Bayesian Analysis of Electron Kinetic Profiles.

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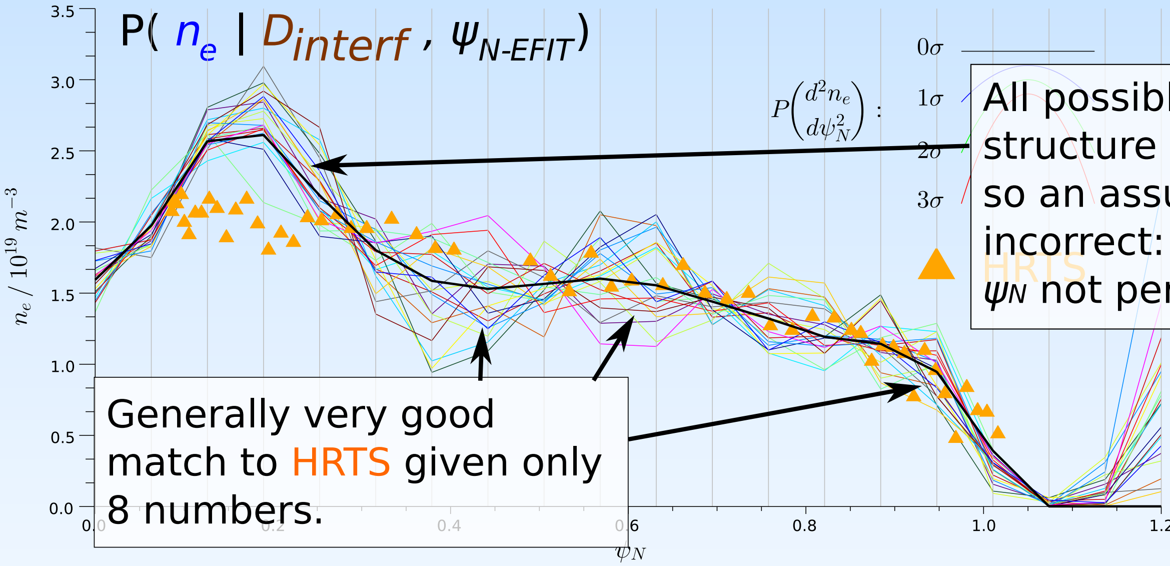
Overall Idea: Add n_e, T_e diagnostics to Bayesian Analysis

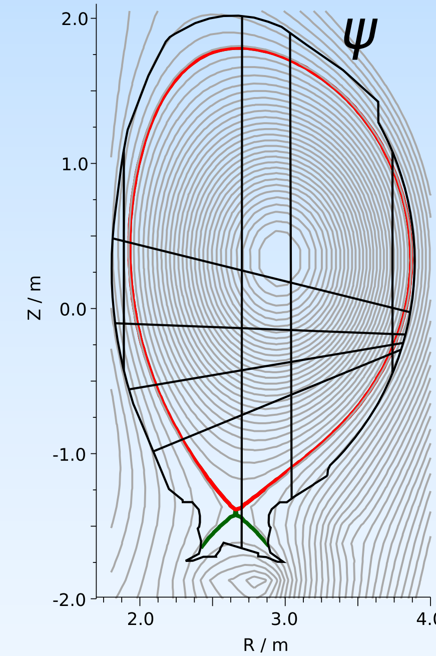
- Polarimetry
  - [Relativistic Model Testing]
- Core LIDAR
- Edge LIDAR
- Equilibrium

Interferometry + Current Tomography I

Invert interferometry data to $n_e(\psi_N)$ using weak smoothing prior based on magnetics only EFIT flux surfaces

