Polarimetry + Interferometry system

Plasma/probing beam

$$E_x^p = E_0^p \cos\theta_p \cos(\omega_0 t + \phi_p + \phi_{px})$$

$$E_y^p = E_0^p \sin\theta_p \cos(\omega_0 t + \phi_p + \phi_{py})$$

$\omega_0 = \dfrac{2\pi c}{195\,\mu m}$ laser frequency

ϕ_p = plasma beam overall phase

$$\phi_p = \frac{\cancel{\lambda} e^2}{2 c m_e \varepsilon_0 \omega^2} \int n_e \, dl + \phi_{p0}$$

ϕ_{py} = phase diff between components of plasma beam

Grated Beam:

$$E_x^g = E_0^g \cos\theta_g \cos(\omega_0 t - \omega_g t + \phi_g + \phi_{gx})$$

$$E_y^g = E_0^g \sin\theta_g \cos(\omega_0 t - \omega_g t + \phi_g + \phi_{gy})$$

ϕ_g = grated beam overall phase = const

ϕ_{gy} = grated beam components phs. diff

ω_g = Grated beam difference in freq = $2 \cdot \pi \cdot 100\,kHz$

'Extra' beam:

$$E_x^e = E_0^e \cos\theta_e \cos(\omega_0 t + \phi_e + \phi_{ex})$$

$$E_y^e = E_0^e \sin\theta_e \cos(\omega_0 t + \phi_e + \phi_{ey})$$

Combined wave: $E_x = E_x^p + E_x^g + E_x^e = A e^{i(\dots)}$

Intensity at detector is $\Psi\Psi^* = A e^{i(\dots)} \times A e^{-i(\dots)}$

Terms are (for y component)

$E_y^p E_y^{p*} = E_0^{p2} \sin^2\theta_p$ 1

$E_y^{g2} E_y^{g*} = E_0^{g2} \sin^2\theta_g$ 2

$E_y^e E_y^{e*} = E_0^{e2} \sin^2\theta_g$ 3

$E_y^g E_y^{p*} = E_0^p E_0^g \sin\theta_p \sin\theta_g \exp i(\cancel{\omega_0 t} - \omega_g t + \phi_g + \phi_{gy} \cancel{- \omega_0 t} - \phi_p - \phi_{py})$ 4

$E_y^g E_y^{e*} = E_0^g E_0^e \sin\theta_g \sin\theta_p \exp i(\cancel{\omega_0 t} - \omega_g t + \phi_g + \phi_{gy} \cancel{- \omega_0 t} - \phi_e - \phi_{ey})$ 5

$E_y^p E_y^{e*} = E_0^p E_0^e \sin\theta_p \sin\theta_e \exp i(\cancel{\omega_0 t} + \phi_p + \phi_{py} \cancel{- \omega_0 t} - \phi_e - \phi_{ey})$ 6

$E_y^p E_y^{g*} = 4^*$ 7

$E_y^e E_y^{g*} = 5^*$ 8

$E_y^e E_y^{p*} = 6^*$ 9

Collecting

$$\Psi\Psi^* = 2E_0^{p^2}\sin^2\theta_p + 2E_0^{g^2}\sin^2\theta_g + 2E_0^{e^2}\sin^2\theta_e$$
$$+ E_0^p E_0^g \sin\theta_p \sin\theta_g \left[e^{i(\alpha-\omega_g t)} + e^{-i(\alpha-\omega_g t)}\right] \quad (1)+(2)$$
$$+ E_0^g E_0^e \sin\theta_g \sin\theta_e \left[e^{i(\beta-\omega_g t)} + e^{-i(\beta-\omega_g t)}\right]$$
$$+ E_0^p E_0^e \sin\theta_p \sin\theta_e \left[e^{i\gamma} + e^{-i\gamma}\right]$$

$$\boxed{\begin{aligned}\alpha_y &= (\phi_g - \phi_p) + (\phi_{gy} - \phi_{py})\\ \beta_y &= (\phi_g - \phi_e) + (\phi_{gy} - \phi_{ey})\\ \gamma_y &= (\phi_p - \phi_e) + (\phi_{py} - \phi_{ey})\end{aligned}}$$

