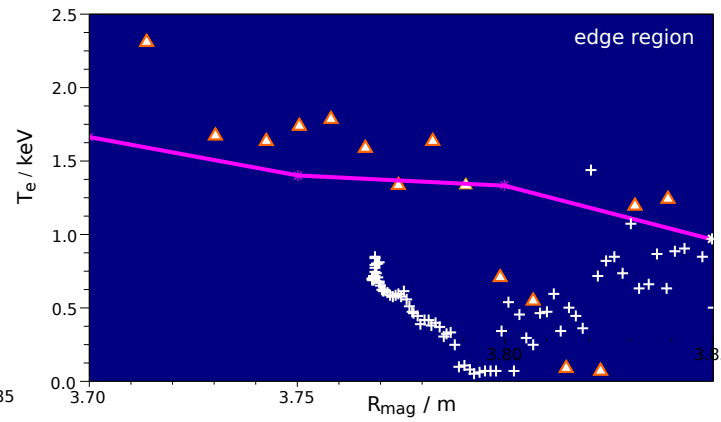
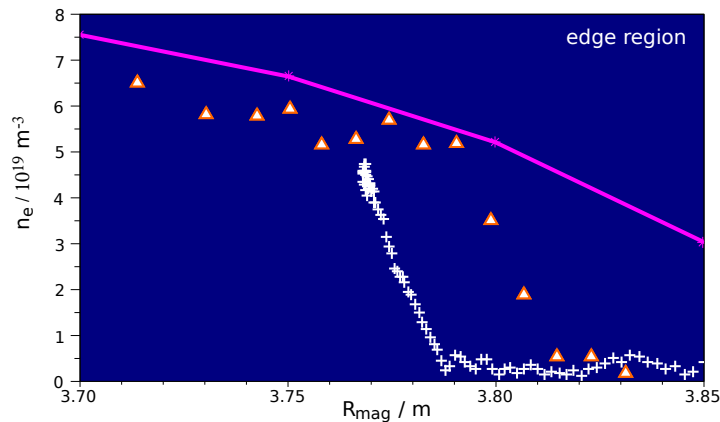
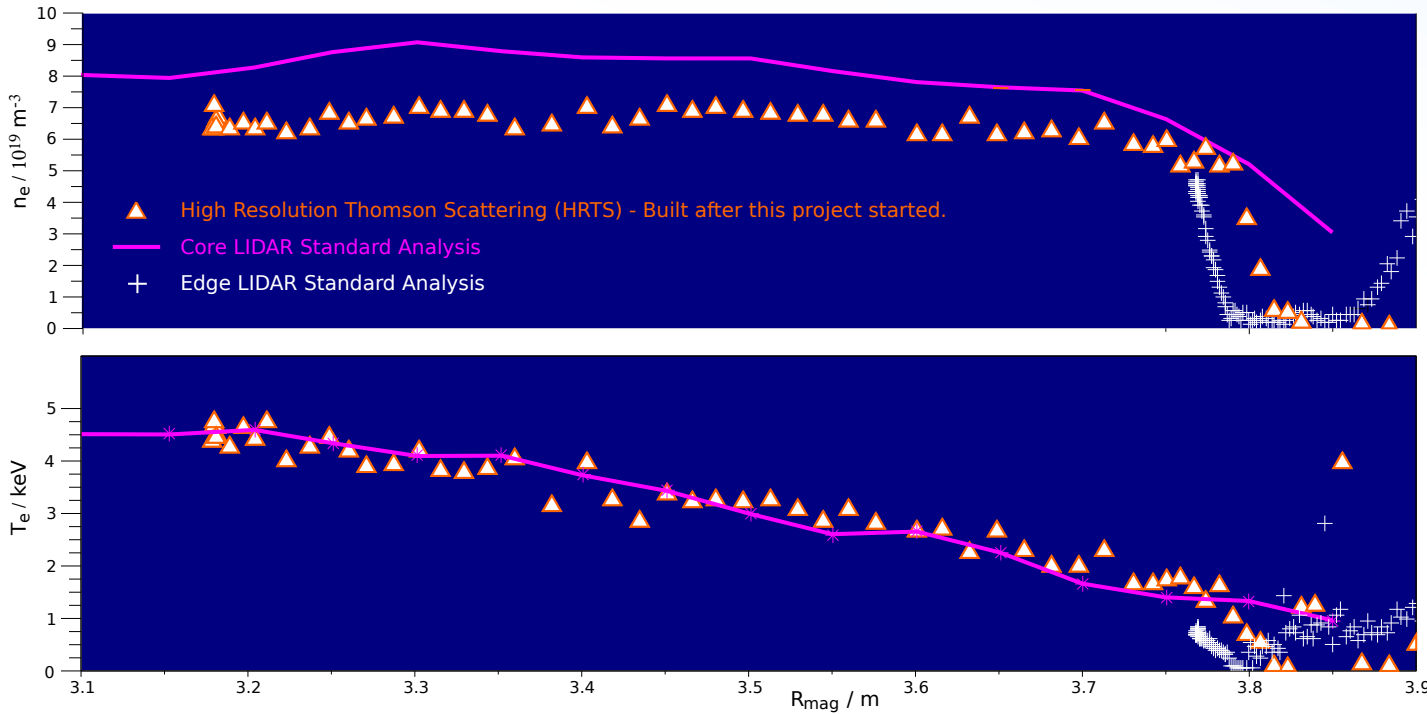
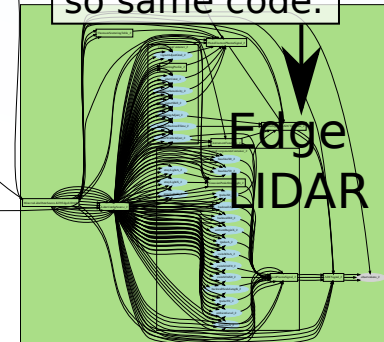
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Equilibrium  
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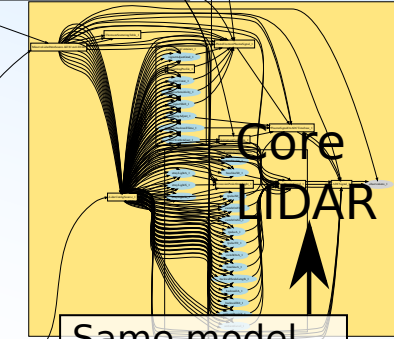
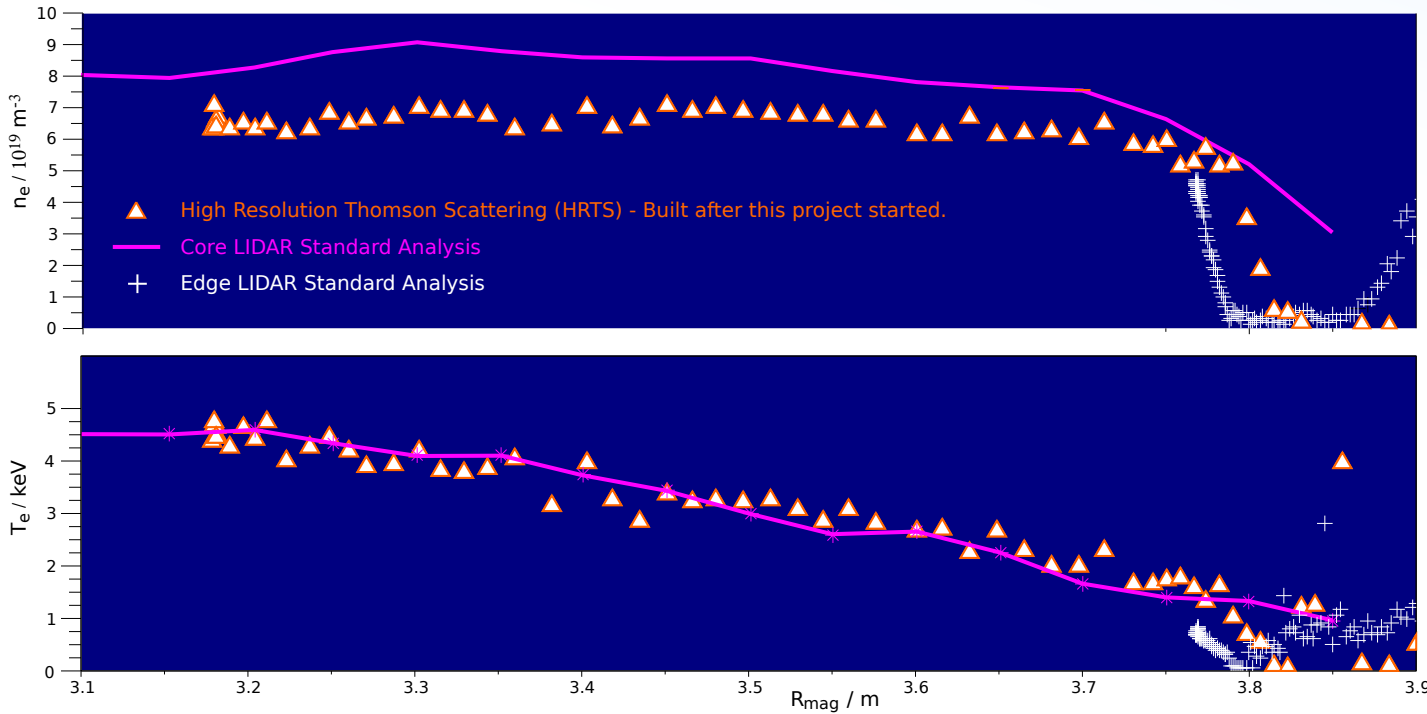
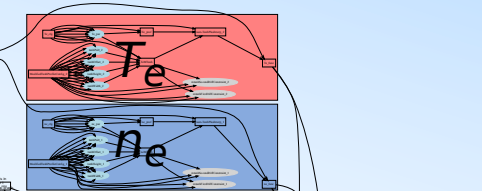
Same model,  
so same code.



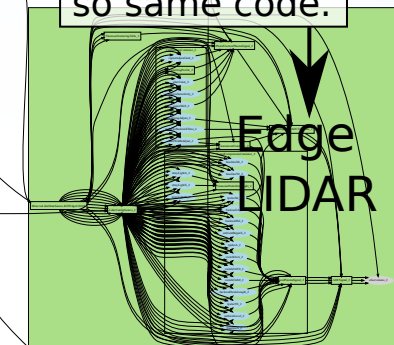
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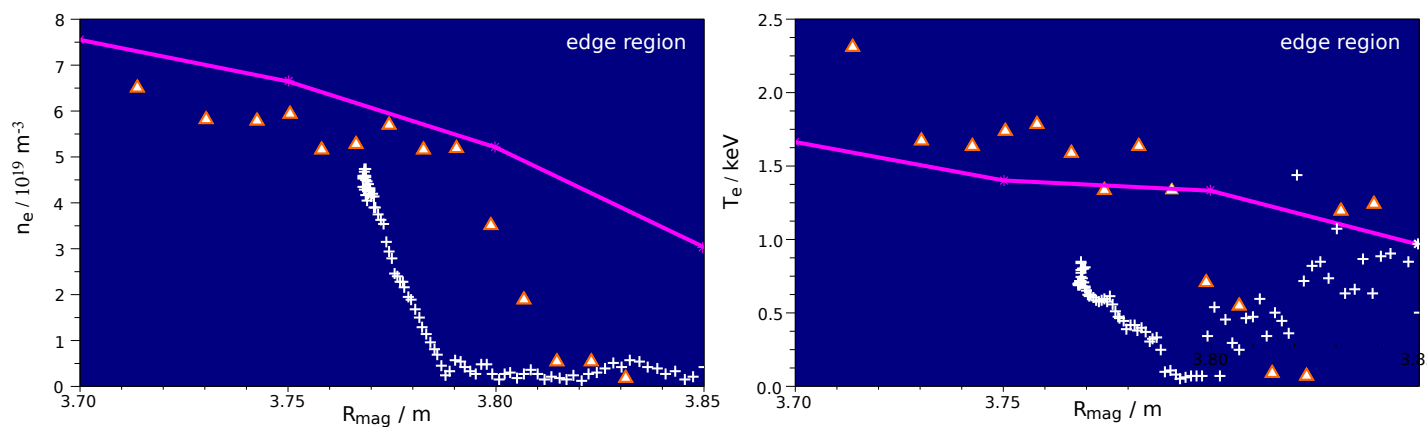
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Interferometry



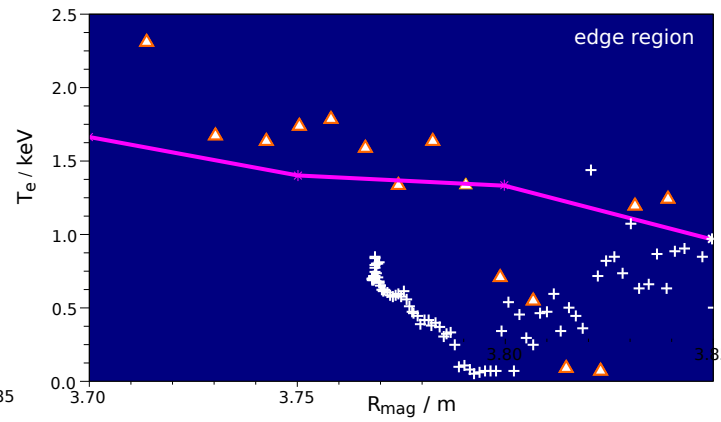
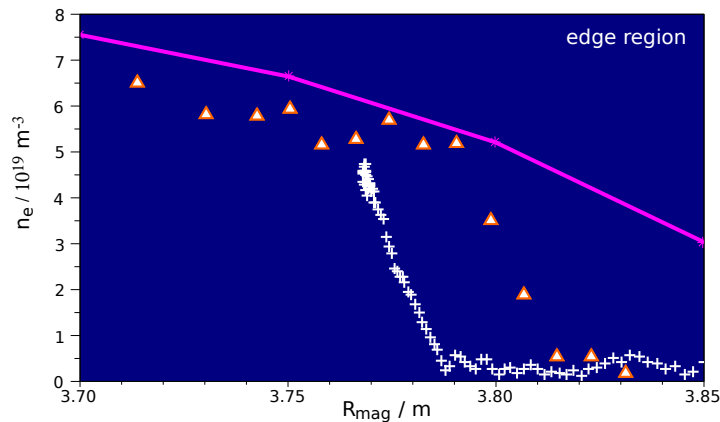
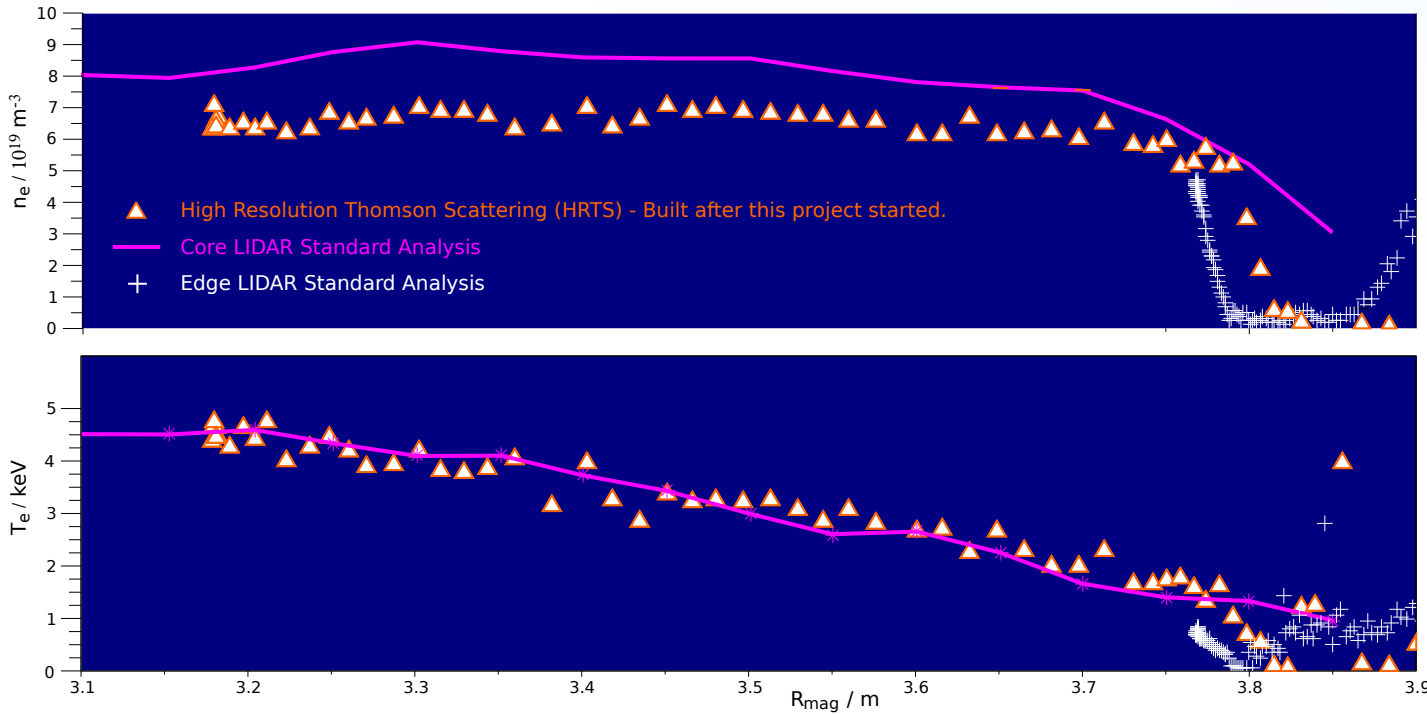
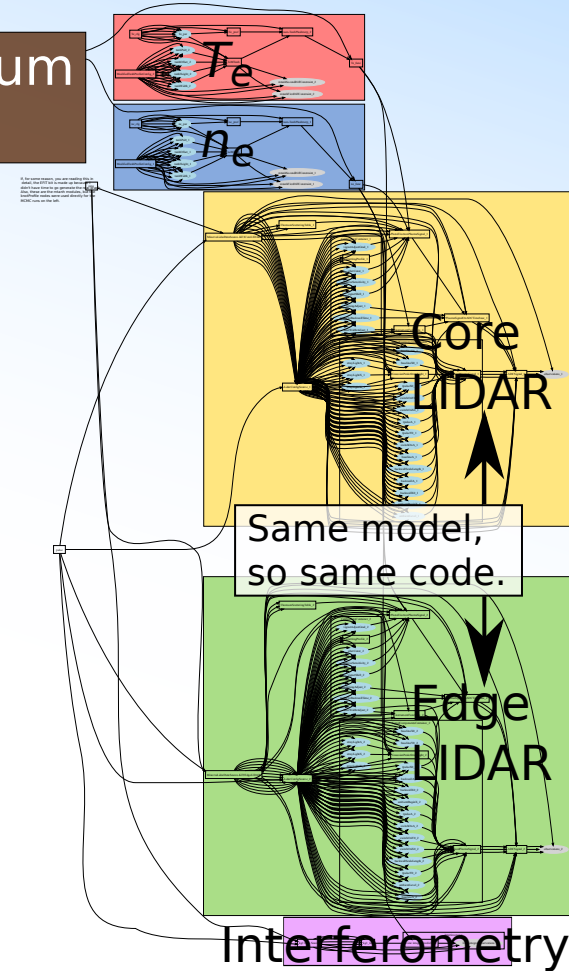