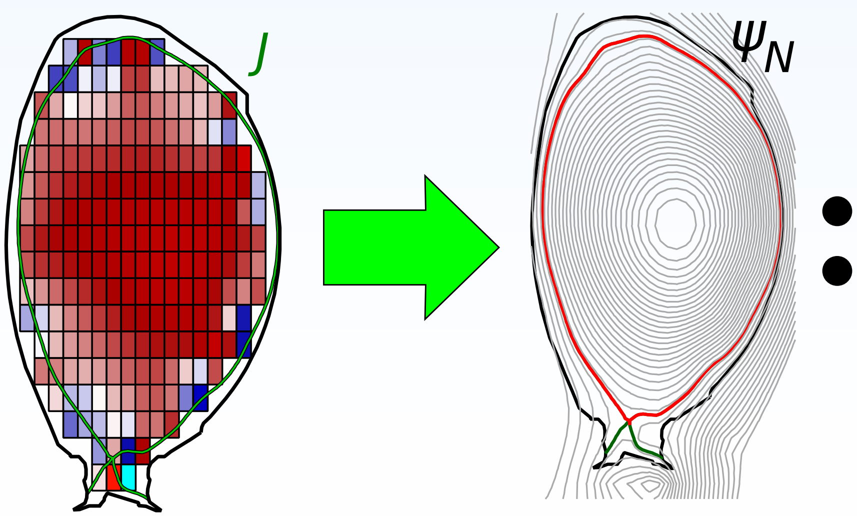


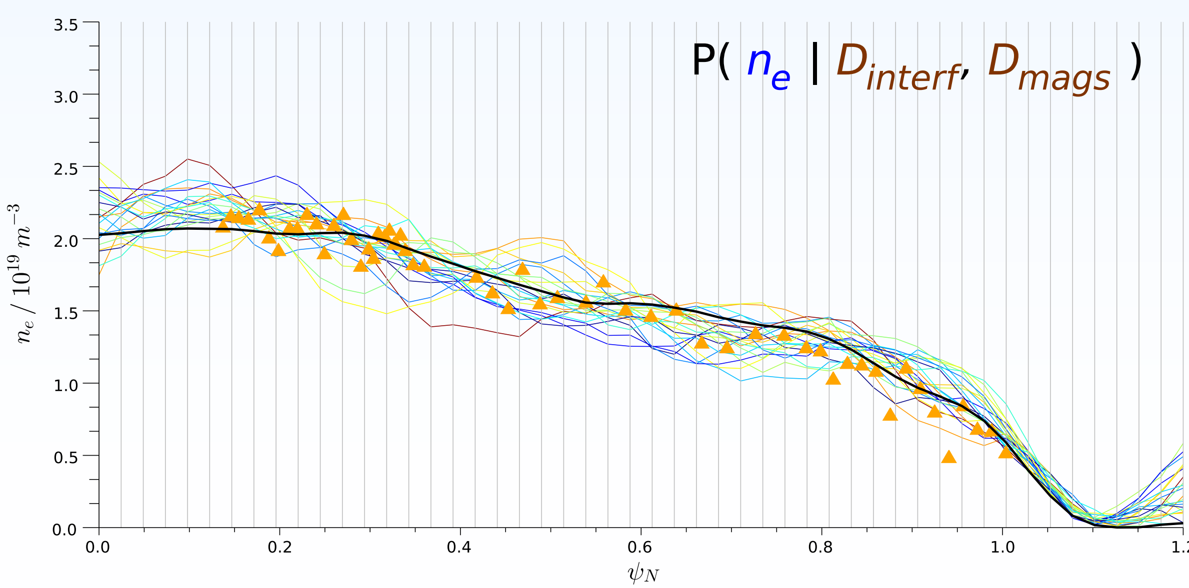
78625
Ohmic
(recovery pulse)



Instead, calculate ψ_N from toroidal currents J, include magnetics diagnostics and invert to full posterior:

Finds combinations of J and n_e that are consistent with both interferometry and magnetics (and with n_e and J priors).