$$\Psi\Psi^* = \text{consts} + 2E_0^p E_0^g \sin\theta_p \sin\theta_g \cos(\alpha_y - \omega_g t)$$
$$+ 2E_0^g E_0^e \sin\theta_g \sin\theta_e \cos(\beta_y - \omega_g t)$$
$$+ 2E_0^p E_0^e \sin\theta_p \sin\theta_e \cos(\gamma_y)$$

post, consts and Last term aren't wave in 100kHz so are lost in AC amplification

x components are same except $\sin\theta_x \rightarrow \cos\theta_x$ and $\phi_{iy} \rightarrow 0$

x component: $i = a_i E_0^g \cos\theta_g \left[E_0^p \cos\theta_p \cos(\alpha_x - \omega_g t) + E_0^e \cos\theta_e \cos(\beta_x - \omega_g t)\right]$

y component: $p = a_p E_0^g \sin\theta_g \left[E_0^p \sin\theta_p \cos(\alpha_y - \omega_g t) + E_0^e \sin\theta_e \cos(\beta_y - \omega_g t)\right]$

$$\alpha_x = (\phi_g - \phi_p) + (\phi_{gx} - \phi_{px})$$
$$\beta_x = (\phi_g - \phi_e) + (\phi_{px} - \phi_{ex})$$

~~Both are~~ Convert i/p to single wave:

$$\Gamma_y \cos(\xi - \omega_g t) = \Gamma_y \cos(\xi)\cos(\omega_g t) + \Gamma_y \sin(\xi)\sin(\omega_g t)$$

$$p = \cancel{a_p E_0^g \sin\theta_g} \; A\cos(\alpha - \omega_g t) + B\cos(\beta - \omega_g t)$$
$$= A\cos(\alpha)\cos(\omega_g t) + A\sin(\alpha)\sin(\omega_g t) + B\cos(\beta)\cos(\omega_g t) + B\sin(\beta)\sin(\omega_g t)$$

cos $\omega_g t$ terms: $\Gamma\cos(\xi) = A\cos\alpha + B\cos\beta$

sin $\omega_g t$ terms: $\Gamma\sin(\xi) = A\sin\alpha + B\sin\beta$

$$\tan(\xi) = \frac{A\sin\alpha + B\sin\beta}{A\cos\alpha + B\cos\beta} \qquad \Gamma^2 = A^2\cos^2\alpha + B^2\cos^2\beta + A^2\sin^2\alpha + B^2\sin^2\beta$$
$$+ 2AB\cos\alpha\cos\beta$$
$$+ 2AB\sin\alpha\sin\beta$$

$$\Gamma^2 = A^2 + B^2 + 2AB\cos(\alpha - \beta)$$

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$$i = \Gamma_x \cos(\xi_x - \omega_g t) \qquad \Gamma_i = A_i^2 + B_i^2 + 2A_iB_i\cos(\alpha_i - \beta_i)$$

$$p = \Gamma_y \cos(\xi_y - \omega_g t) \qquad \xi_i = \frac{A_i \sin\alpha_i + B_i \sin\beta_i}{A_i \cos\alpha_i + B_i \sin\beta_i}$$

$$A_x = a_i E_0^g \cos\theta_g \; E_0^p \cos\theta_p$$

$$B_x = a_i E_0^g \cos\theta_g \; E_0^e \cos\theta_e$$

$$A_y = a_p E_0^g \sin\theta_g \; E_0^p \sin\theta_p$$

$$B_y = a_p E_0^g \sin\theta_g \; E_0^e \sin\theta_e$$

$\int \cos^2 \rightarrow 0.5$

$$RMS = \langle i . i \rangle = \int_0^{\frac{2\pi}{\omega_g}} \Gamma_x^2 \cancelto{0.5}{\cos^2(\xi_x - \omega_g t)}\, dt$$

$$= 0.5\,\Gamma_x^2$$

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1 introduce $i' = i$ phase shifted by $\frac{\pi}{2}$ – and $\times$ by unknown amplitude K

$$i' = K\Gamma_x \sin(\xi_x - \omega_g t)$$

$$RMP = \langle i' . i' \rangle = \int_0^{2\pi/\omega_g} K^2 \Gamma_x^2 \cancelto{0.5}{\sin^2(\xi_x - \omega_g t)}\, dt$$