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Equilibrium  
Code

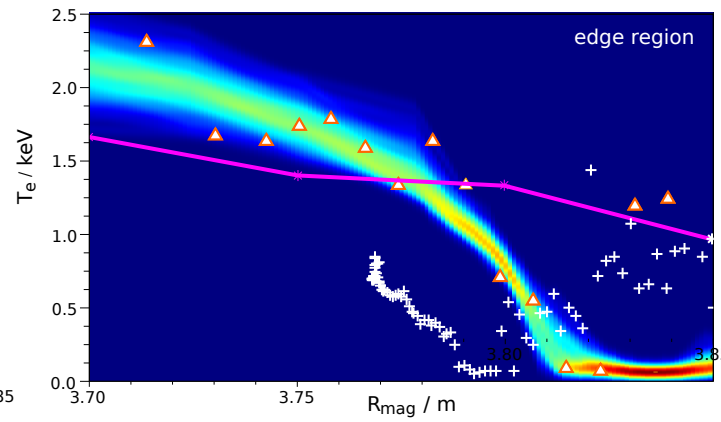
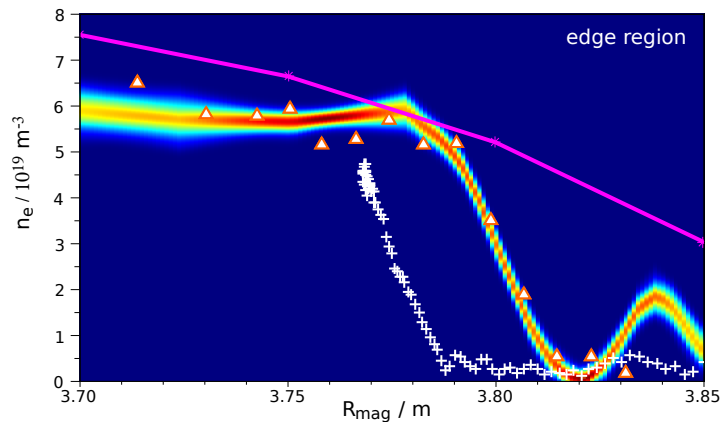
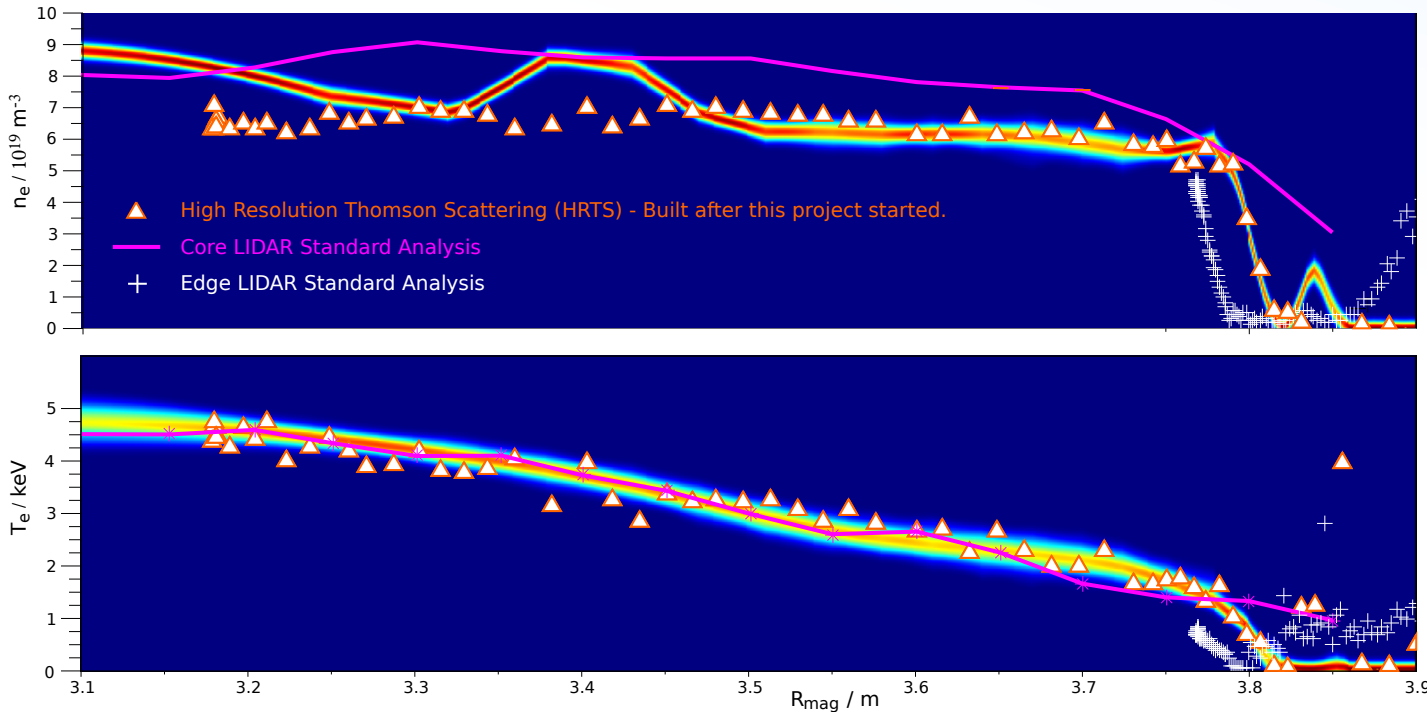
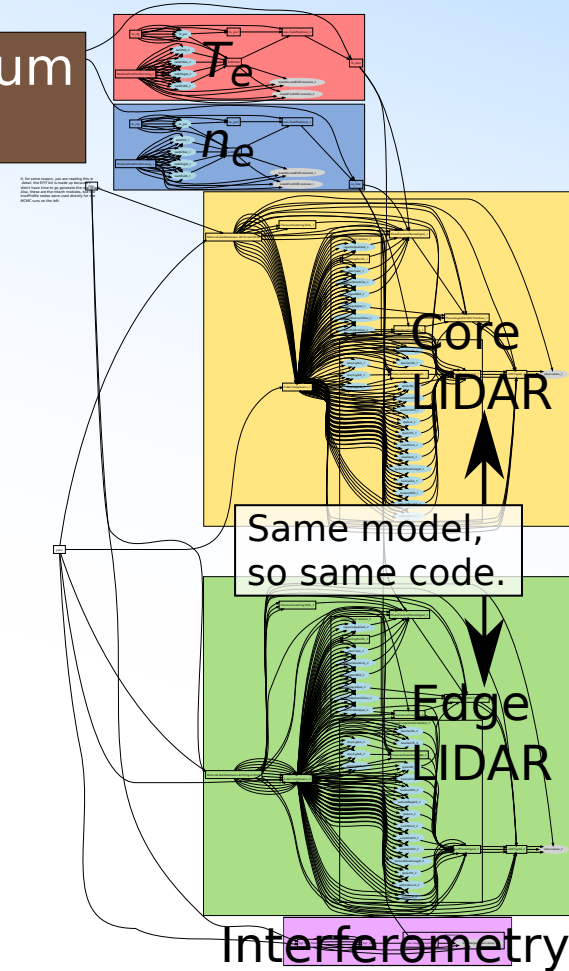


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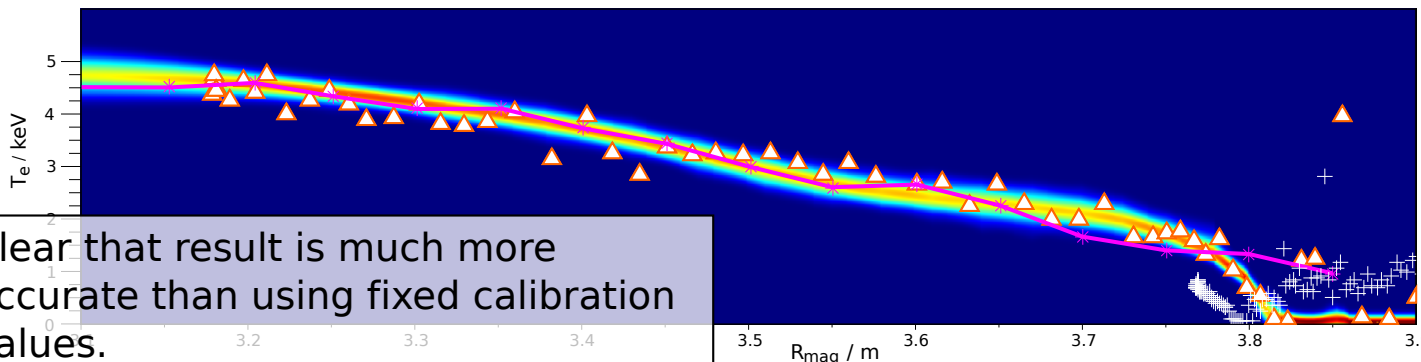
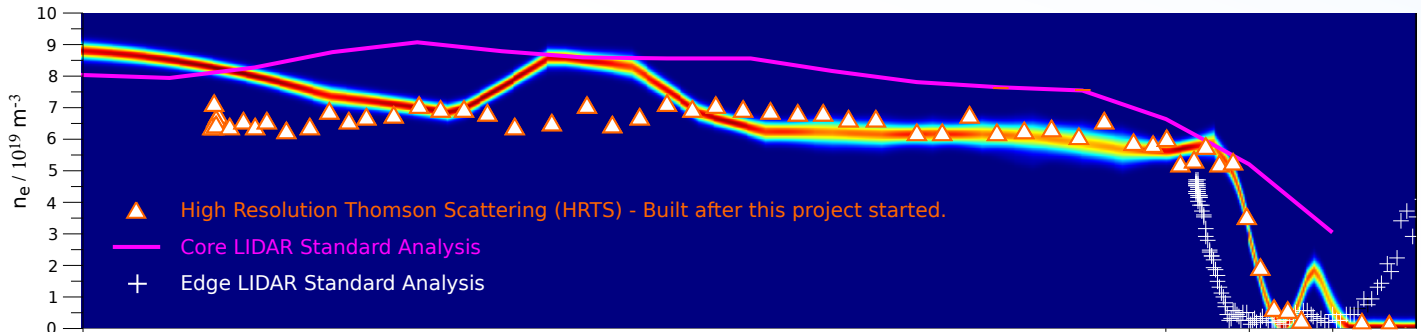
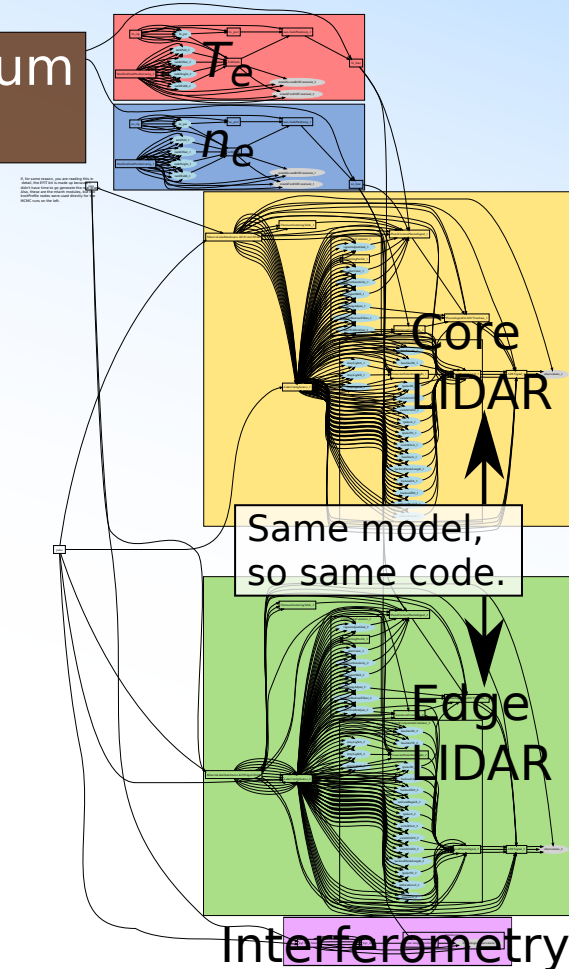


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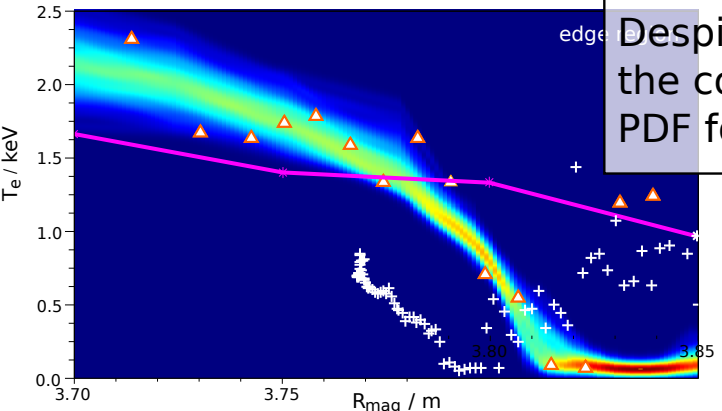
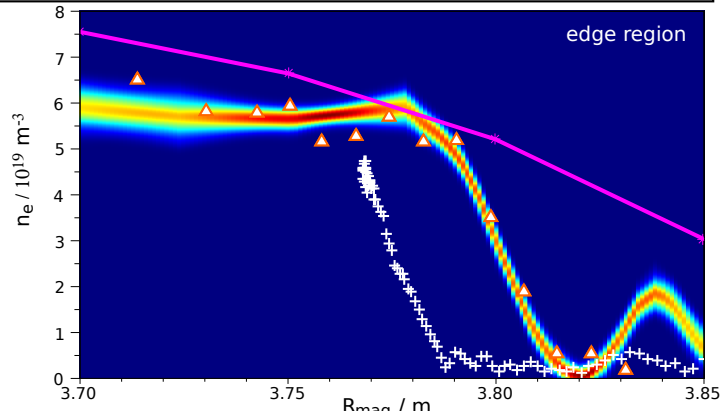
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Clear that result is much more accurate than using fixed calibration values.



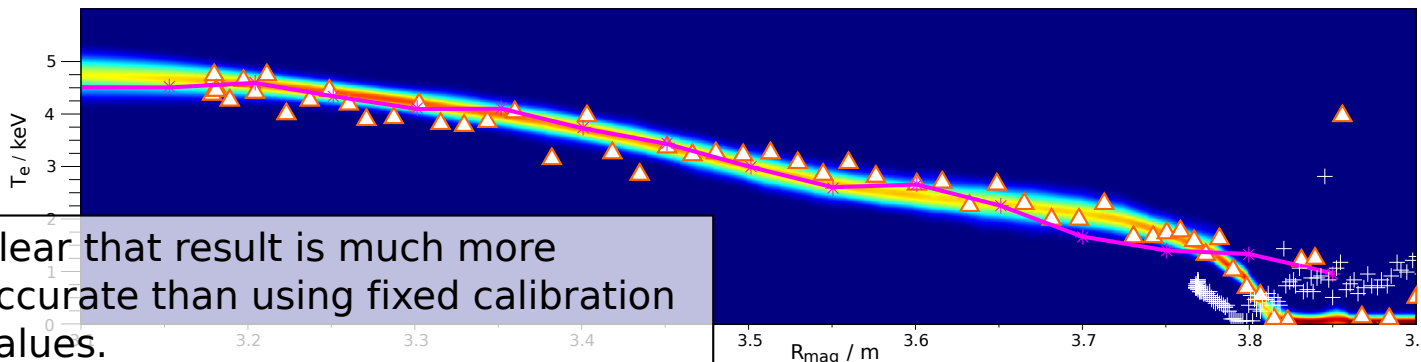
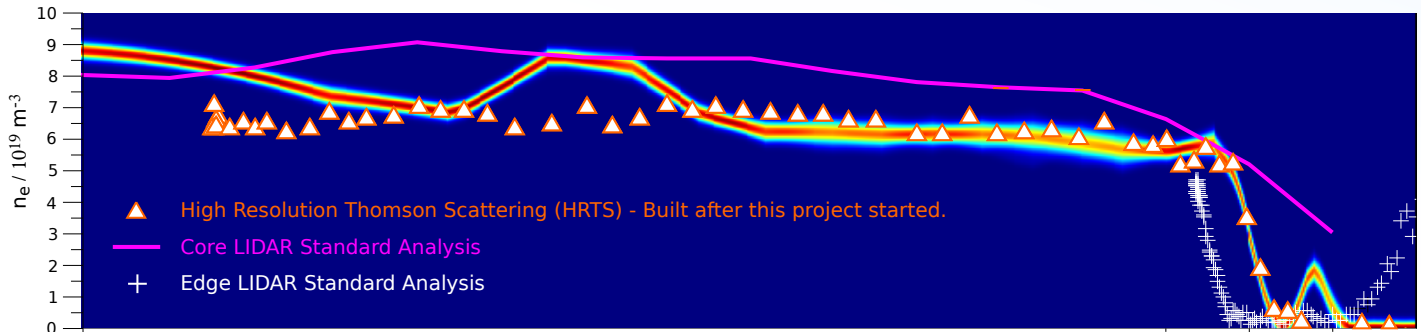
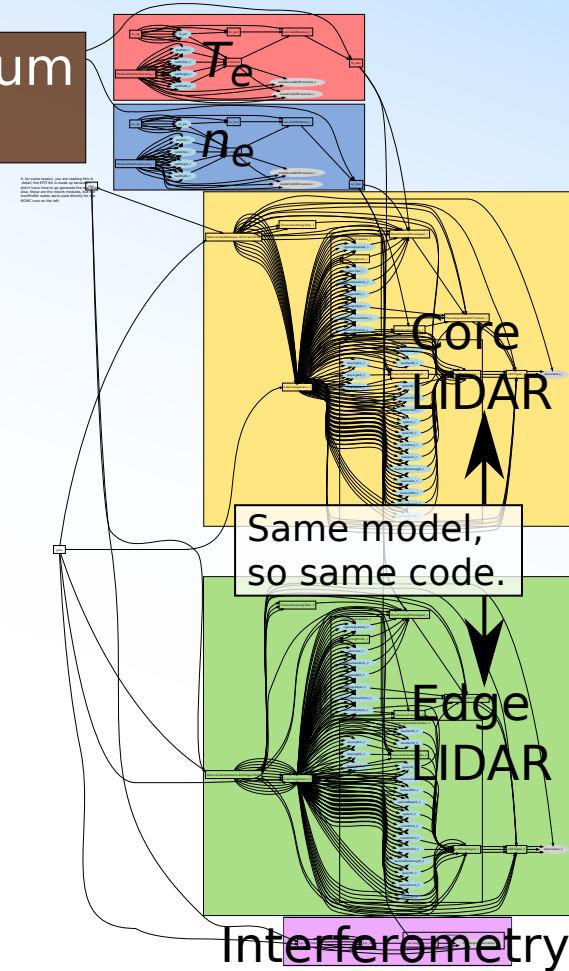
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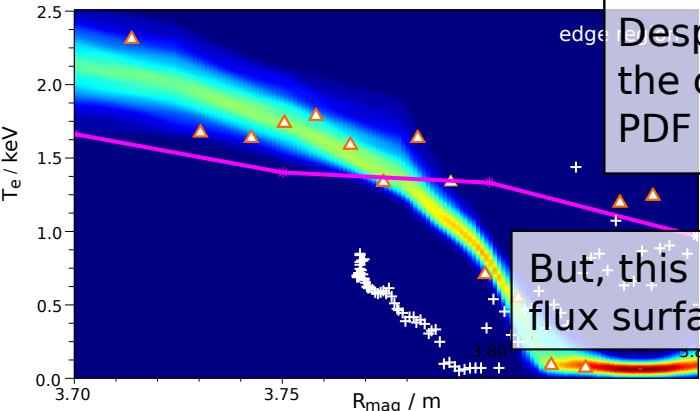
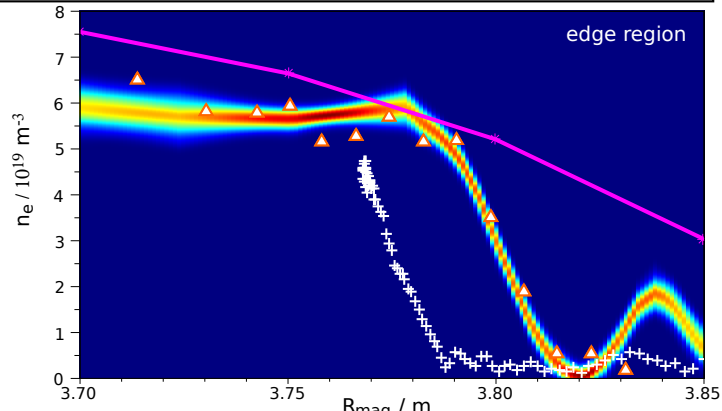
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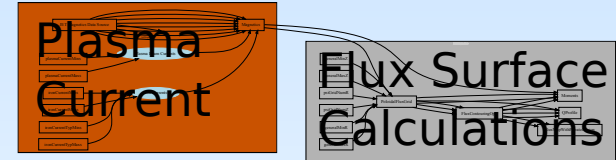
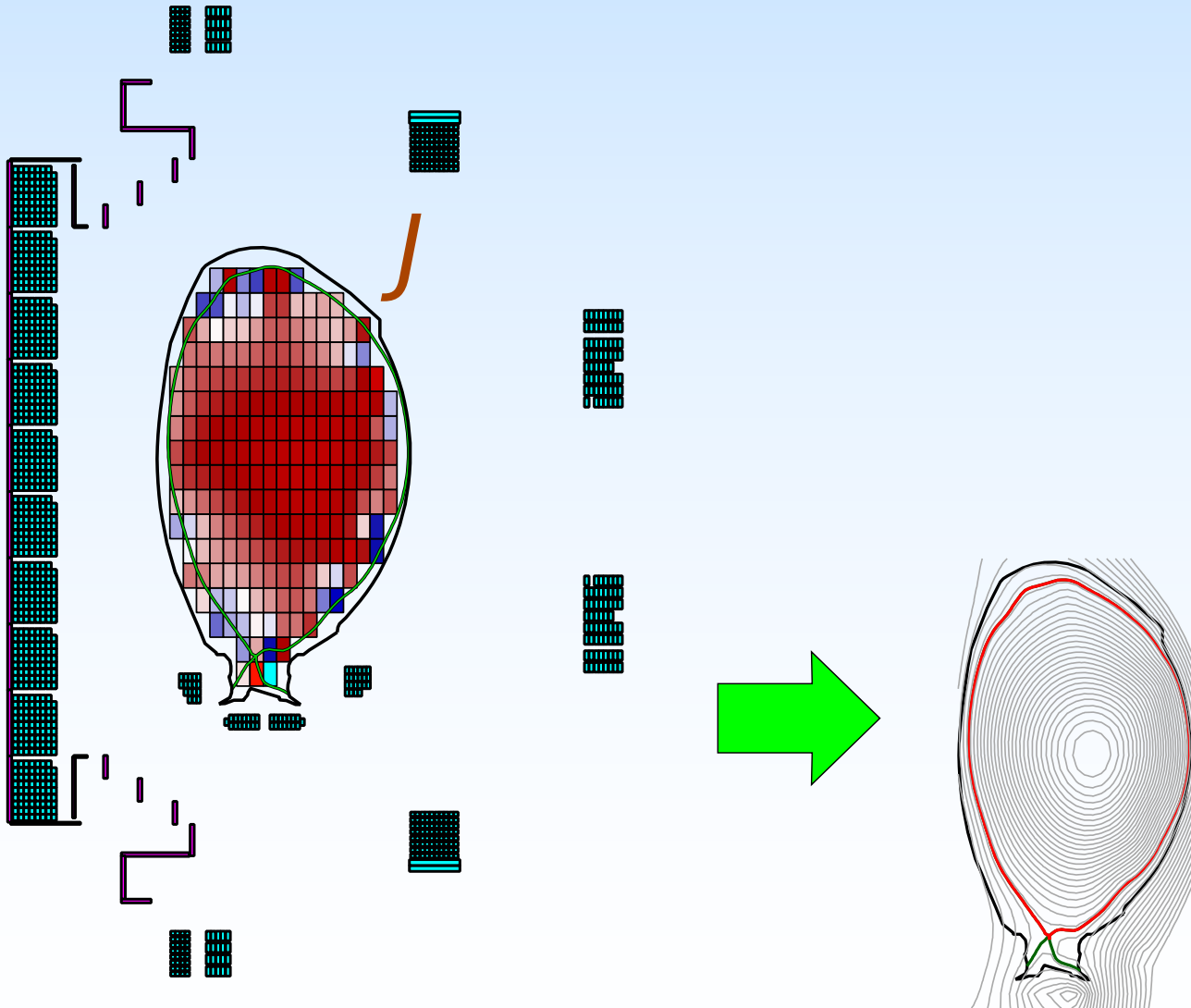


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# Interferometry + Current Tomography I