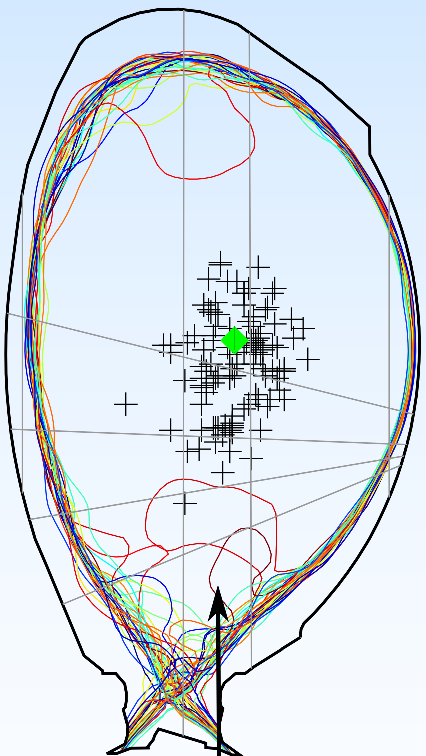




Interferometry + Current Tomography II

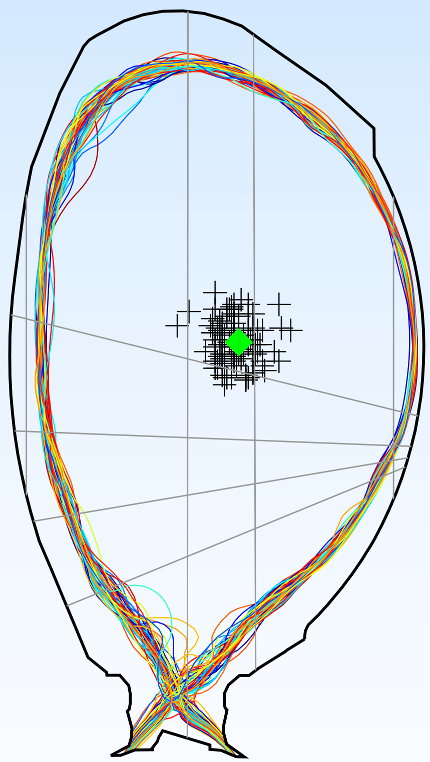
Each sample is also a possible set of J given magnetics **and interferometry.**
Deliberatly using over-weak currents priors, that with only magnetics gives:

$P(J \mid D_{mags})$



Magnetics
+Weak CAR prior

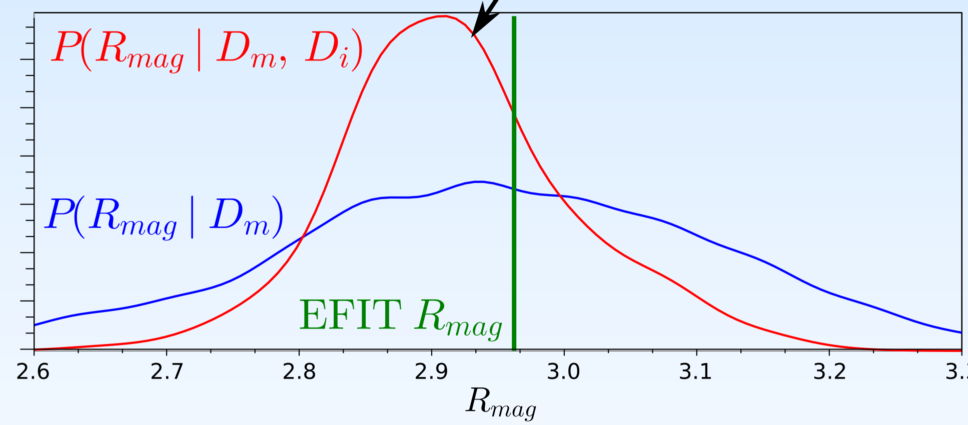
$P(J \mid D_{mags}, D_{interf})$

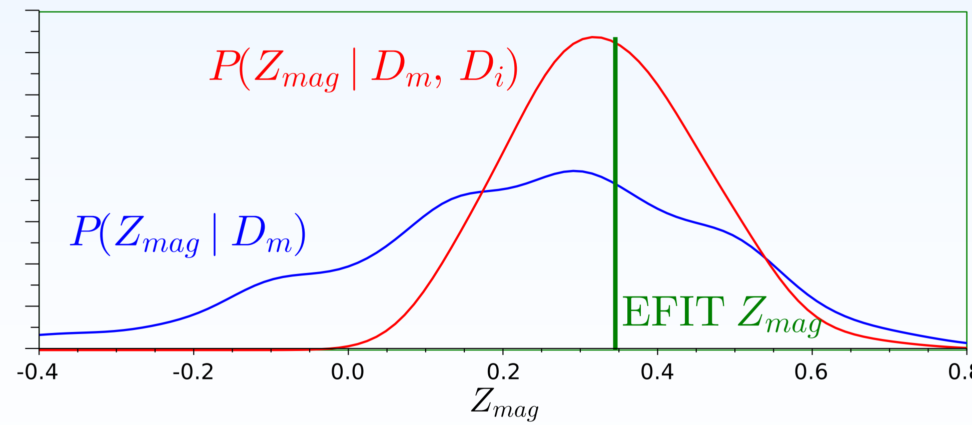


Magnetics + Weak CAR prior
+ Interferometry + Smooth *ne*

Interferometry combined with *ne* assumptions provides some information about plasma current: i.e: Some currents give flux surfaces for which no *ne* profile can make interferometry data make sense.

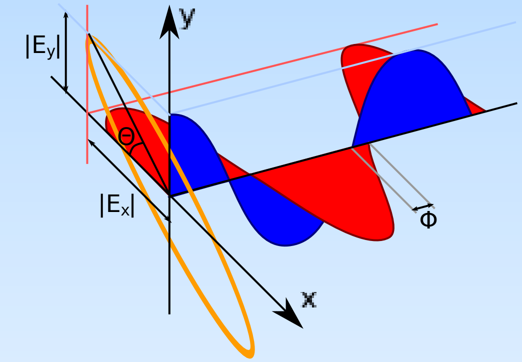
Less obviously, gives higher certainty magnetic axis





Polarimetry I

Use well known full plasma polarisation evolution equation.
Depends primarily on n_e and $\boldsymbol{B}$.

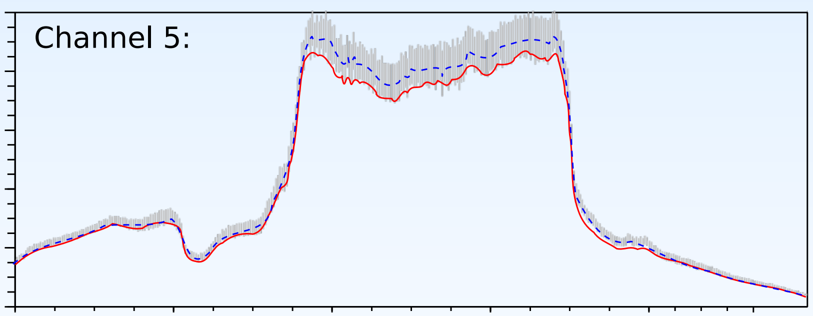


To test model:

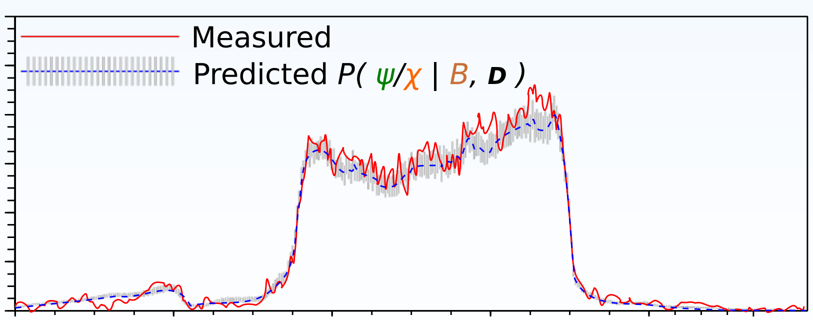
- Predict final polarisation from samples of P(n_e | D_{interf}, $\boldsymbol{B}_{EFIT}$).
- Take mean and standard deviation of rotation ψ and ellipticity χ.
- Compare to measured data.