$$= 0.5\, K^2 \Gamma_x^2$$

$$PSD = \langle i . p \rangle = \Gamma_x \Gamma_y \int_0^{\frac{2\pi}{\omega_g}} \cos(\xi_x - \omega_g t)\cos(\xi_y - \omega_g t)\, dt$$

$$= \Gamma_x\Gamma_y \int_0^{2\pi/\omega_g} \big(\underset{①}{\cos(\xi_x)\cos(\omega_g t)} + \underset{②}{\sin(\xi_x)\sin(\omega_g t)}\big)\big(\underset{③}{\cos(\xi_y)\cos(\omega_g t)} + \underset{④}{\sin(\xi_y)\sin(\omega_g t)}\big)$$

$$= \Gamma_x\Gamma_y \Big[\overset{①③}{\cos(\xi_x)\cos(\xi_y)} \int \cancelto{0.5}{\cos^2(\omega_g t)\, dt} + \overset{②④}{\sin(\xi_x)\sin(\xi_y)} \int \cancelto{0.5}{\sin^2(\omega_g t)}\, dt$$

$$+ \overset{①④}{\cos(\xi_x)\sin(\xi_y)} \int \cancelto{0}{\cos(\omega_g t)\sin(\omega_g t)}\, dt + \overset{②③}{\sin(\xi_x)\cos(\xi_y)} \int \cancelto{0}{\sin(\omega_g t)\cos(\omega_g t)}\, dt \Big]$$

$$= 0.5\,\Gamma_x\Gamma_y \cancel{\cos(\xi_x - \xi_y)} \left[\cos(\xi_x)\cos(\xi_y) + \sin(\xi_x)\sin(\xi_y)\right]$$

$$\cos(\tan^{-1}(x)) = \frac{1}{\sqrt{1+x^2}} \qquad \sin(\tan^{-1}(x)) = \frac{x}{\sqrt{1+x^2}}$$

$$\cos \tan^{-1}(\zeta) = \frac{1}{\sqrt{1+\left(\frac{A\sin\alpha + B\sin\beta}{A\cos\alpha + B\cos\beta}\right)^2}} = \frac{1}{\sqrt{\frac{(A\cos\alpha + B\cos\beta)^2 + (A\sin\alpha + B\sin\beta)^2}{(A\cos\alpha + B\cos\beta)^2}}}$$

$$= \frac{1}{\sqrt{\frac{A^2\cos^2\alpha + B^2\cos^2\beta + A^2\sin^2\alpha + B^2\sin^2\beta + 2AB\cos\alpha\cos\beta + 2AB\sin\alpha\sin\beta}{(A\cos\alpha + B\cos\beta)^2}}}$$

$$\cos(\zeta) = \frac{(A\cos\alpha + B\cos\beta)}{\sqrt{A^2+B^2+2AB\cos(\alpha-\beta)}} = \frac{(A\cos\alpha + B\cos\beta)}{\Gamma}$$

$$\sin(\zeta) = \left(\frac{A\sin\alpha + B\sin\beta}{\cancel{A\cos\alpha + B\cos\beta}}\right)\frac{\cancel{(A\cos\alpha + B\cos\beta)}}{\Gamma} = \frac{(A\sin\alpha + B\sin\beta)}{\Gamma}$$

Back to PSD

$$PSD = 0.5\,\Gamma_x\Gamma_y\left[\cos(\zeta_x)\cos(\zeta_y) + \sin(\zeta_x)\sin(\zeta_y)\right]$$

$$= 0.5\,\cancel{\Gamma_x}\cancel{\Gamma_y}\left[\frac{(A_x\cos(\alpha_x) + B_x\cos(\beta_x))}{\cancel{\Gamma_x}}\frac{(A_y\cos\alpha_y + B_y\cos(\beta_y))}{\cancel{\Gamma_y}} + \frac{(A_x\sin\alpha_x + B_x\sin\beta_x)}{\cancel{\Gamma_x}}\frac{(A_y\sin\alpha_y + B_y\sin\beta_y)}{\cancel{\Gamma_y}}\right]$$

$$\cancel{= 0.5[A_xA_y\cos(\alpha_x)\cos(\alpha_y) + B_xB_y\cos(\beta_x)\cos(\beta_y) + A_xB_y\cos(\alpha_x)\cos(\beta_y) + A_yB_x\cos(\alpha_y)\cos(\beta_x)}$$