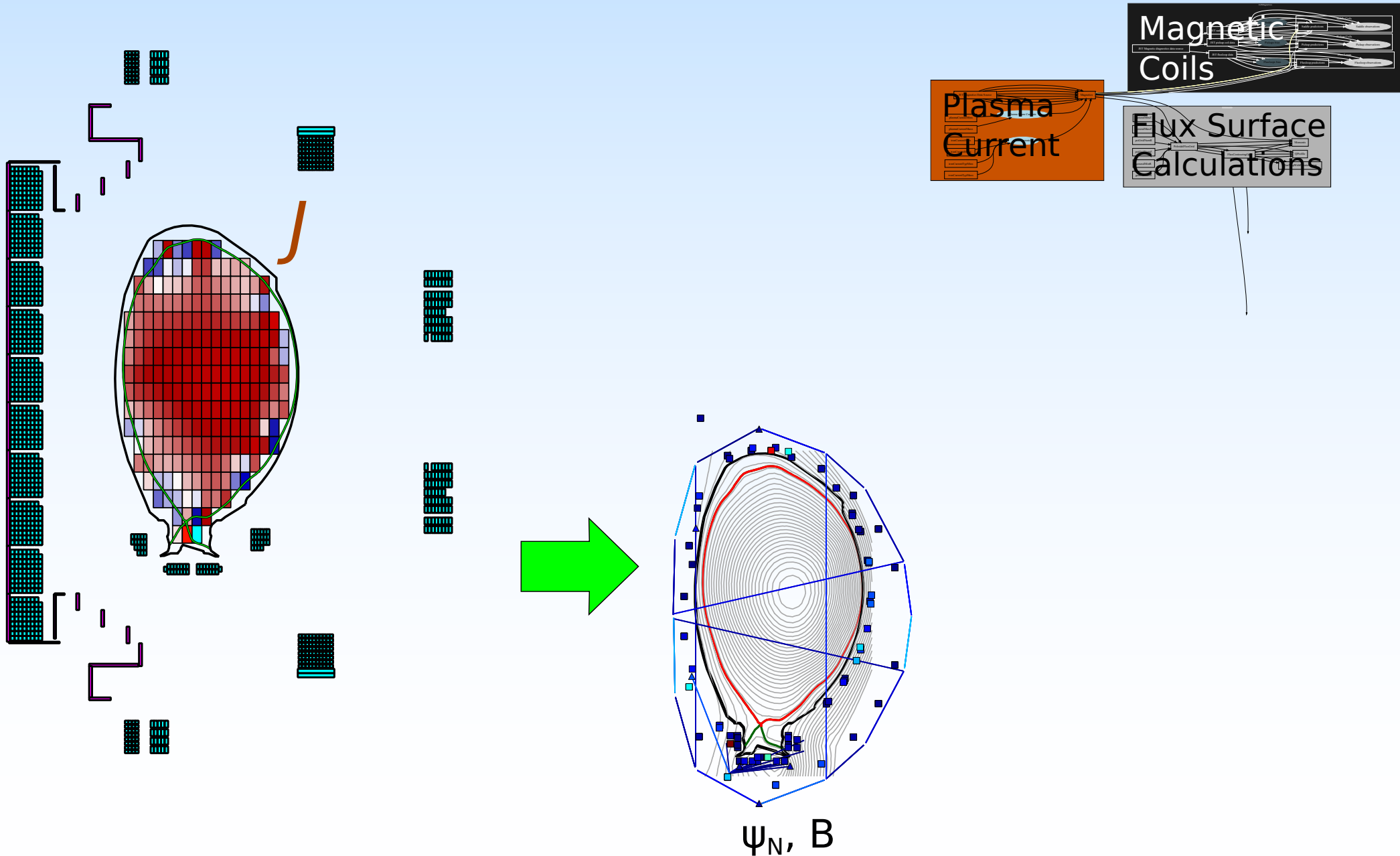
Instead, calculate  $\psi_N$  from toroidal currents  $J$



$\psi_N, B$

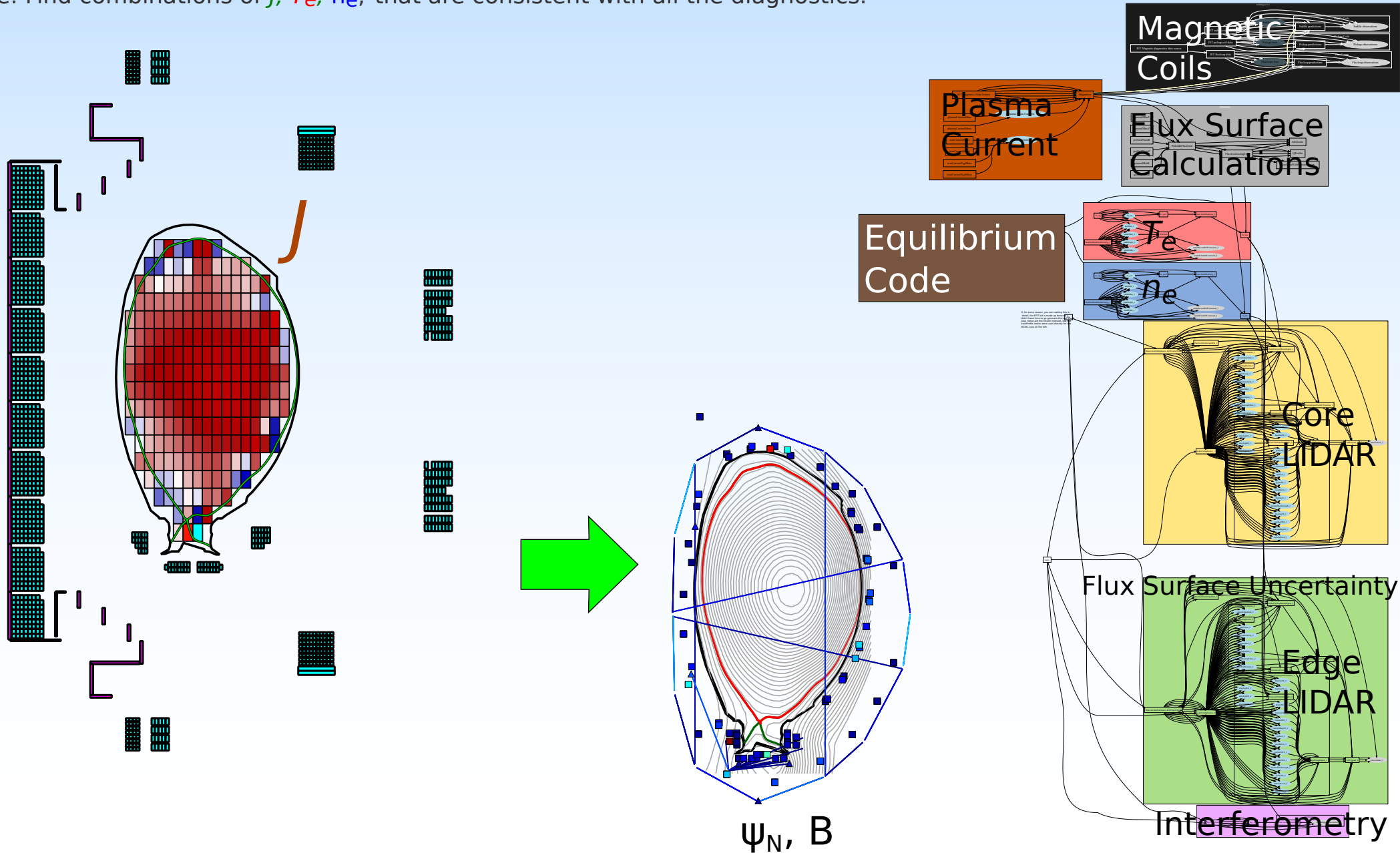
## Interferometry + Current Tomography I

Instead, calculate  $\psi_N$  from toroidal currents  $J$ , include magnetics diagnostics and invert to full posterior:



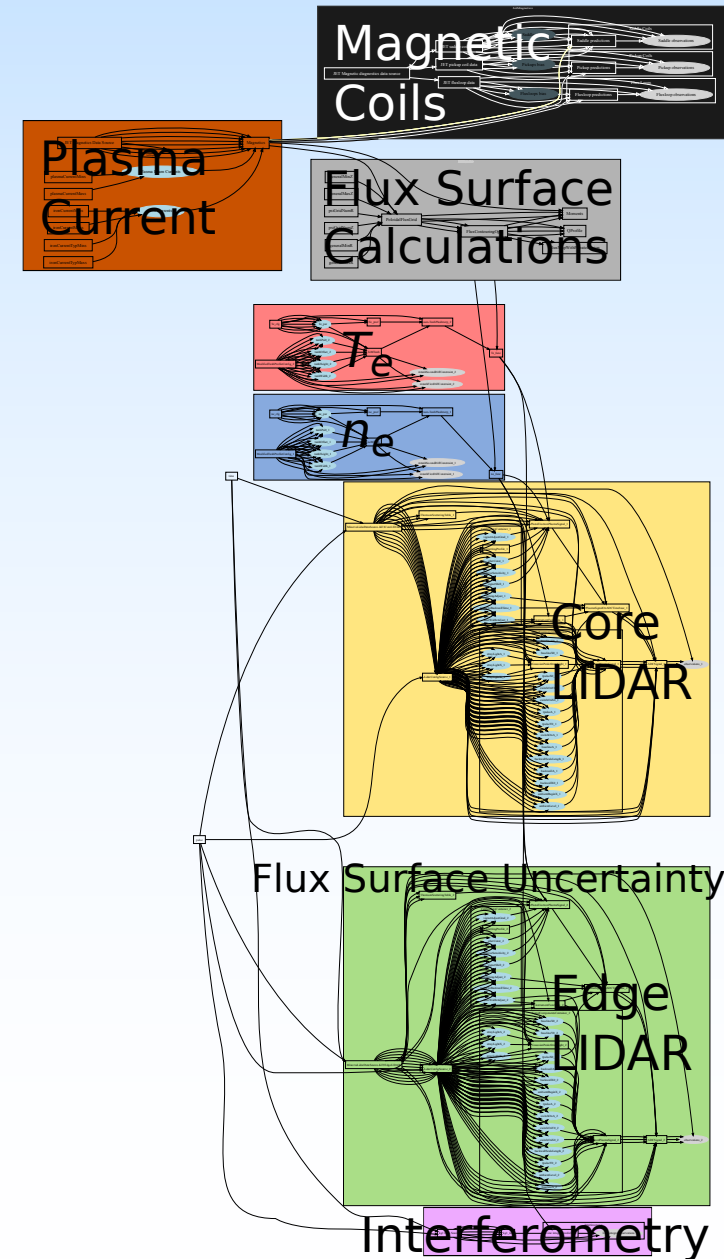
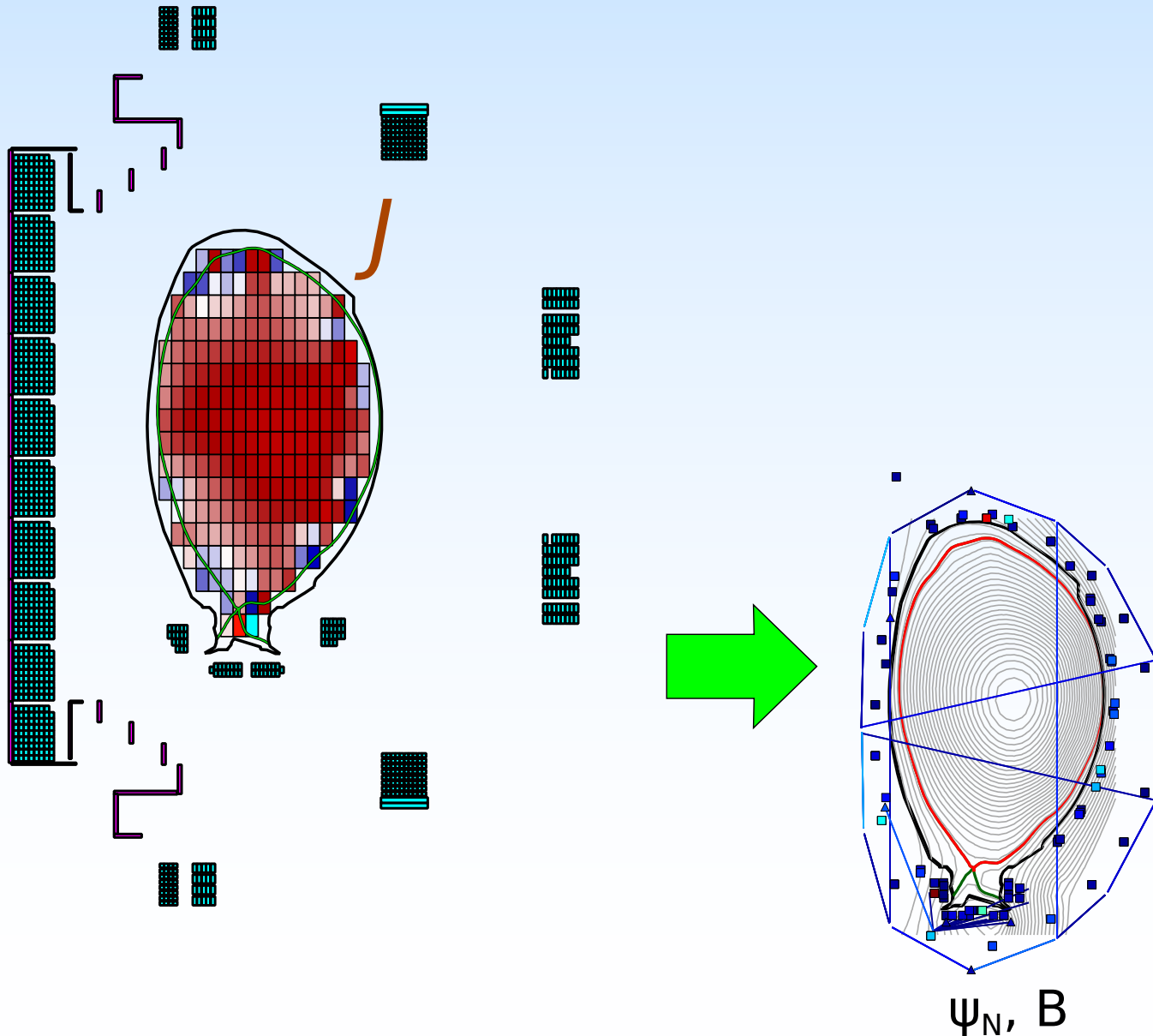
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 i.e: Find combinations of  $J$ ,  $T_e$ ,  $n_e$ , that are consistent with all the diagnostics.



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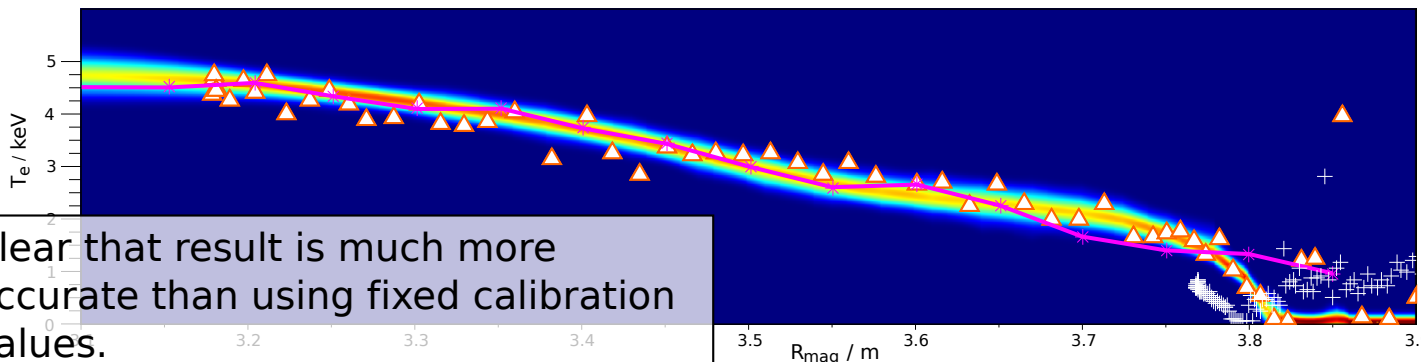
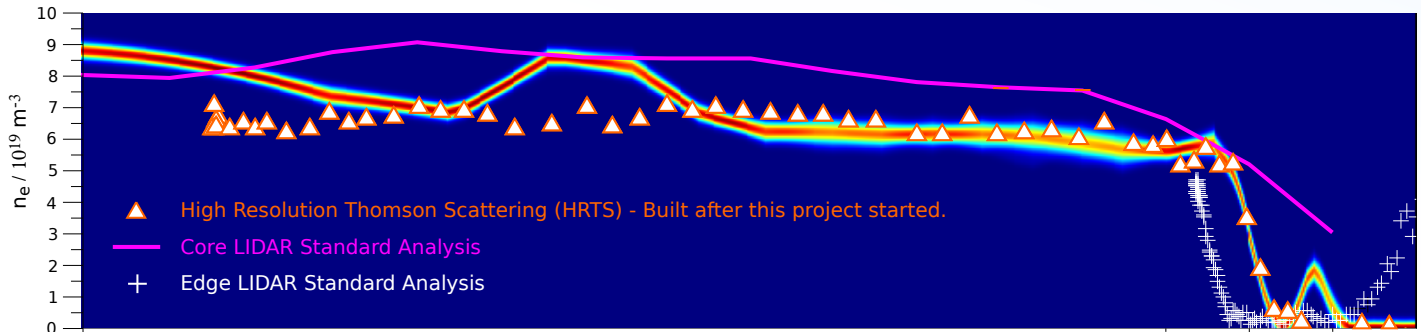
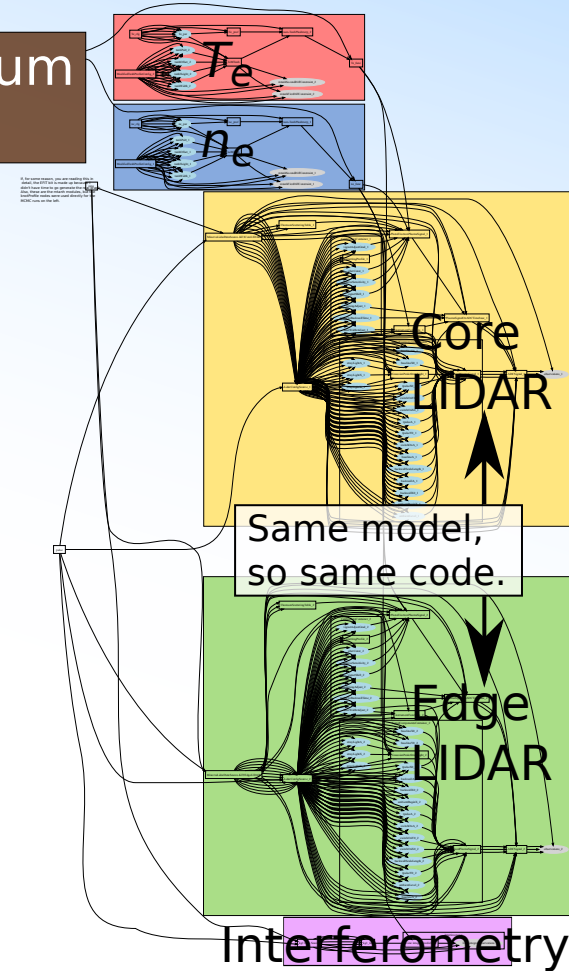