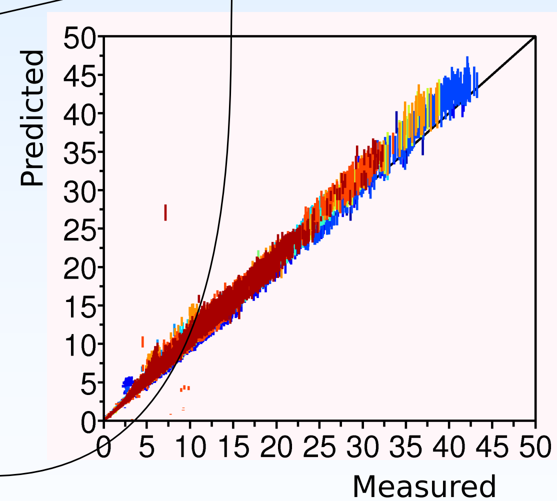
Grey bands represent 2σ of $P(\psi/\chi \mid \boldsymbol{B}, D)$. Despite la
variation in n_e profiles used, predictions are well det

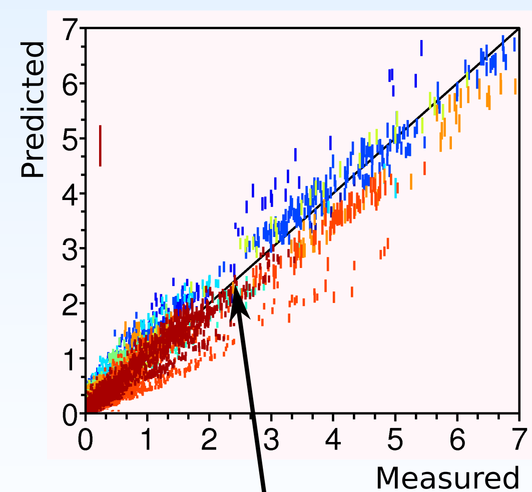
ψ



χ







Good agreement for channel 5. Only the full forward model can calculate χ for lateral channels.

Often, ψ and χ approximated by 'Faraday' and 'Cotton-Mouton' effects, each valid in specific cases not generally true on JET. **Lots** of effort spent trying to 'correct' the calculations back to the full model. Leads to confusing mix of terminology and unnecessary inaccuracy that gets confused with real diagnostic uncertainty.

Polarimetry II - High Temperature Models A

As well as 'cold plasma' model (fluid approx), two papers gave 'corrections' for high-T_e effects (quoted as large for T_e > ~5keV) derived from kinetic theory.

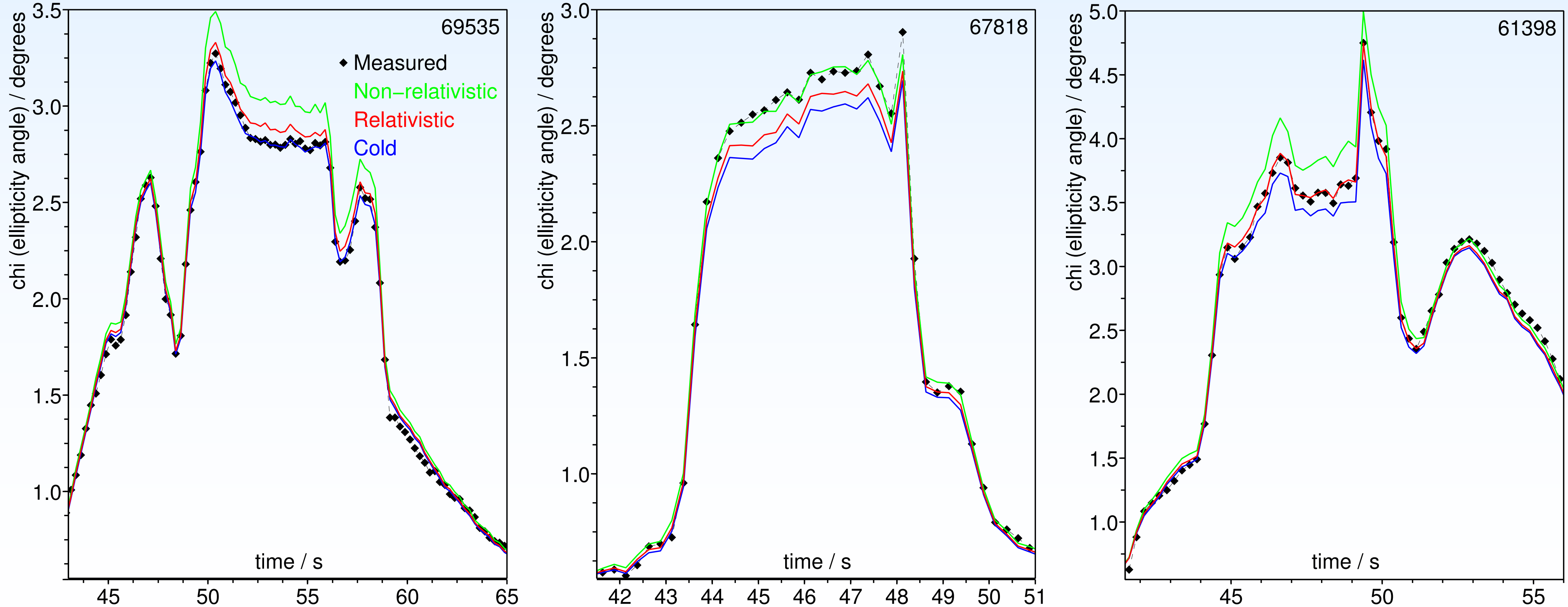
a) S.E. Segre (2002): Argues non-relativistic kinetic approximation is sufficient.