$$= 0.5[A_xA_y\cos(\alpha_x)\cos(\alpha_y) + A_xB_y\cos(\alpha_x)\cos(\beta_y) + B_xA_y\cos(\beta_x)\cos(\alpha_y) + B_xB_y\cos\beta_x\cos\beta_y$$
$$+ A_xA_y\sin\alpha_x\sin\alpha_y + A_xB_y\sin\alpha_x\sin\beta_y + B_xA_y\sin\beta_x\sin\alpha_y + B_xB_y\sin\beta_x\sin\beta_y]$$

$$PSD = 0.5[A_xA_y\cos(\alpha_x - \alpha_y) + A_xB_y\cos(\alpha_x - \beta_y) + B_xA_y\cos(\beta_x - \alpha_y) + B_xB_y\cos(\beta_x - \beta_y)]$$

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Define

① $(\alpha_x - \alpha_y) = \cancel{\phi_g} - \cancel{\phi_p} - \cancel{\phi_g} + \cancel{\phi_p} + (\phi_{py} - \phi_{gy}) \cancel{-(\phi_{px}-\phi_{gx})} = \Delta\phi_y - \Delta\phi_x$

② $(\alpha_x - \beta_y) = \cancel{\phi_g} - \phi_p - \cancel{\phi_g} + \phi_e + (\phi_{ey} - \phi_{gy}) -(\phi_{px}-\phi_{gx}) = \Delta\phi_{ey} - \Delta\phi_p + \Delta\phi_{ex}$

③ $\beta_x - \alpha_y = \cancel{\phi_g} - \phi_e - \cancel{\phi_g} + \phi_p + (\phi_{py} - \phi_{gy}) = \Delta\phi_y - \Delta\phi_p + \Delta\phi_x$

④ $\beta_x - \beta_y = \cancel{\phi_g} - \cancel{\phi_e} - \cancel{\phi_g} + \cancel{\phi_e} - \phi_{gy} + \phi_{ey} = \Delta\phi_{ey} + \Delta\phi_{ex}$

$\Delta\phi_y = (\phi_{py} - \phi_{gy})$

$\Delta\phi_{ey} = (\phi_{ey} - \phi_{gy})$ also define $E_0^e = \delta E_0^p$

$\Delta\phi_p = (\phi_p - \phi_e)$

$\Delta\phi_x = (\phi_{px} - \phi_{gx})$

$\cancel{PSD = 0.5(a_i a_p E_0^2 \cos^2\theta_g)}$ $\Delta\phi_{ex} = (\phi_{ex} - \phi_{gx})$

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$$PSD = 0.5\, a_i a_p E_0^{g^2} \cos^2\theta_g E_0^{p^2} \Big[\cos\theta_p \sin\theta_p \cos(\Delta\phi_y)$$
$$+ \delta \cos\theta_p \sin\theta_e \cos(\Delta\phi_p - \Delta\phi_{ey})$$
$$+ \delta \sin\theta_p \cos\theta_e \cos(\Delta\phi_p - \Delta\phi_y)$$
$$+ \delta^2 \cos\theta_e \sin\theta_e \cos(\Delta\phi_{ey}) \Big]$$

$$RMS = 0.5\, a_i^2 E_0^{g^2} \cos^2\theta_g E_0^{p^2} \Big[\cos^2\theta_p$$
$$+ \delta \cos\theta_p \cos\theta_e \cos(\Delta\phi_p)$$
$$+ \delta^2 \cos^2\theta_e \Big]$$

$$RMP = K^2 RMS$$

following same procedure as for PSD.. $PSP = \langle i'.p \rangle$

$$PSP = 0.5 K\, a_i a_p E_0^{g^2} \cos^2\theta_g E_0^{p^2} \Big[\cos\theta_p \sin\theta_p \sin(\Delta\phi_y)$$
$$+ \delta \cos\theta_p \sin\theta_e \sin(\Delta\phi_p - \Delta\phi_{ey})$$
$$+ \delta \sin\theta_p \cos\theta_e \sin(\Delta\phi_p - \Delta\phi_y)$$
$$+ \delta^2 \cos\theta_e \sin\theta_e \sin(\Delta\phi_{ey}) \Big]$$