# Core LIDAR + Edge LIDAR + Interferometry

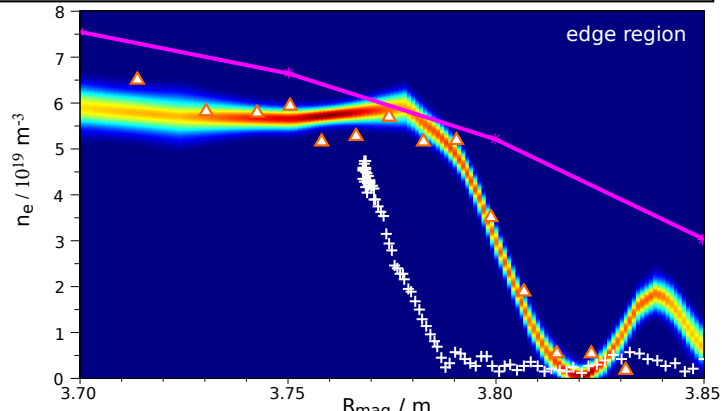
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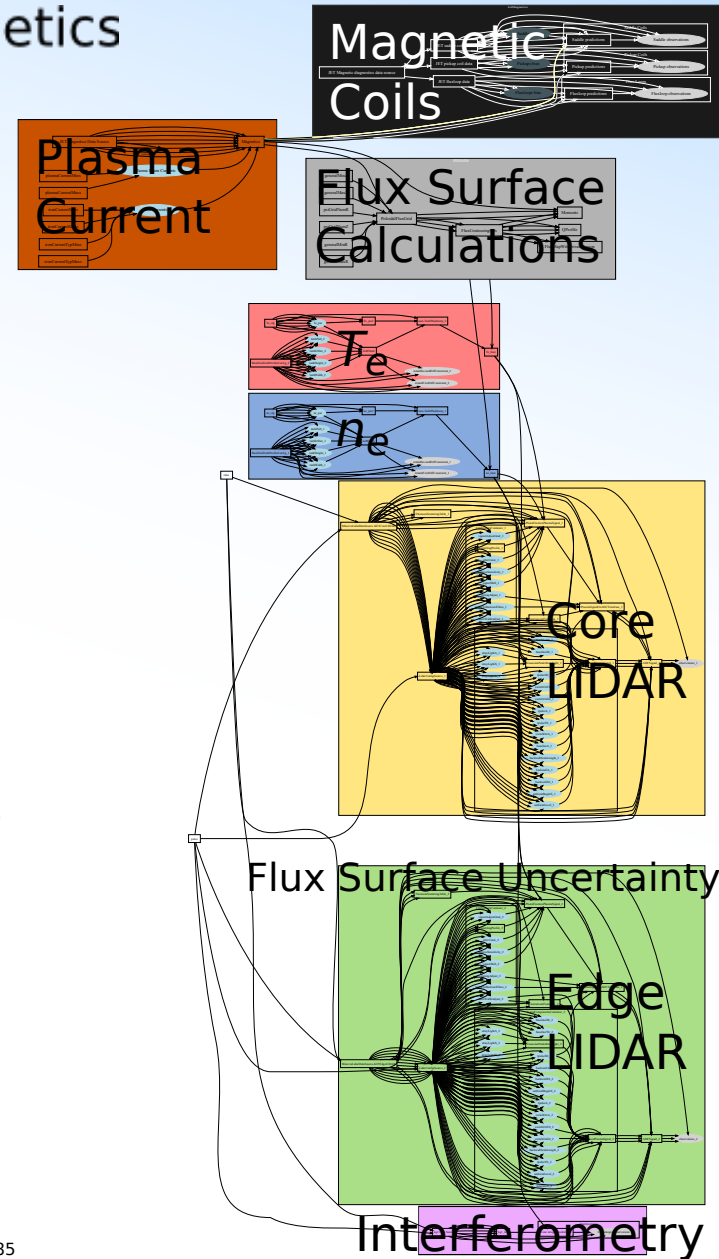
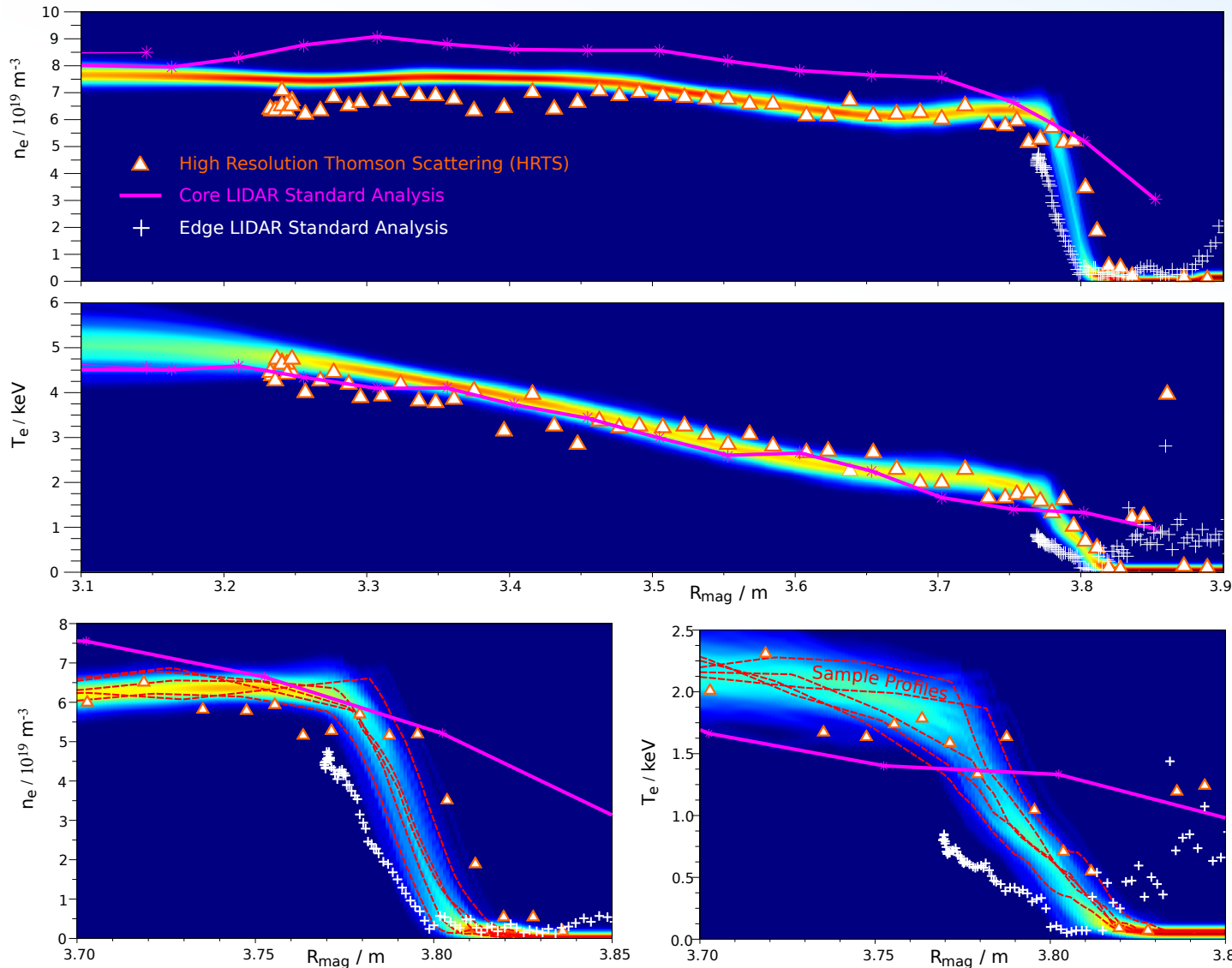


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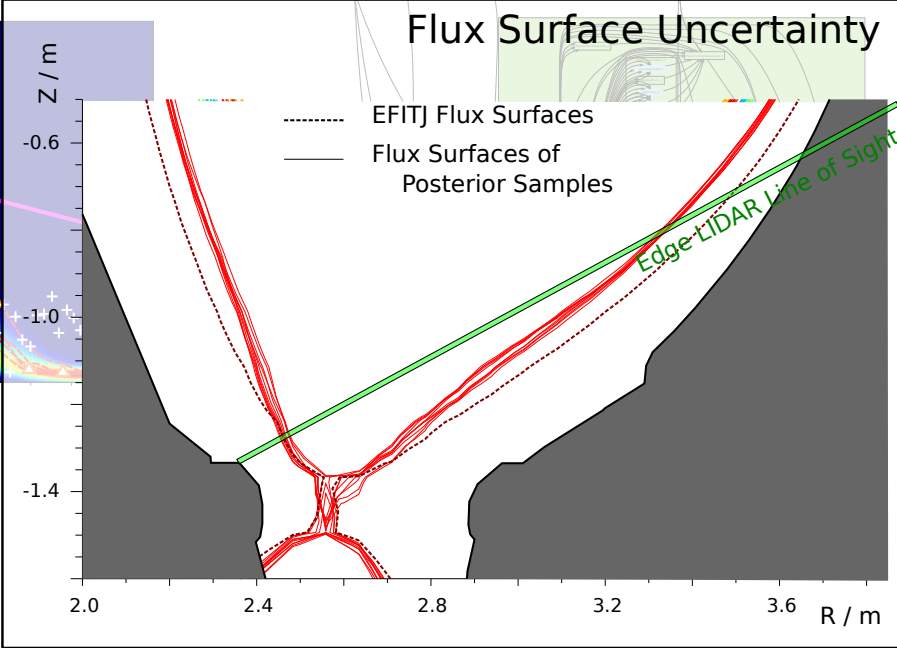
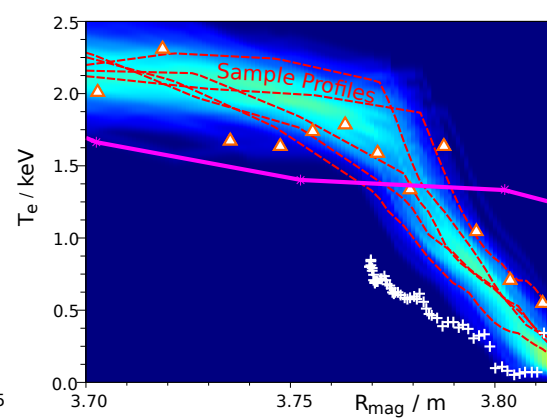
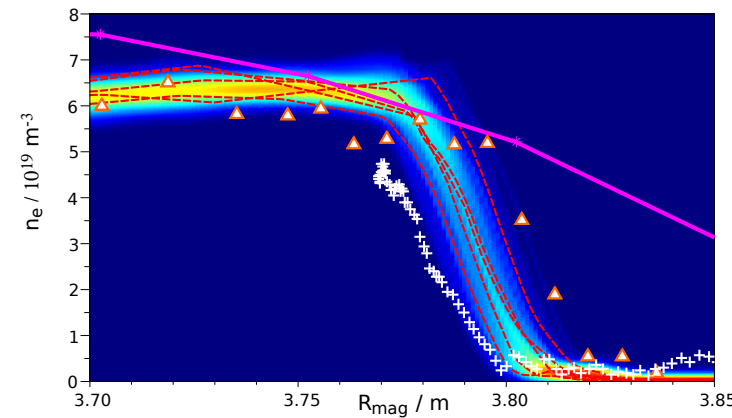
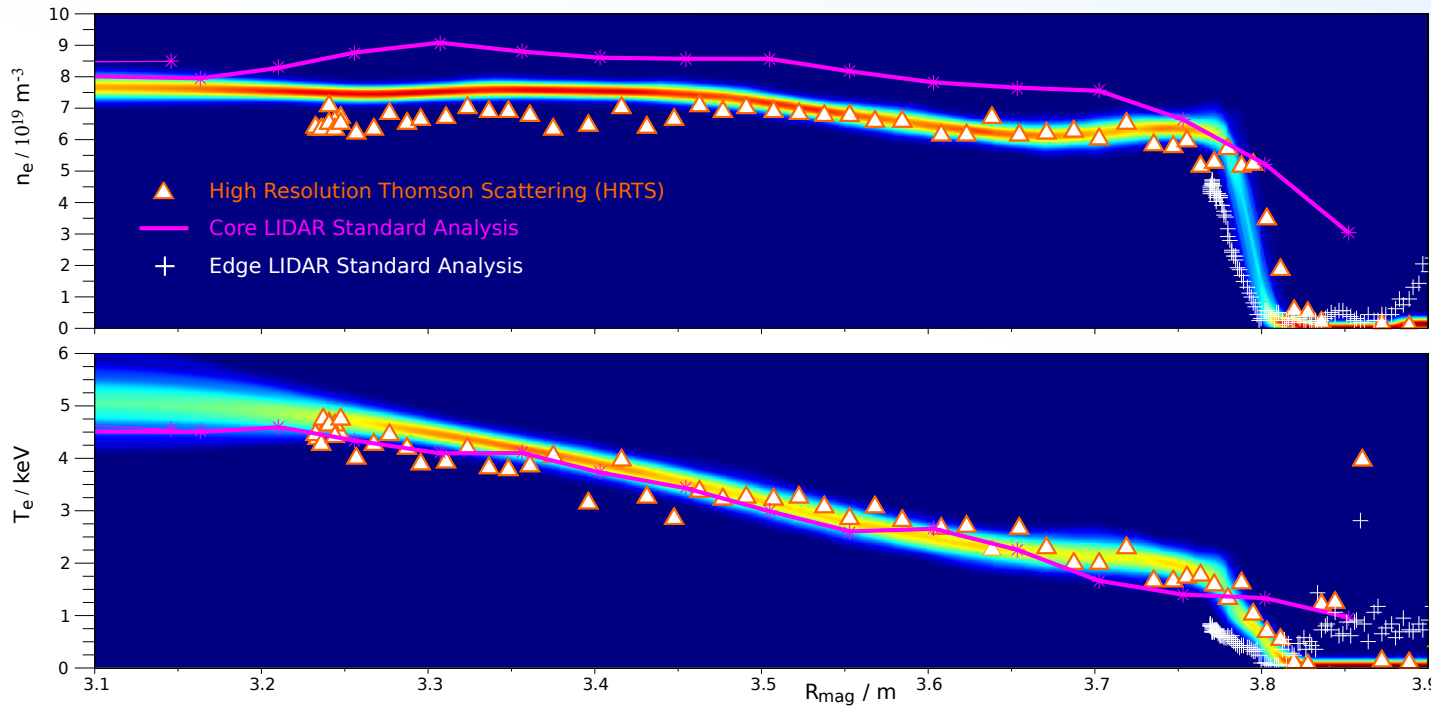
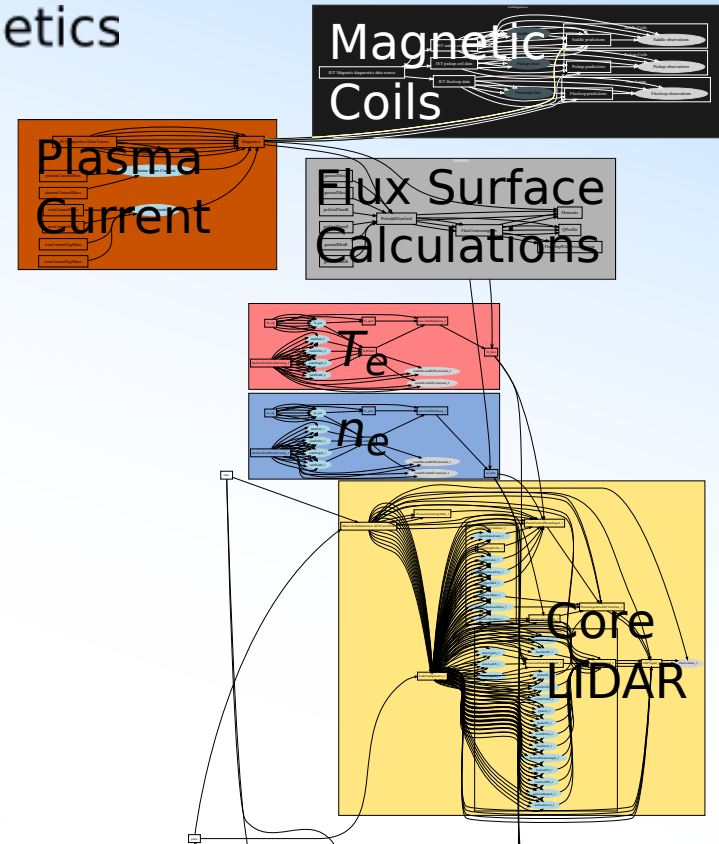
# Core LIDAR + Edge LIDAR + Interferometry + Magnetics

Connect magnetics model and run inversion.



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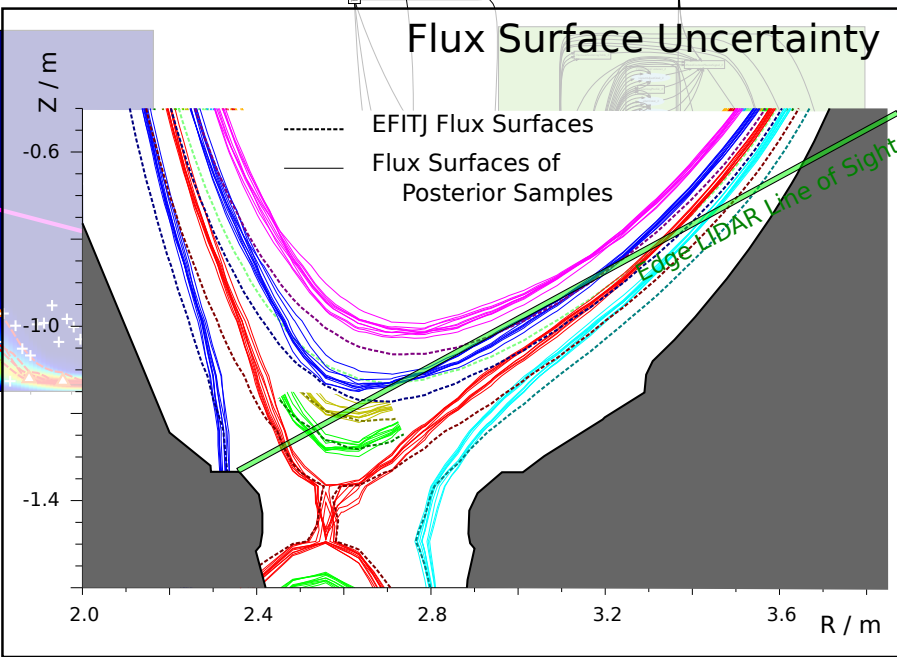
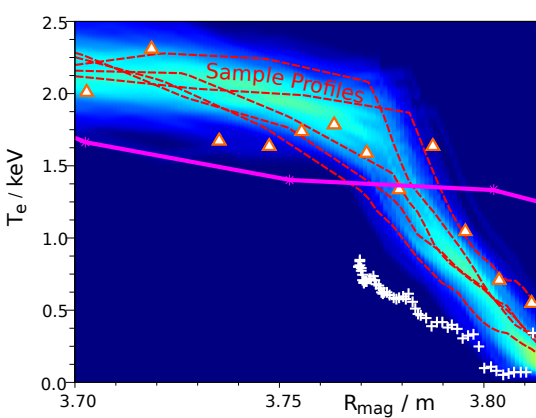
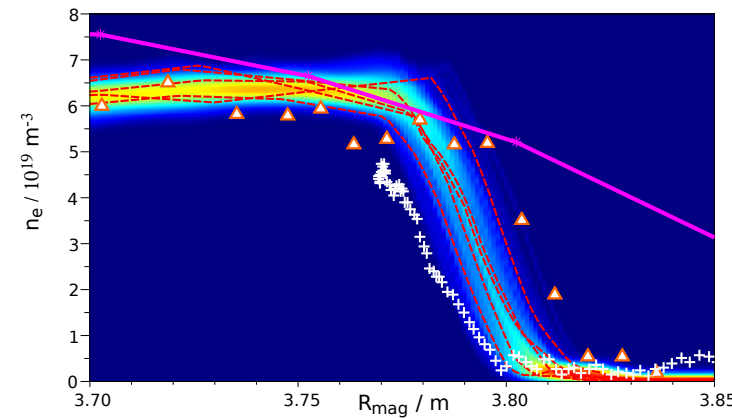
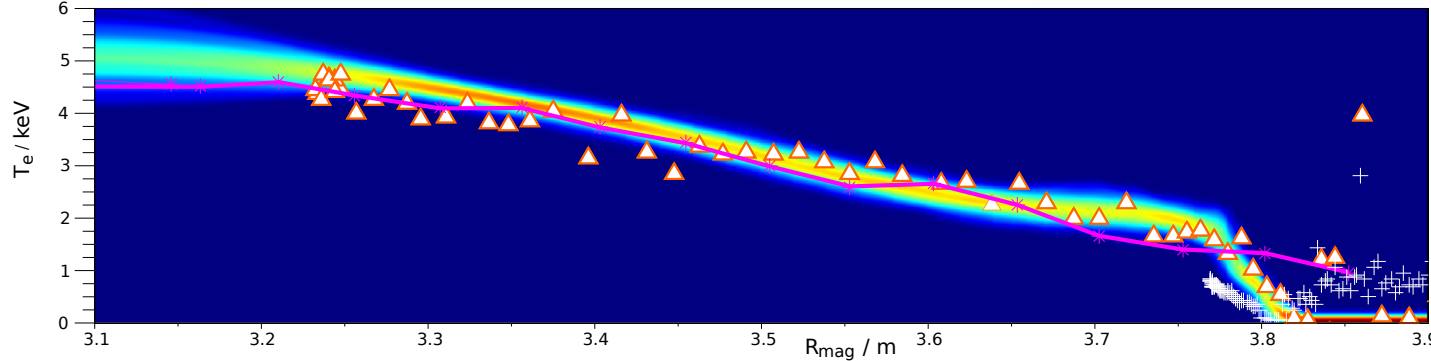
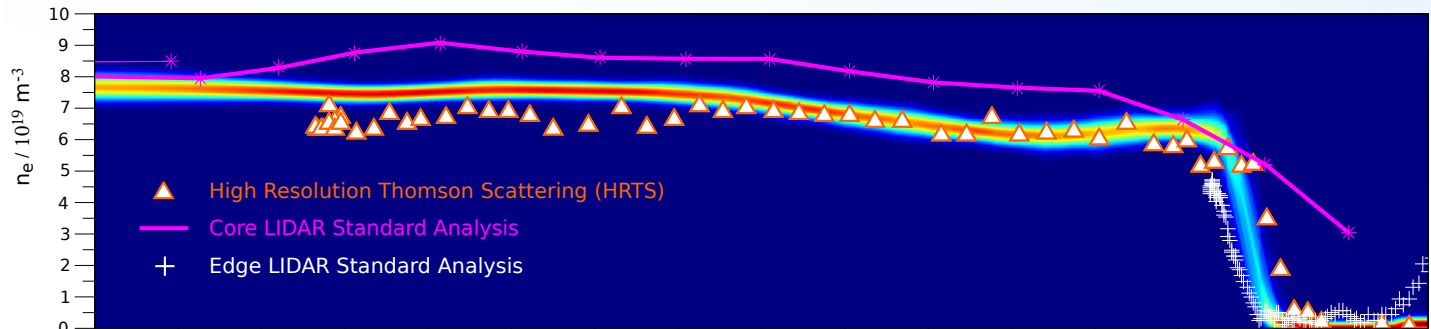
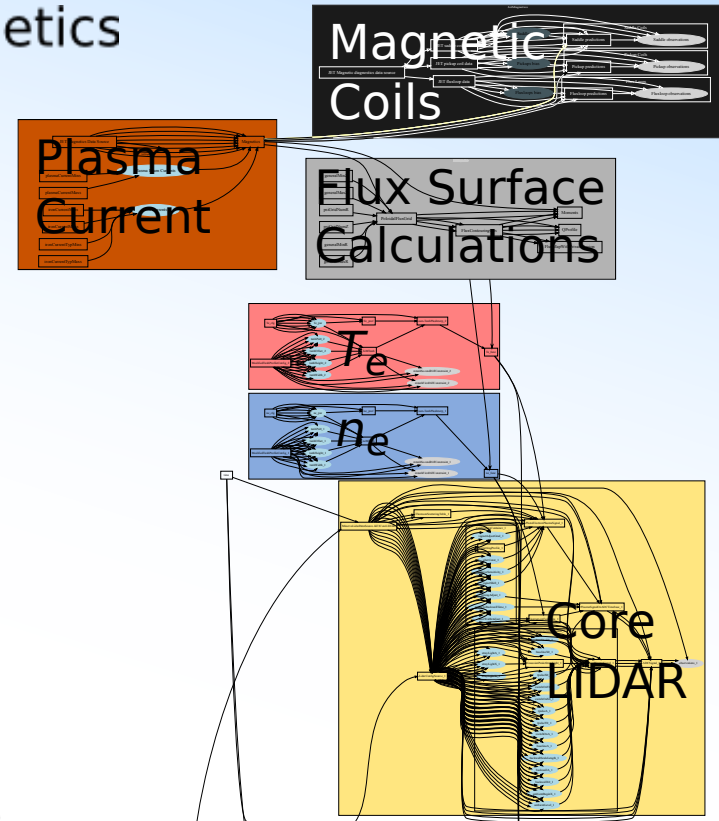
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The TS diagnostics provide information on plasma current near LOS.  
Plasma current one of the most important and least diagnosed parameters in Tokamaks.

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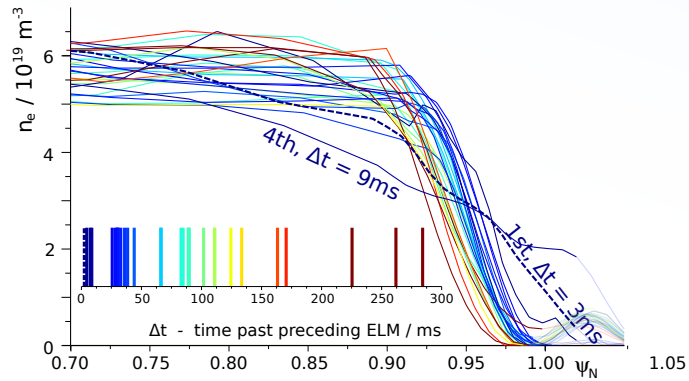
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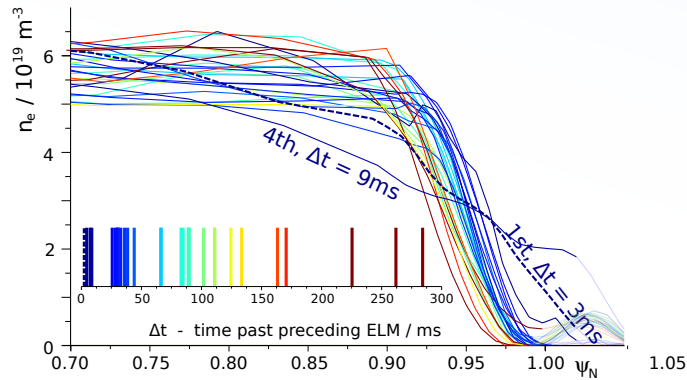
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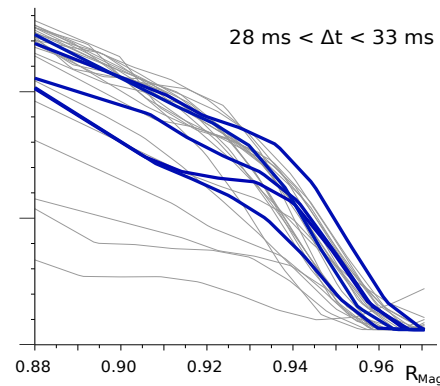
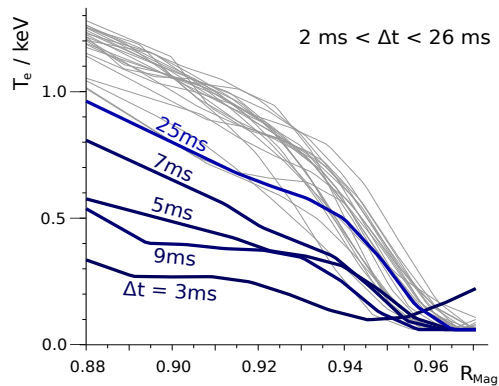
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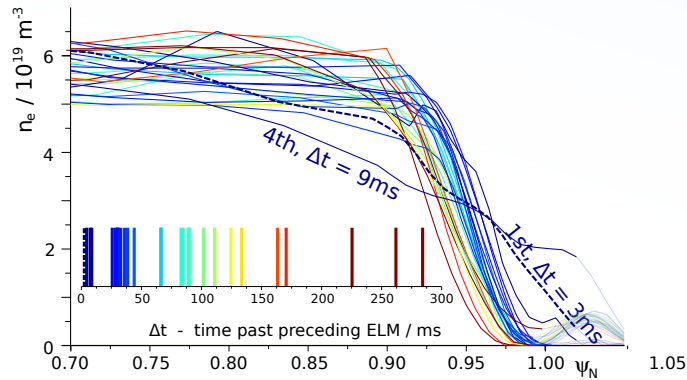
Appears to be two distinct phases for  $T_e$ :

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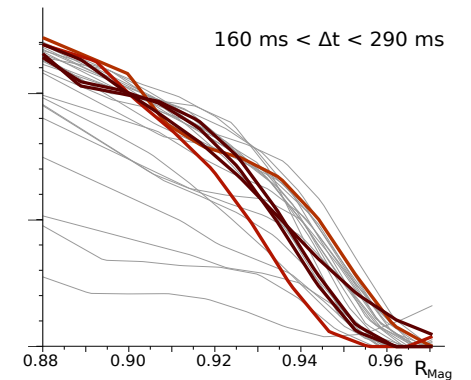
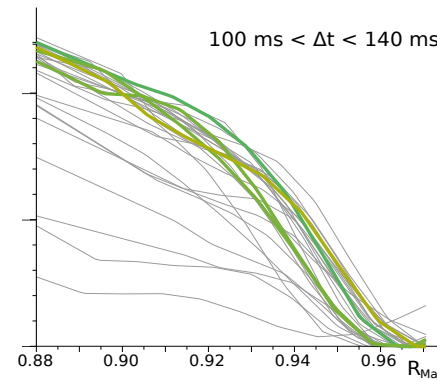
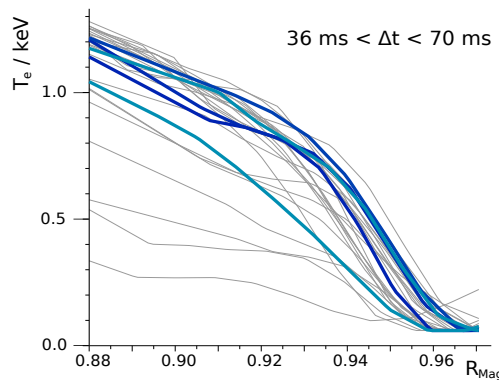
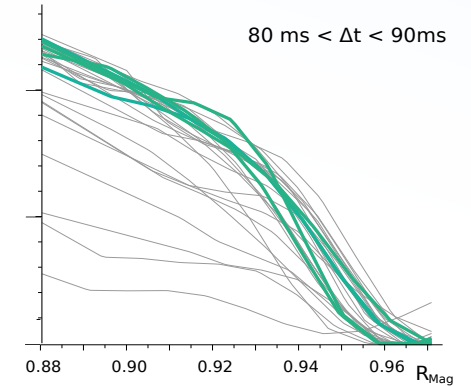
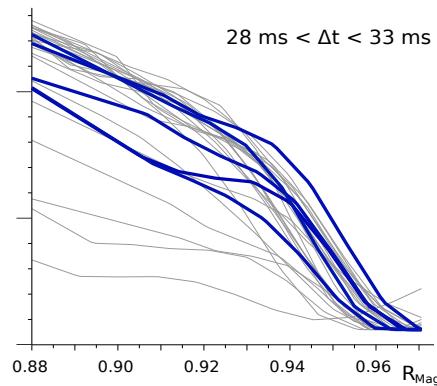
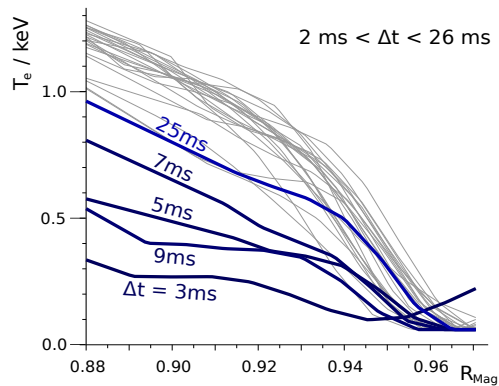
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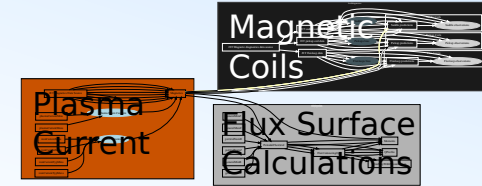
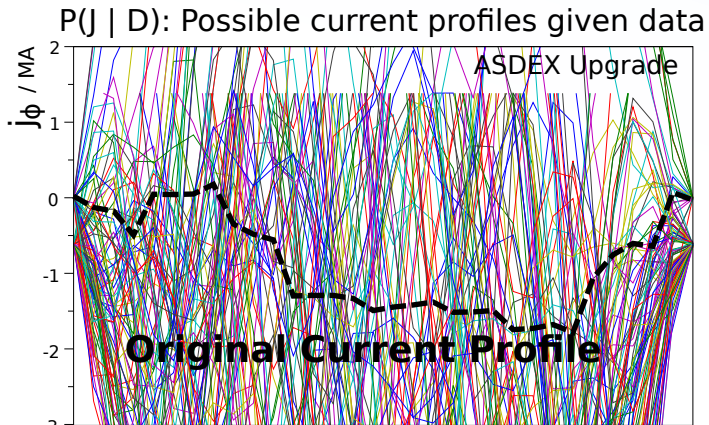
- 1) Rapid rise in height and gradient during first 50ms.
- 2) Slow rise in height and width at fixed gradient until next ELM.



# Current and Flux Surfaces

Inference of plasma current and flux surfaces  $P(\psi_N | \dots)$  is the big problem.

## 1) Current Tomography and external magnetics:

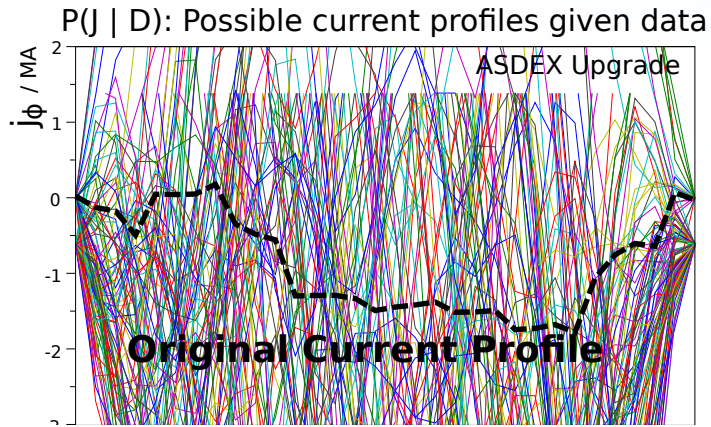




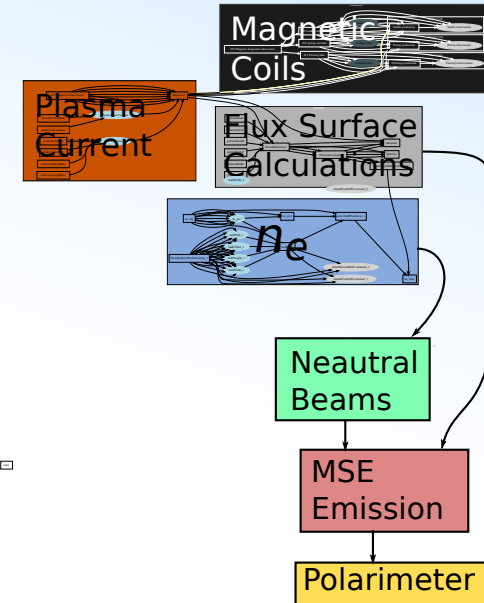
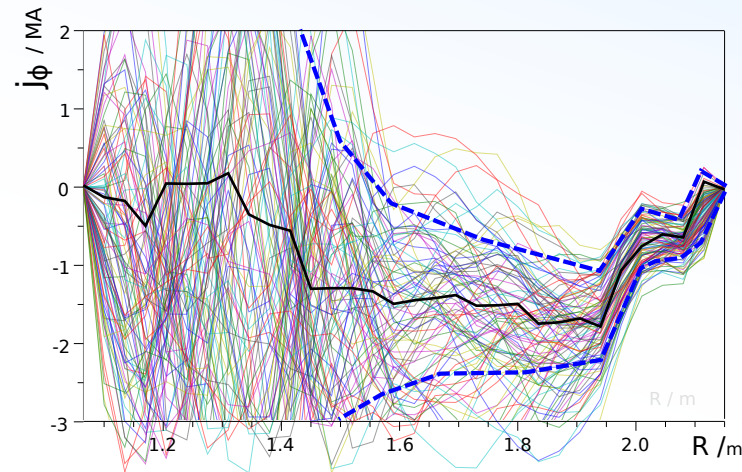
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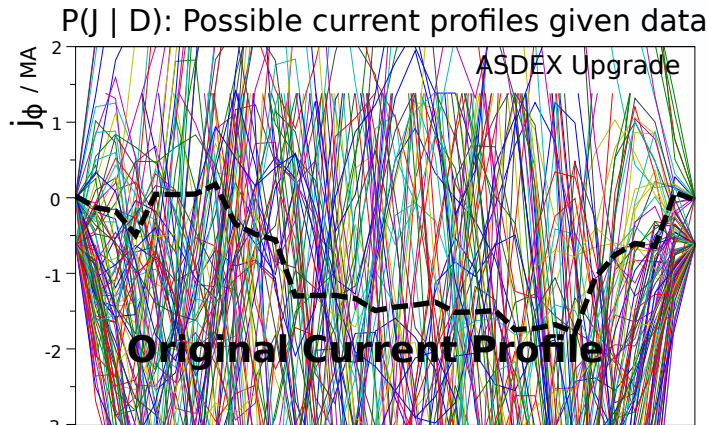
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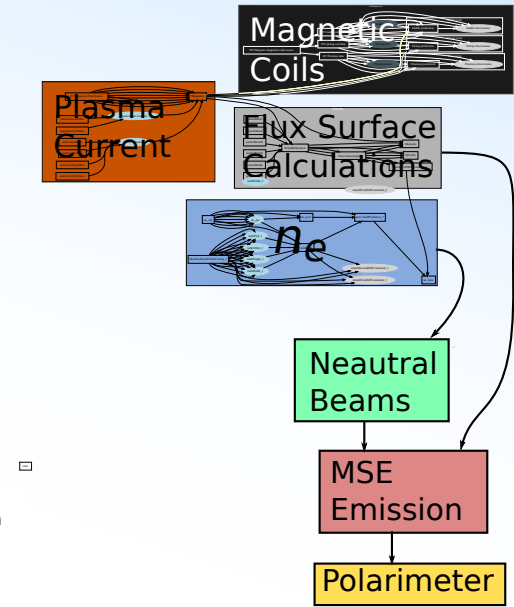
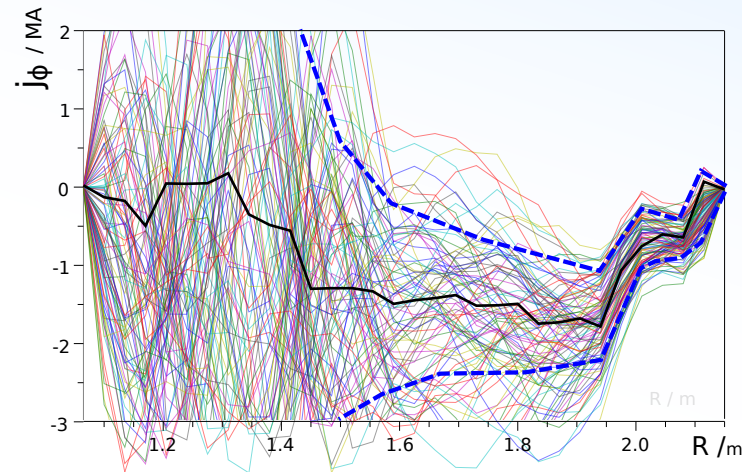
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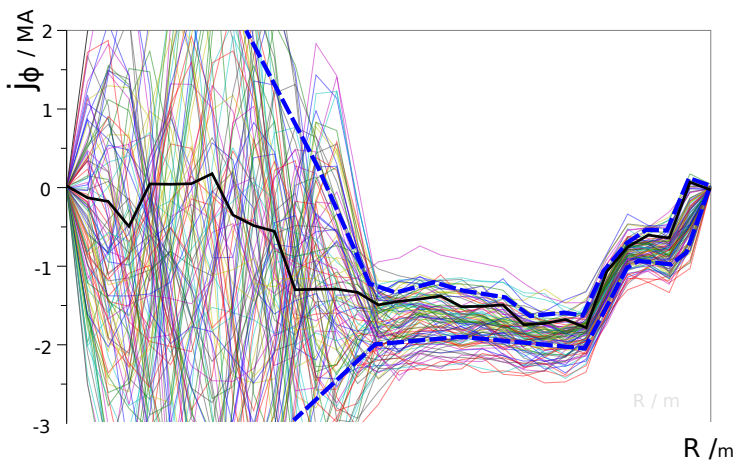
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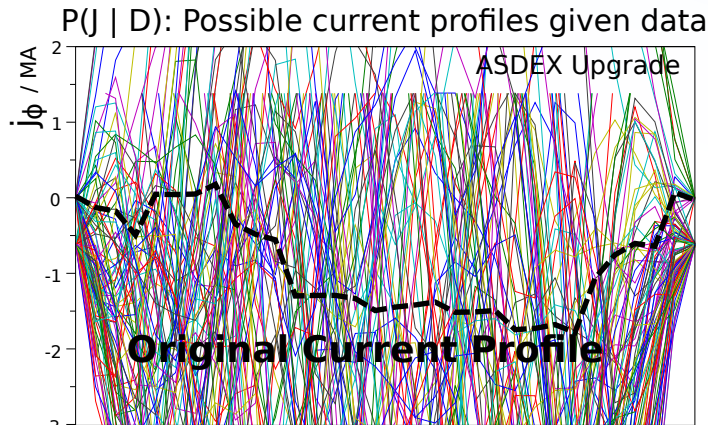
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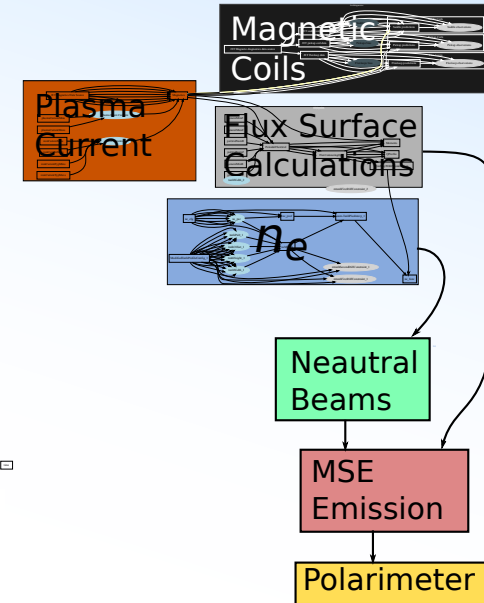
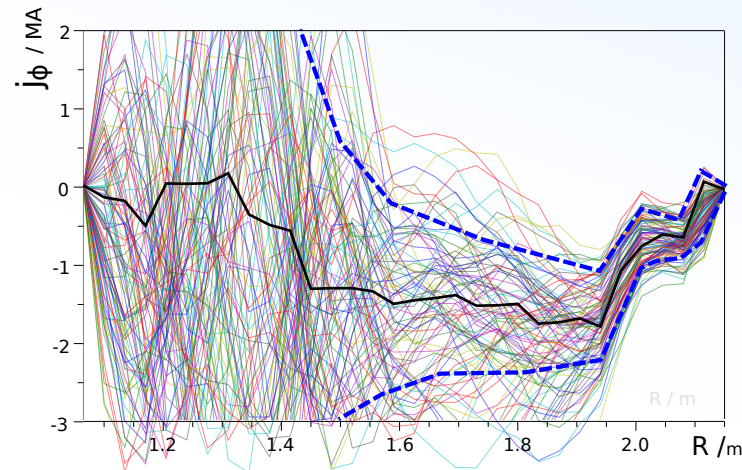
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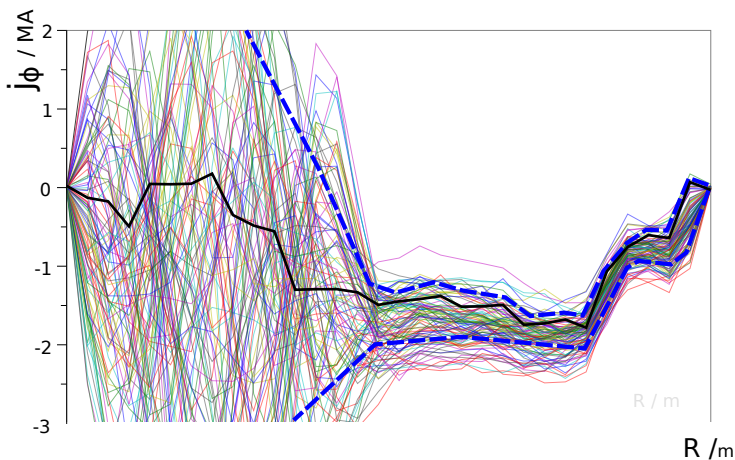
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In the meantime....

Add the 'prior knowledge' that the plasma should be somewhere near, something like an equilibrium....

$$J_\phi = Rp' + \frac{\mu_0}{R} f f'$$

(We can also think of this as the observation that the plasma hasn't exploded).

What can this and the external magnetics tell us?

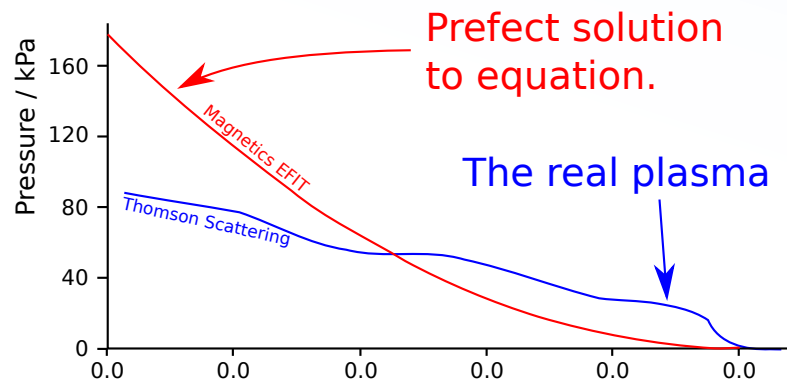
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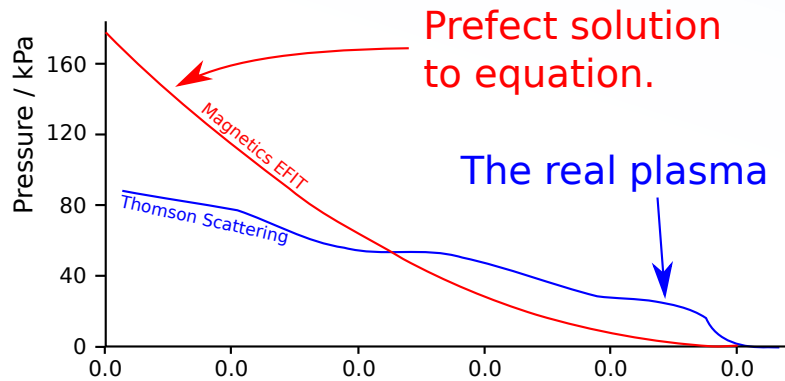
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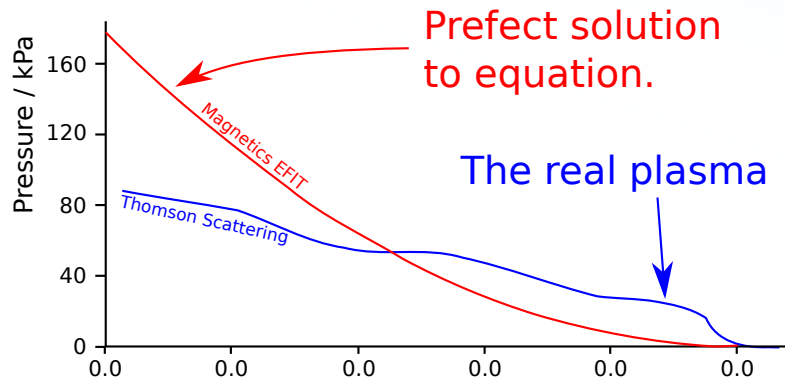
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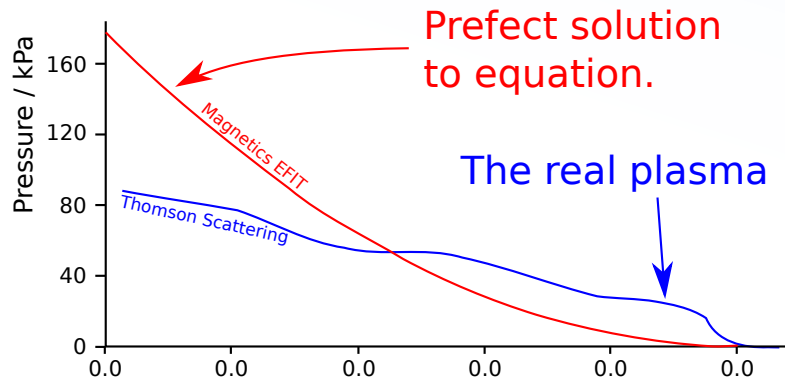
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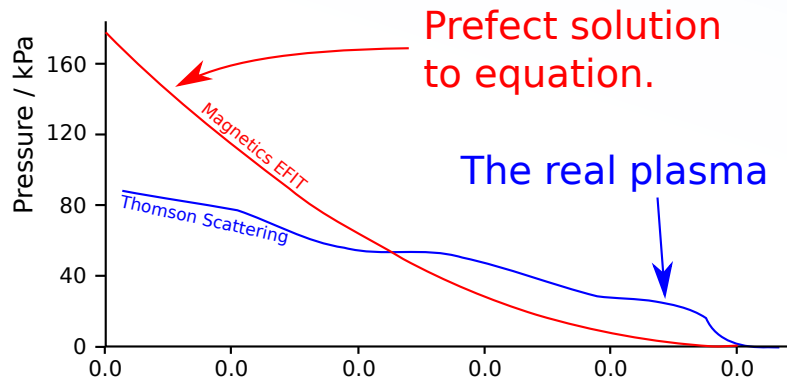
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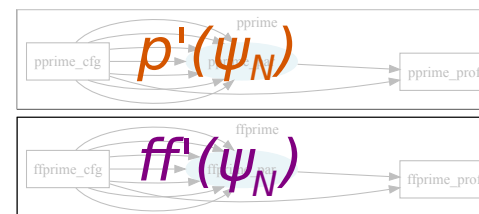
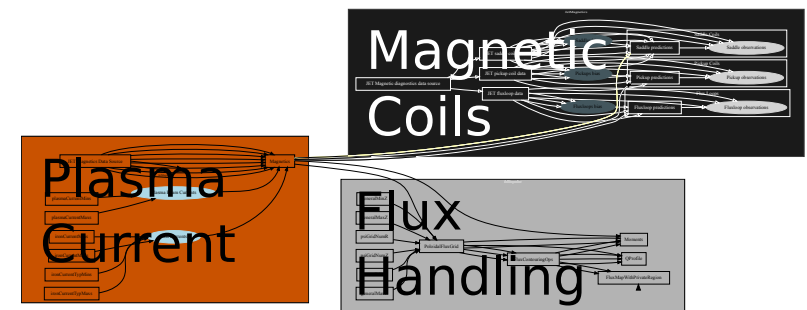


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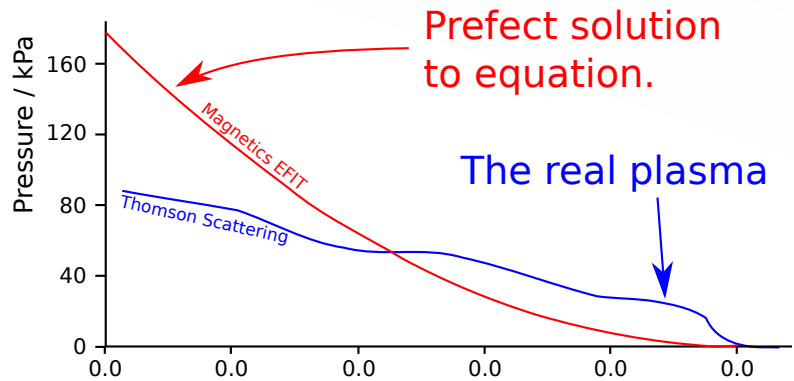
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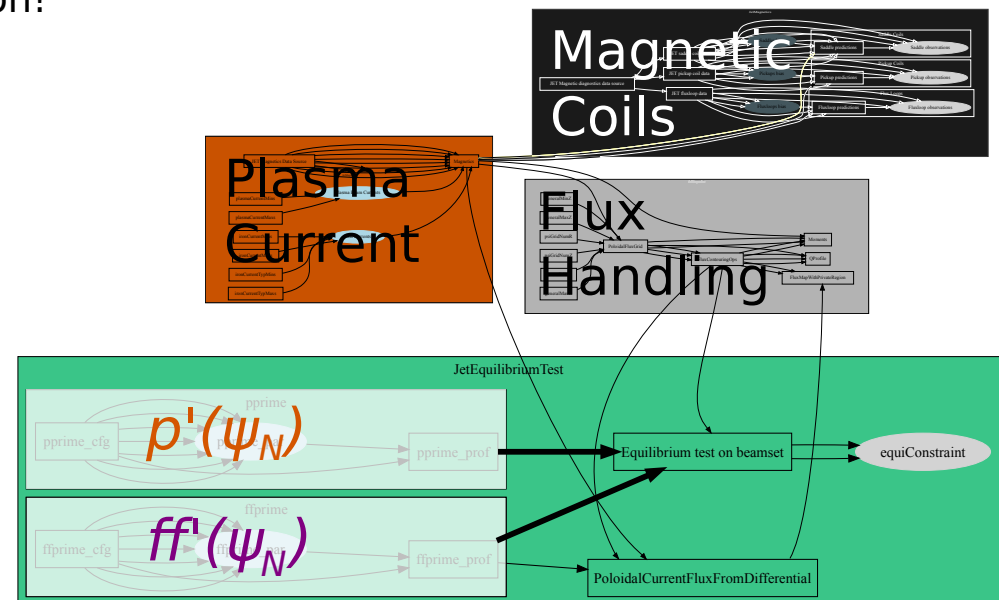
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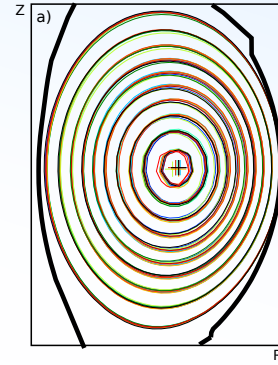
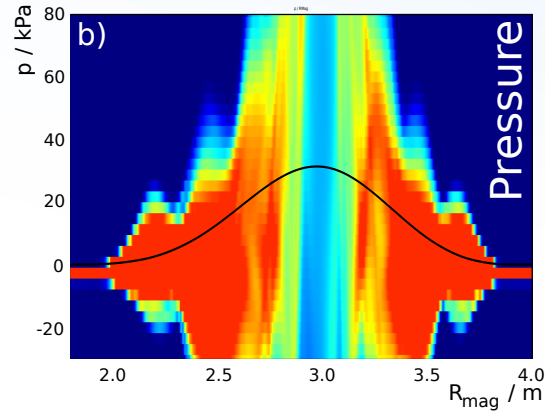
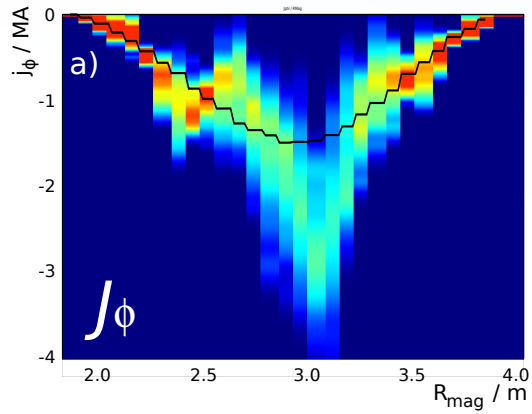
$$P(j_\phi - Rp' - \frac{\mu_0}{R} f f') = G(0, \sigma_{equi})$$

The posterior  $P(J, p', ff' | D_{diags}, \text{Equilibrium})$  will include all possible combinations of  $J, p'$  and  $ff'$  that are consistent with the diagnostics, the priors and describe a plasma very close to equilibrium.



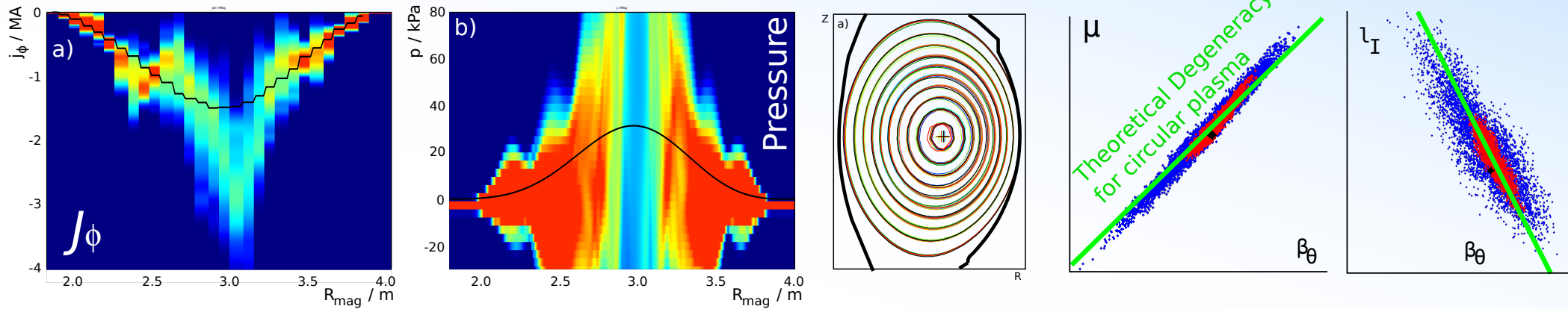
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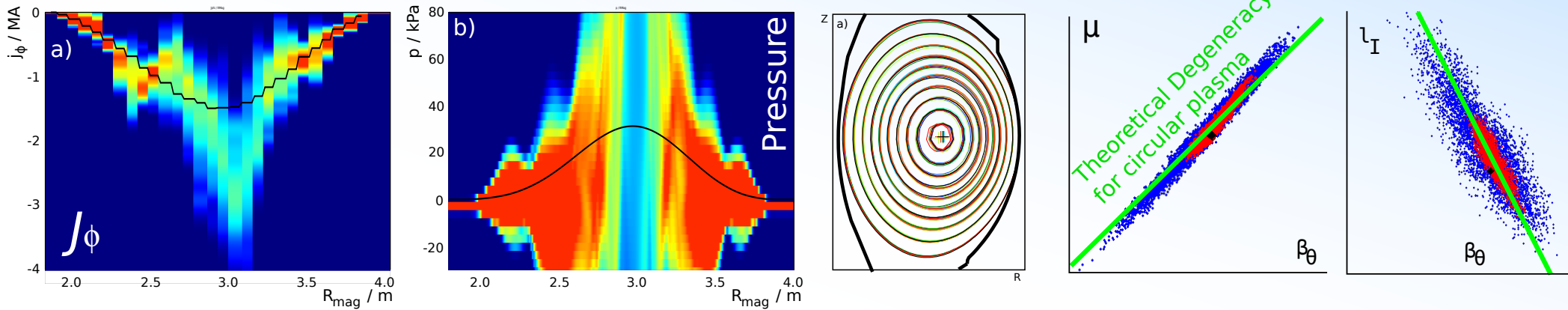
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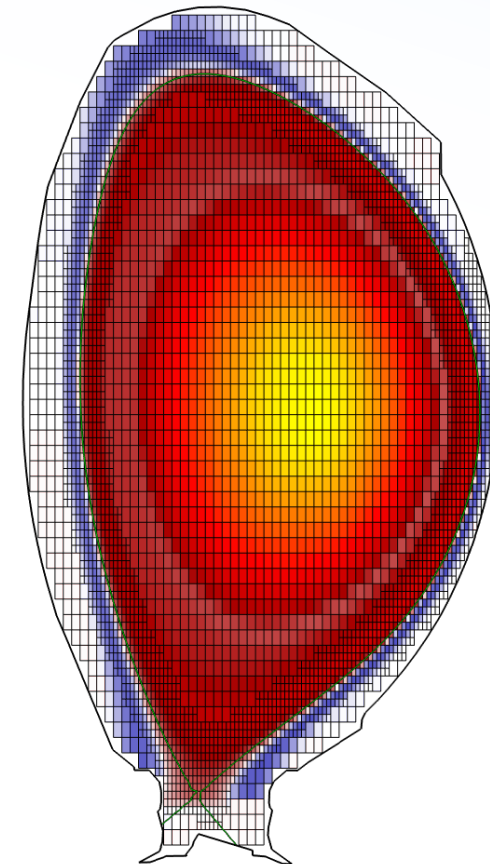
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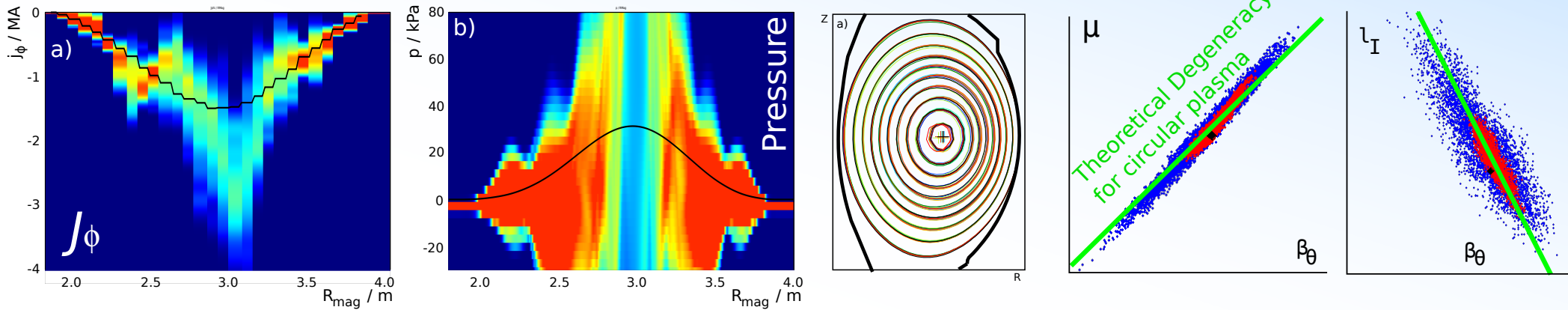
Because of modularity, we can switch parametrisation and priors of  $J$ ,  $p'$  and  $ff'$  at will and on-the-fly.  
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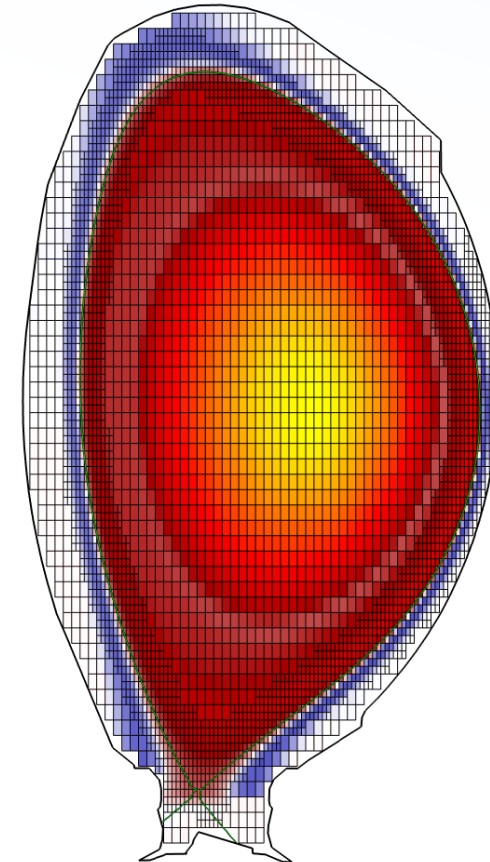
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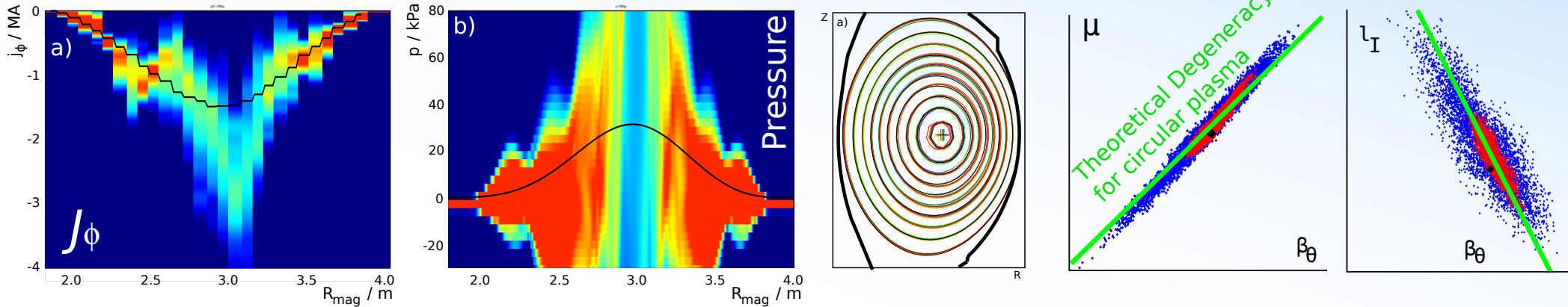
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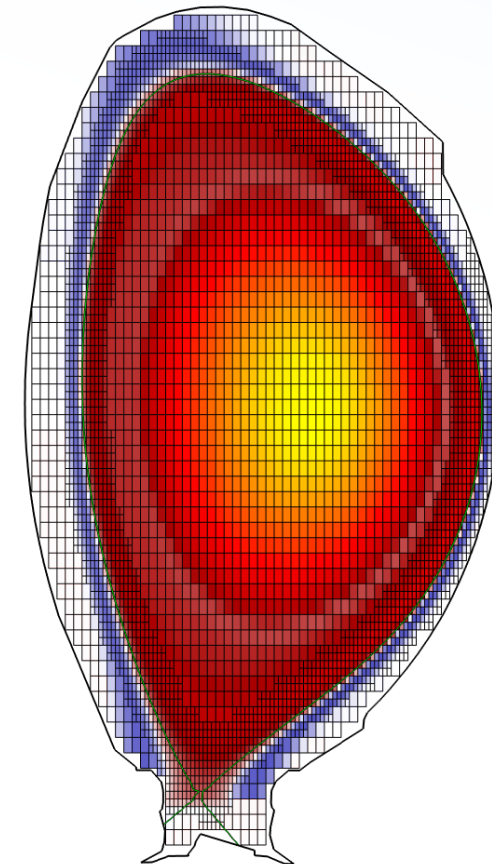
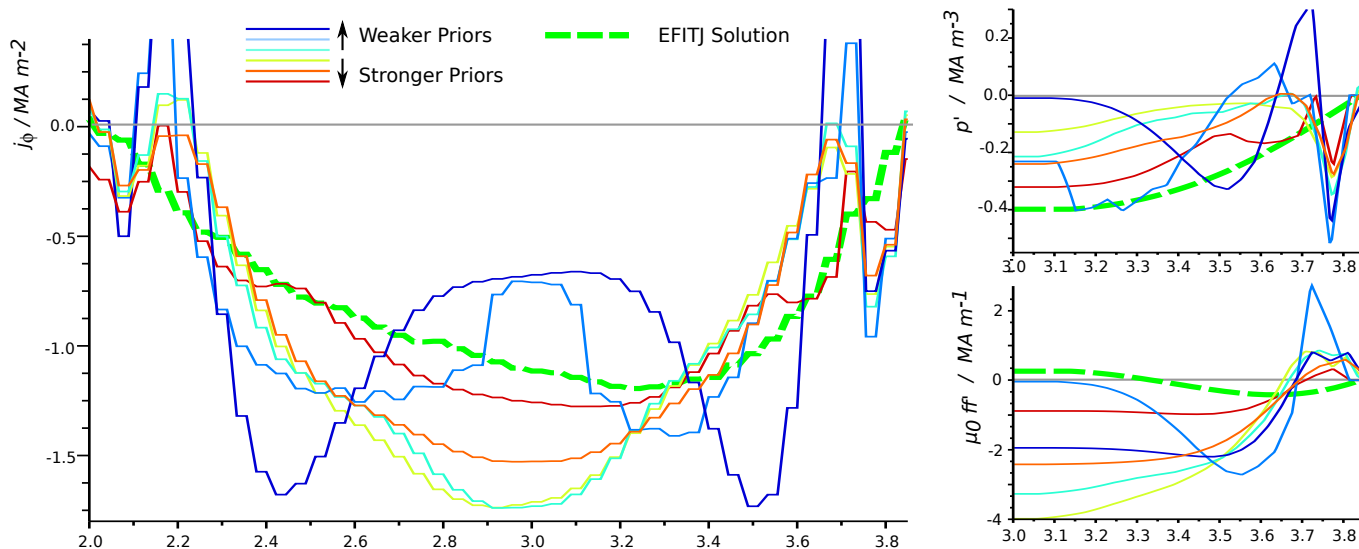
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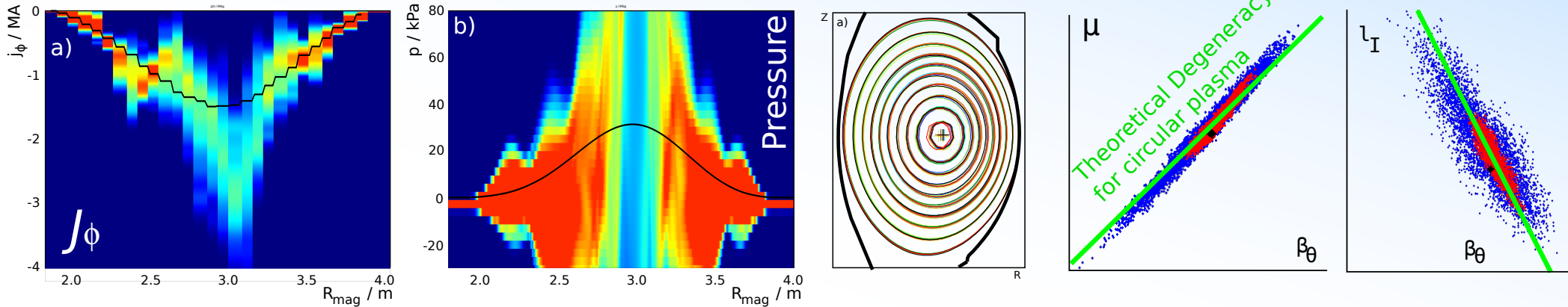
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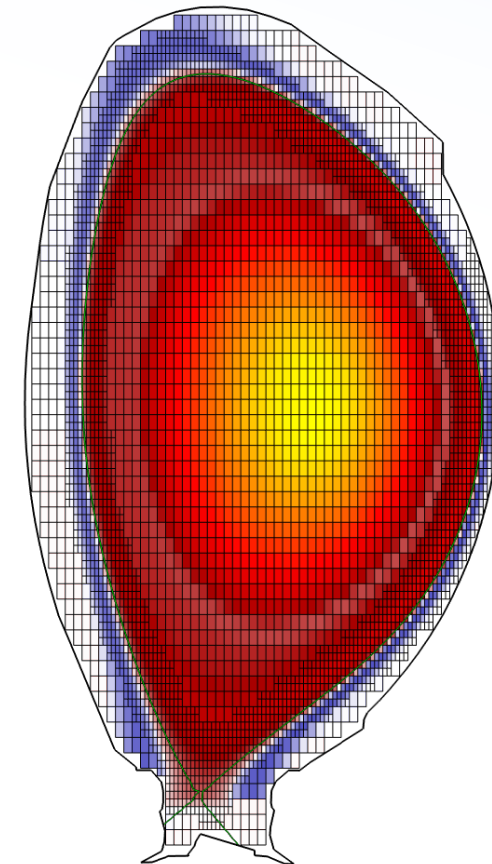
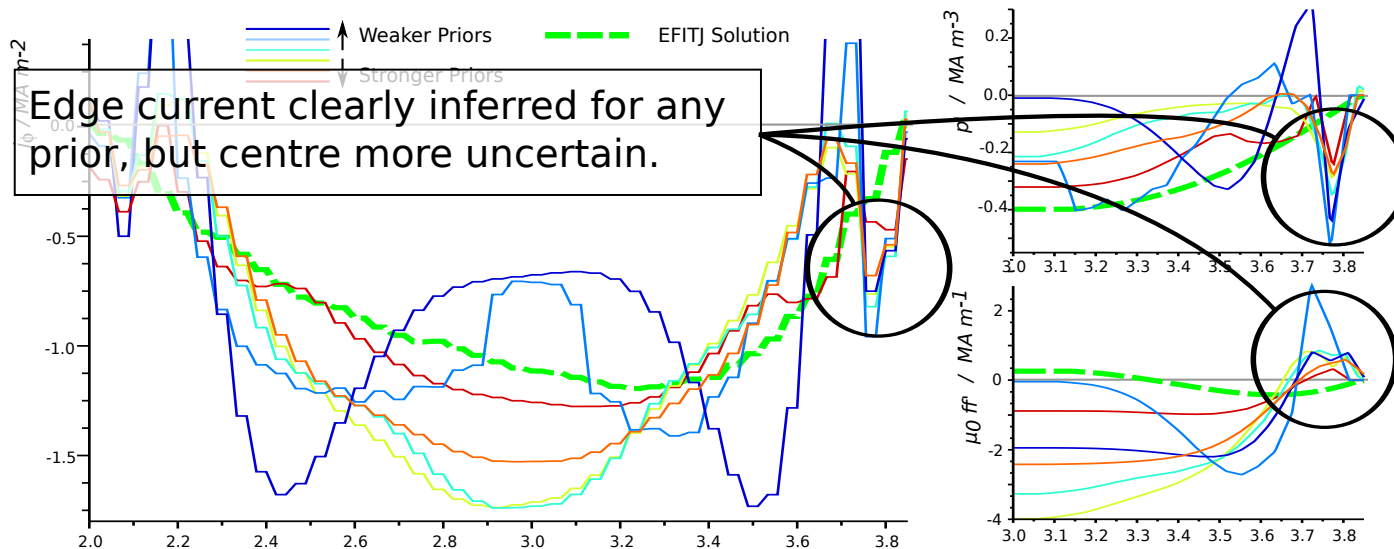
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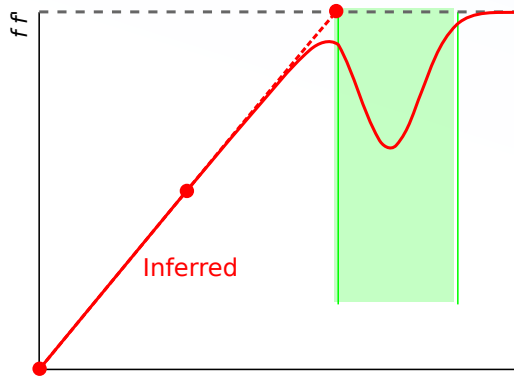


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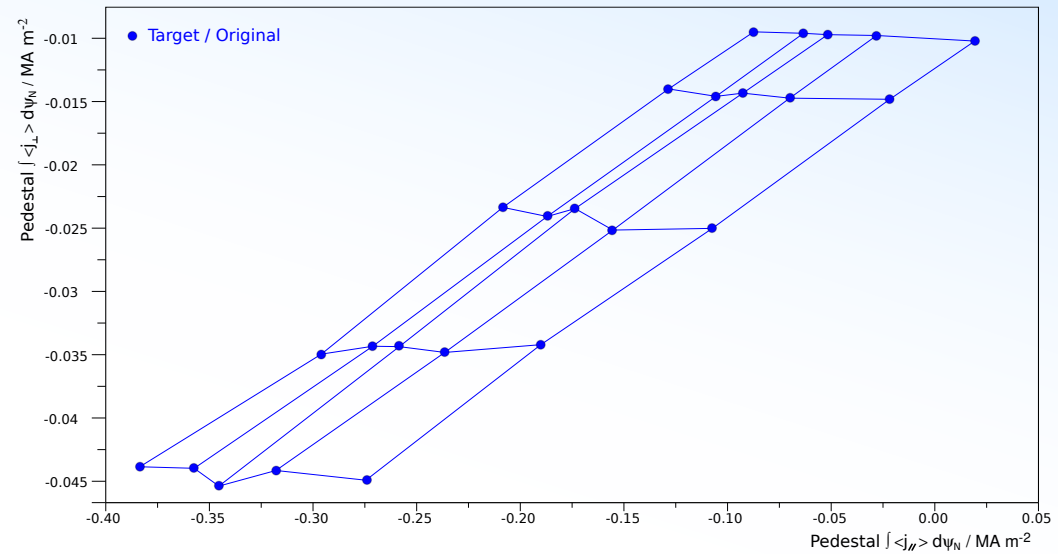
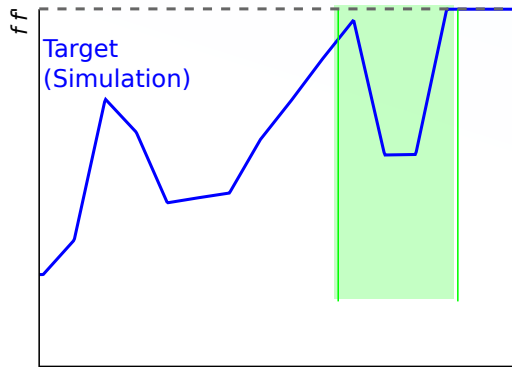
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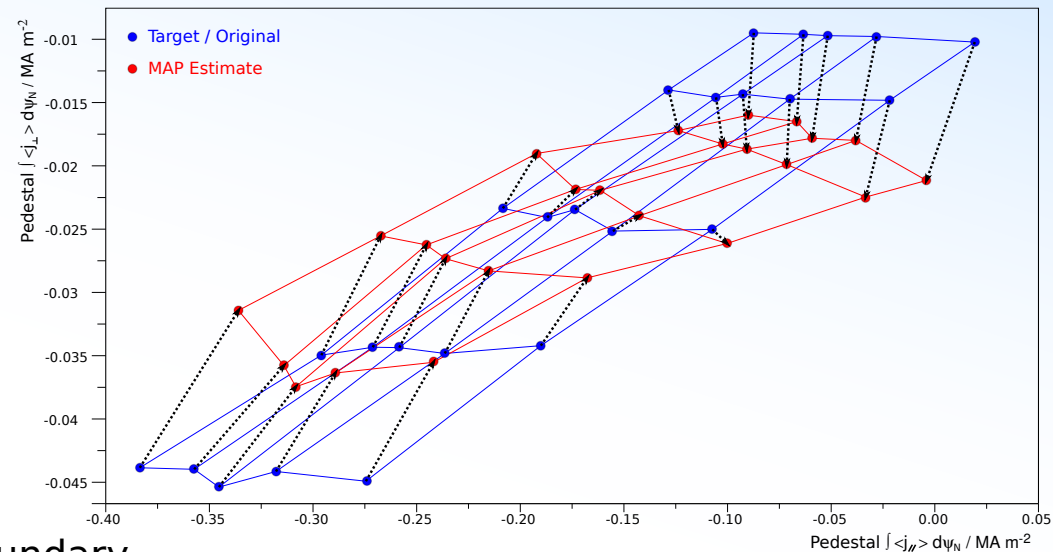
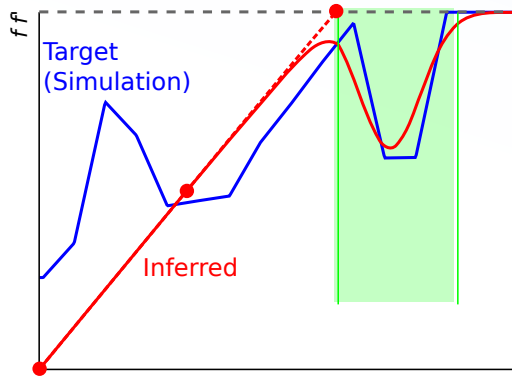
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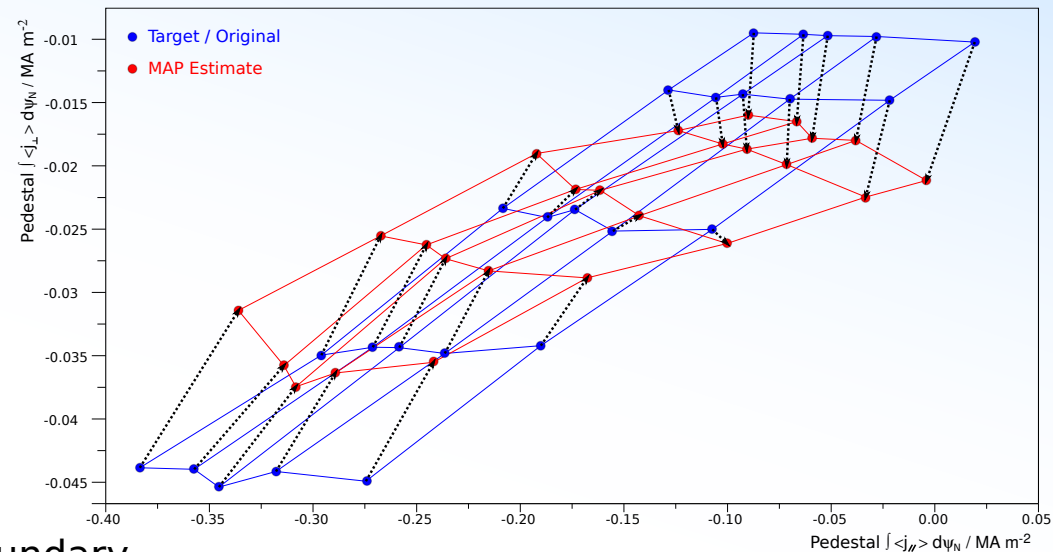
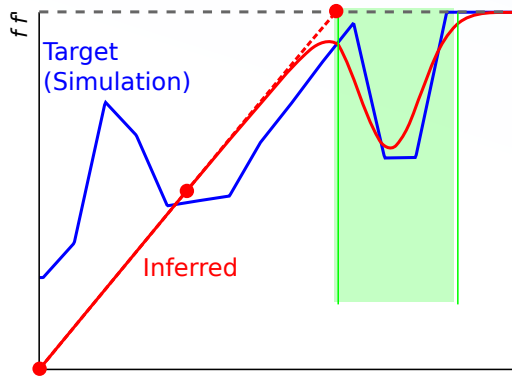
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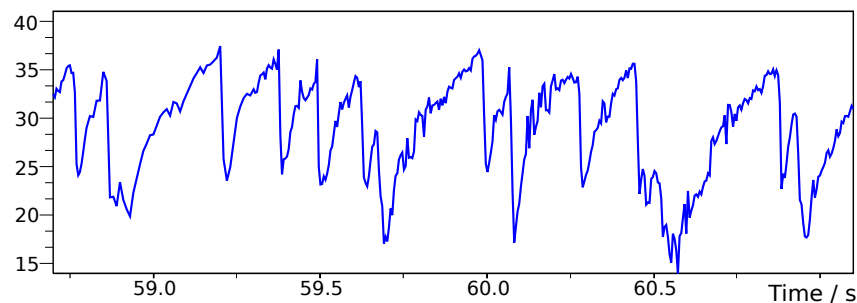
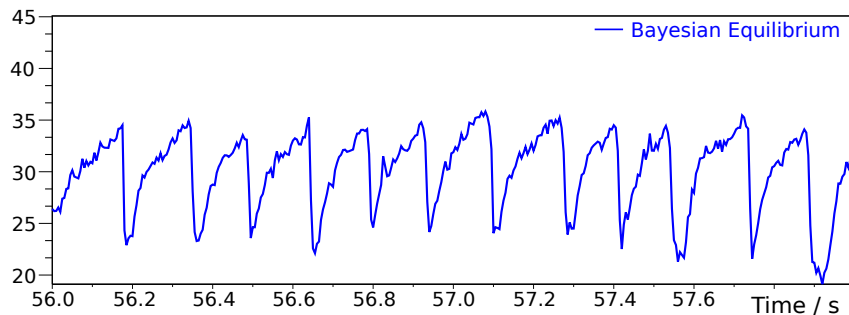


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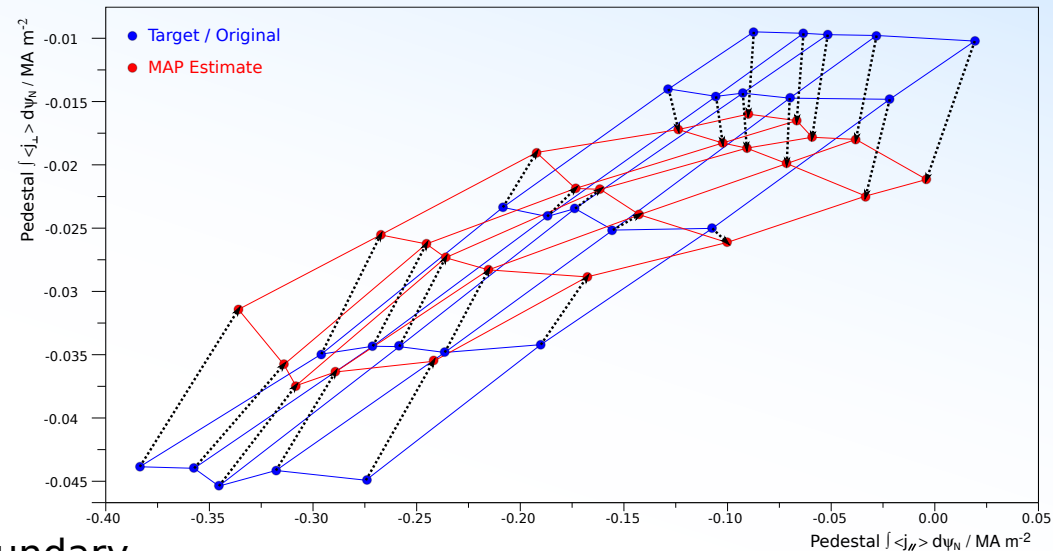
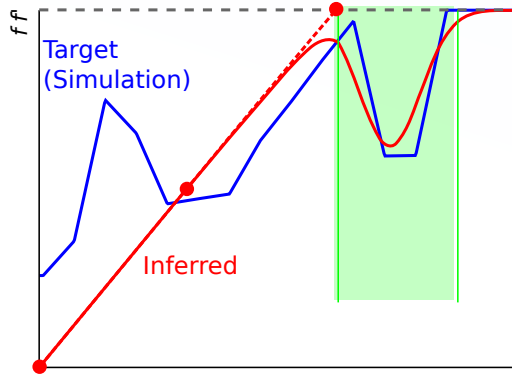
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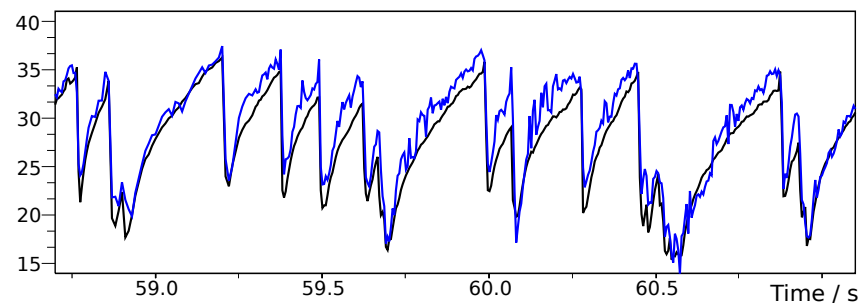
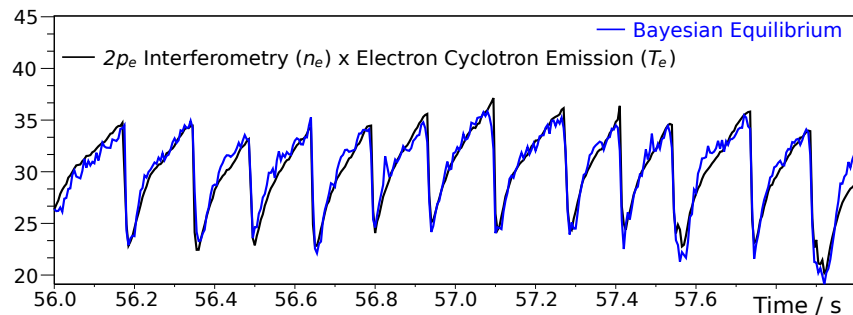


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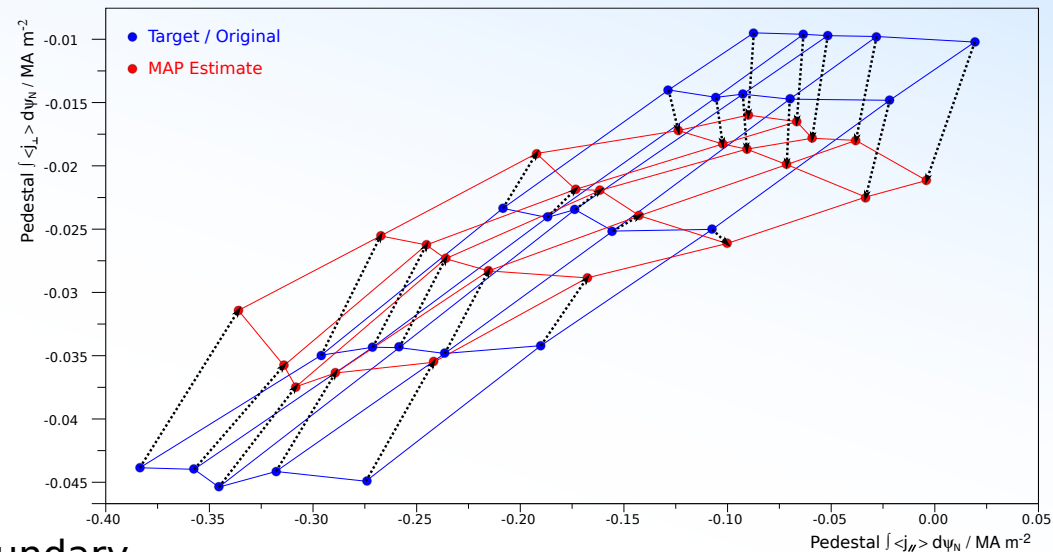
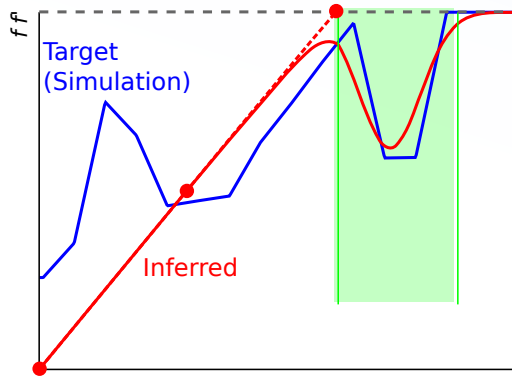
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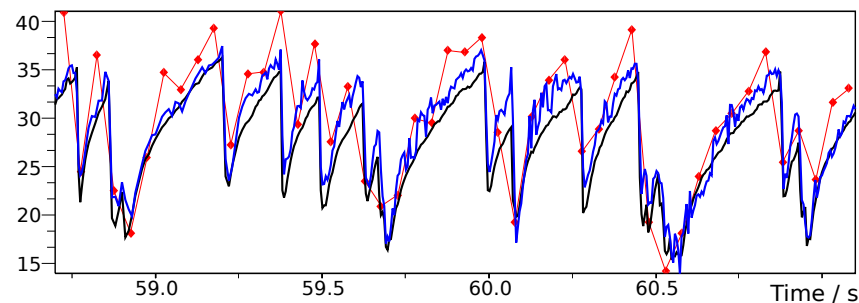
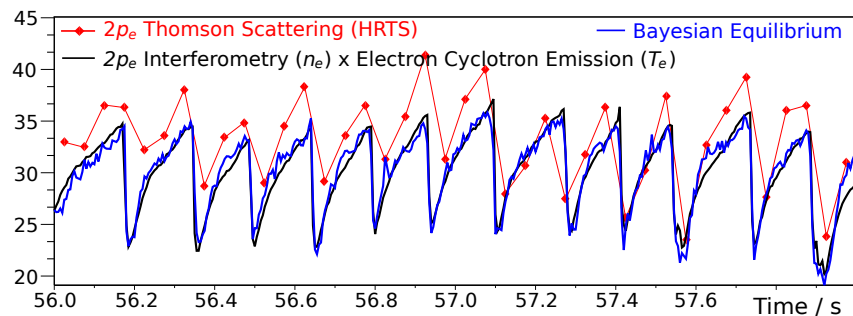


Surprising results:

We CAN reconstruct information inside boundary.

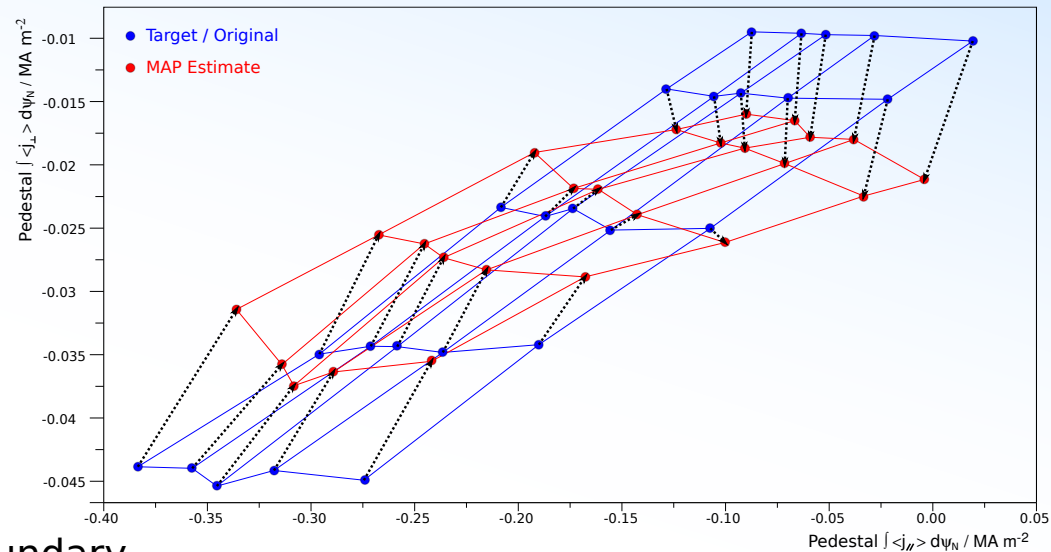
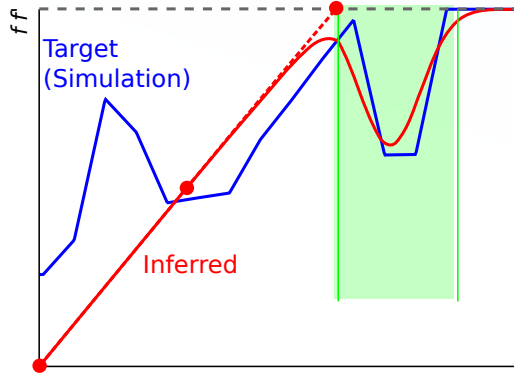
Can recover some information about pedestal current both the parallel AND perpendicular (i.e pressure) to the magnetic field.

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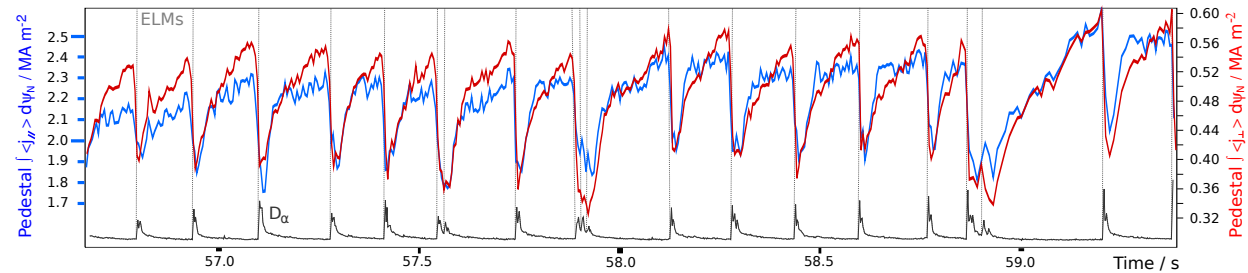
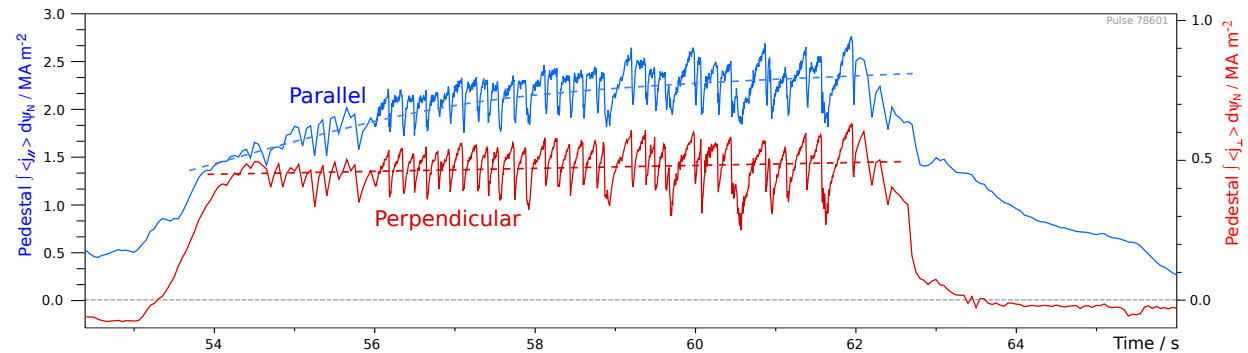
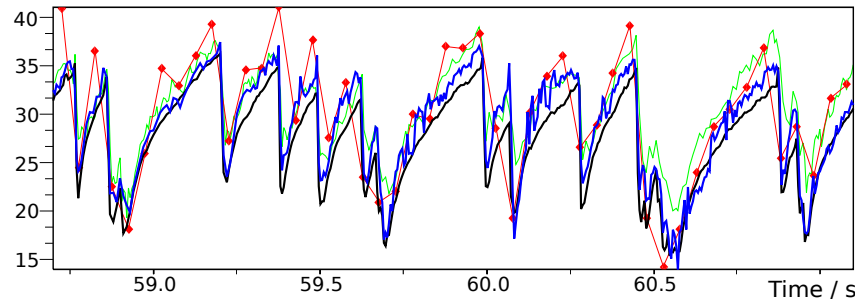
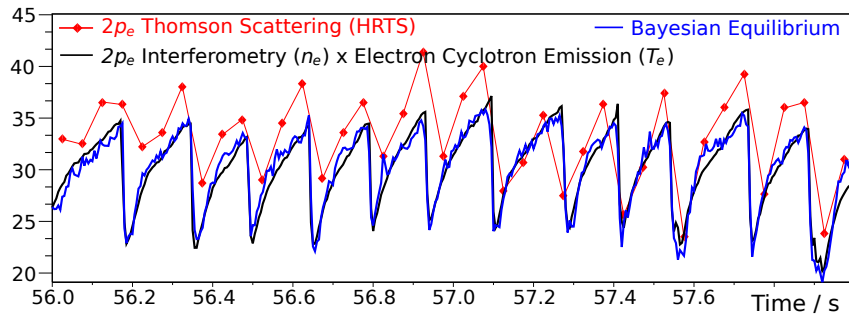


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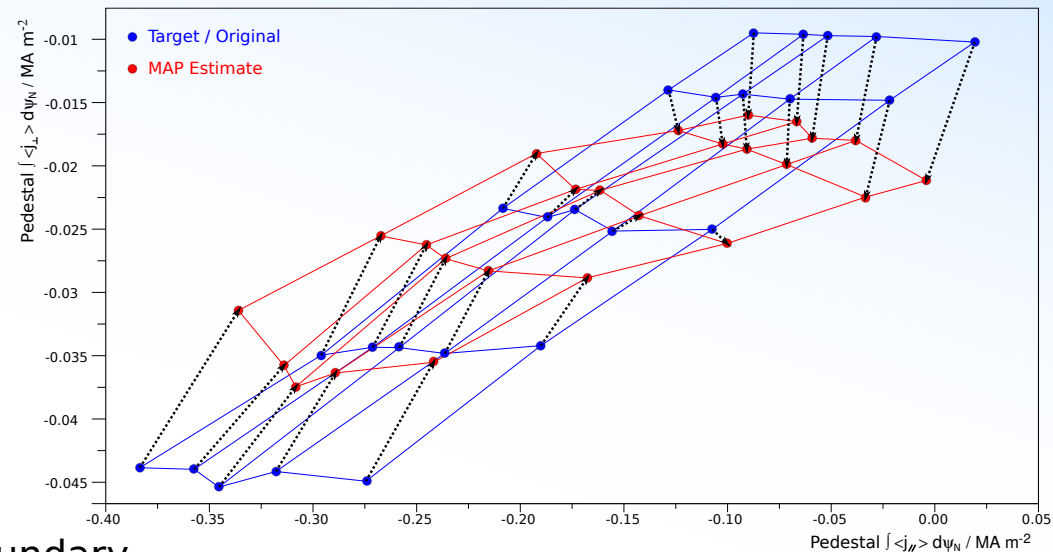
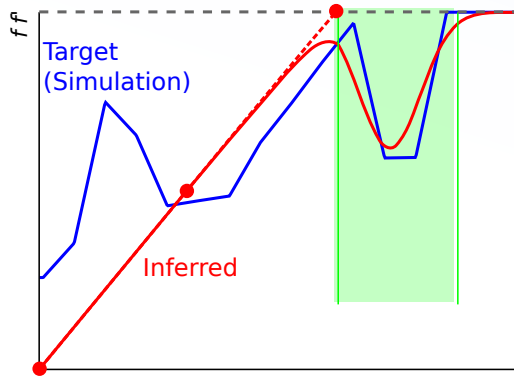
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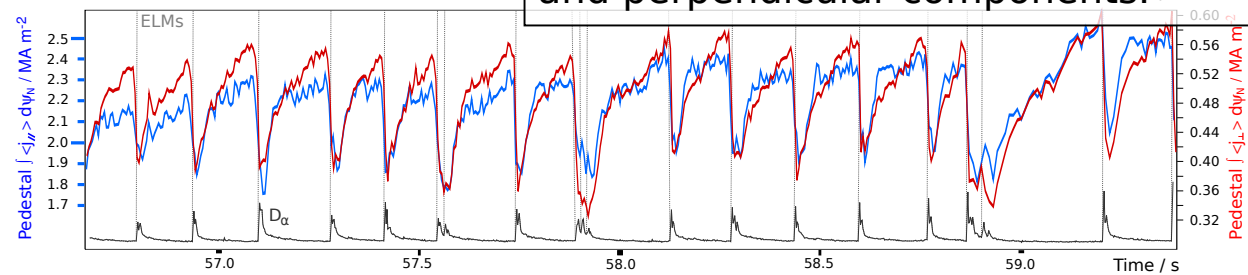
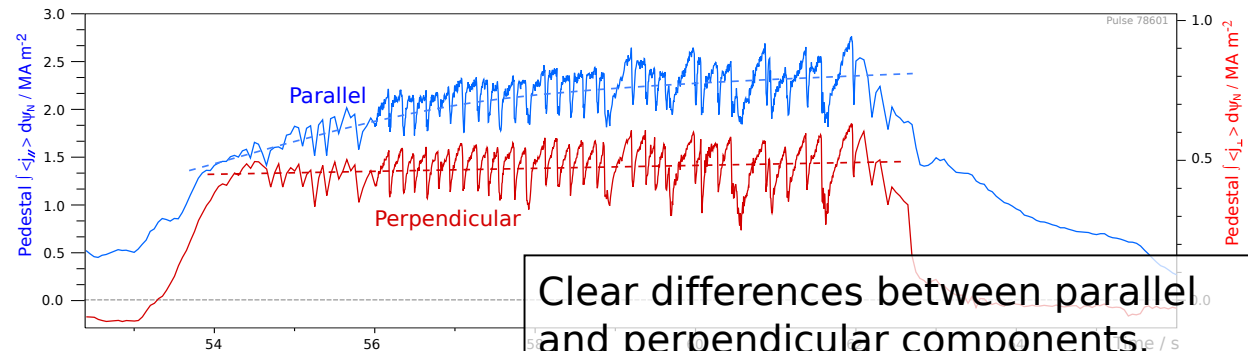
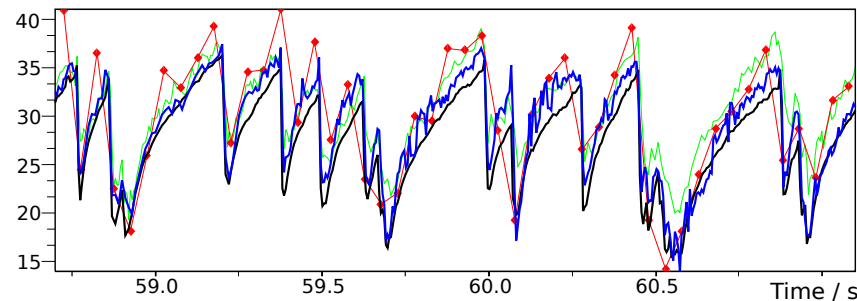
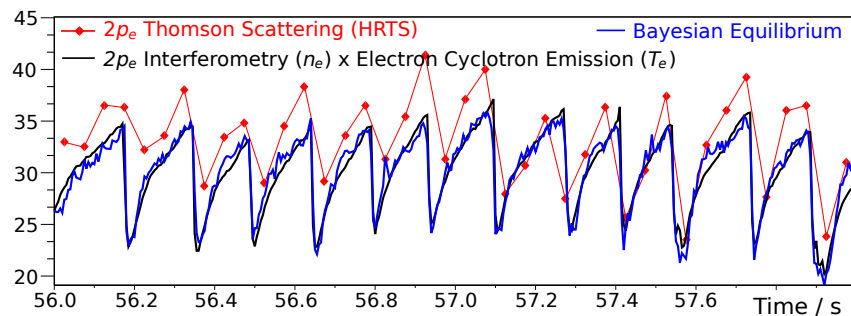


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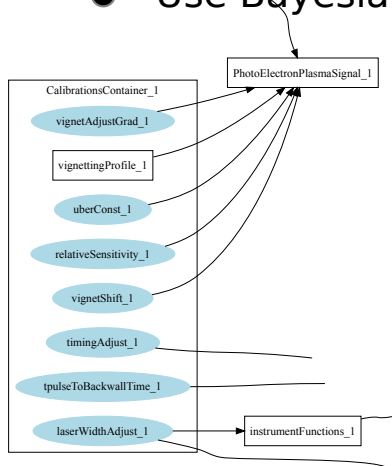
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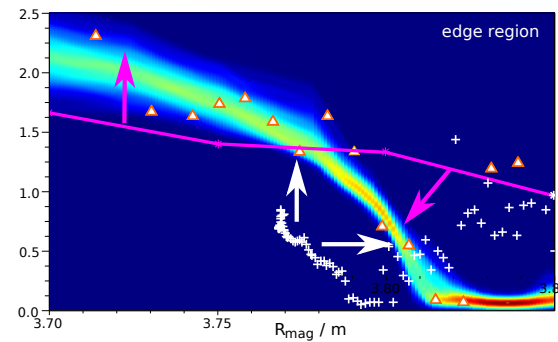
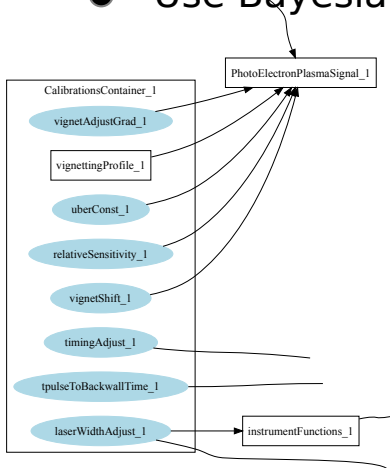
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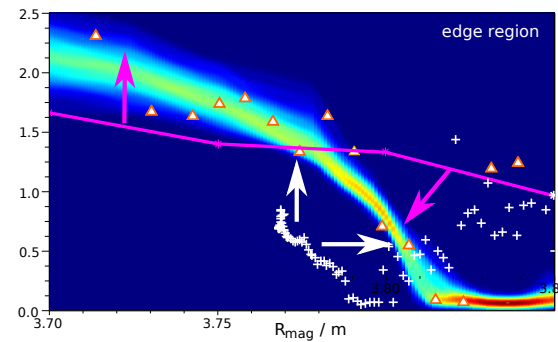
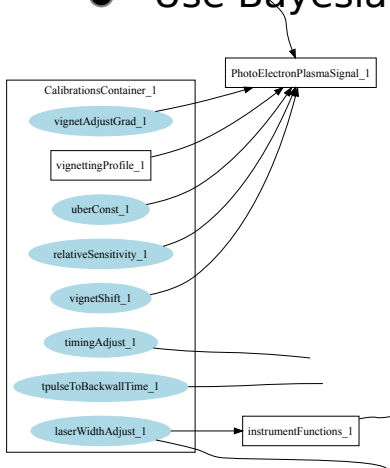


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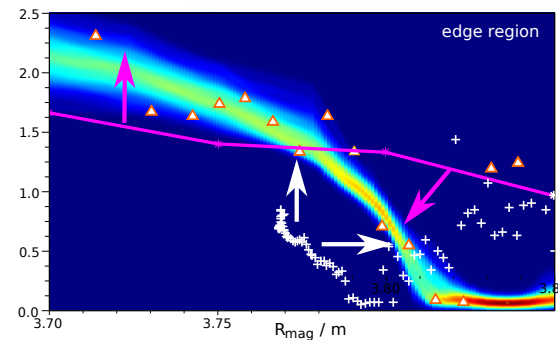
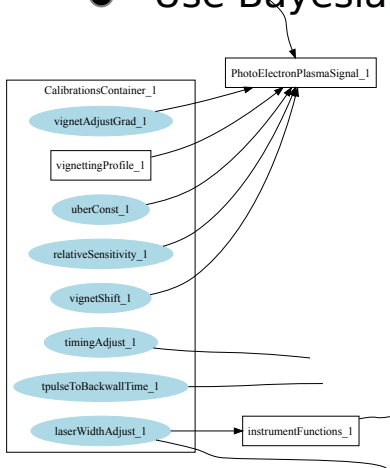


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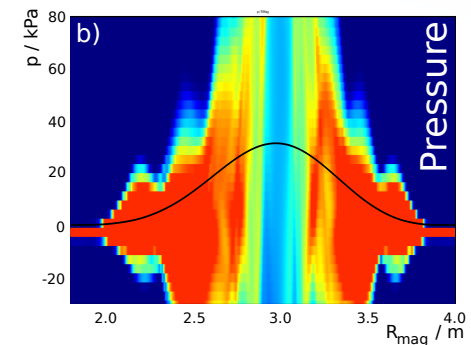
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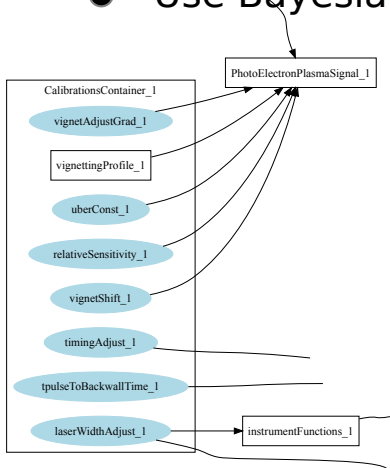




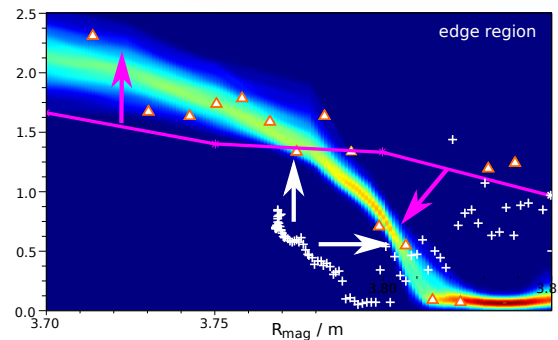
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