b) V.V. Mirnov (2007): Argues mass increase of electron is important and derives a weakly relativistic approx.

We should be able to test, but...

For core (high-T_e) channels, measurement and prediction for cold plasmas differ systematically over entire pulses and campaigns. Partly due to inaccurate knowledge of ***B*** (from magnetics EFIT here), but diagnostic behaviour is not fully understood (optics etc) and the calibration varies significantly.

Uncertainty due to calibration is much larger than model differences and is systematic for entire pulses:



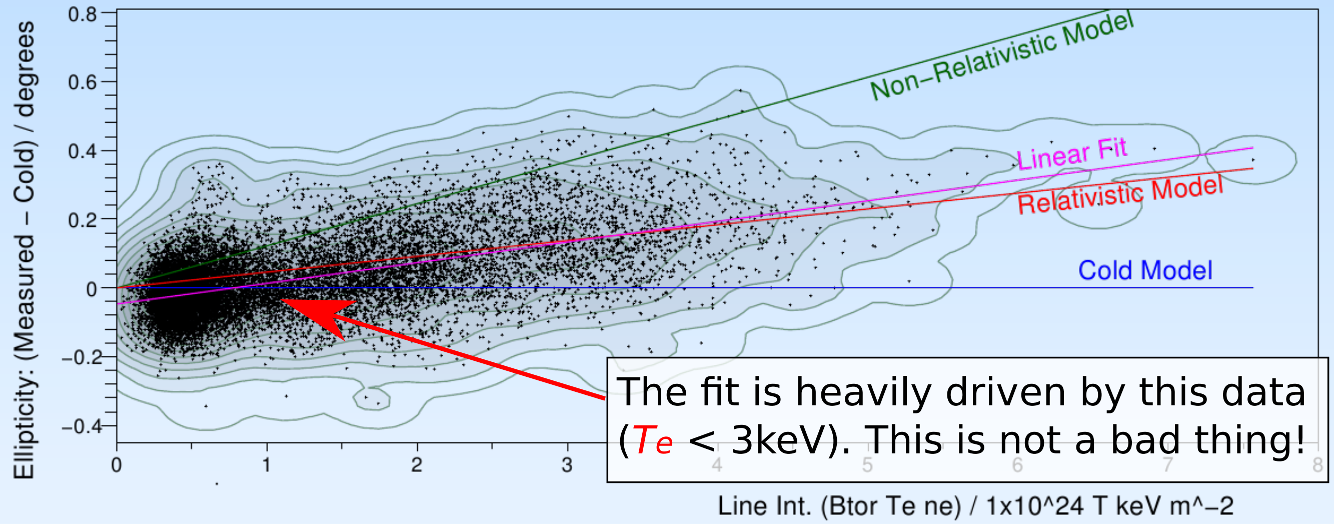
Can easily find pulses that agree with any model.

Polarimetry III - High Temperature Models B

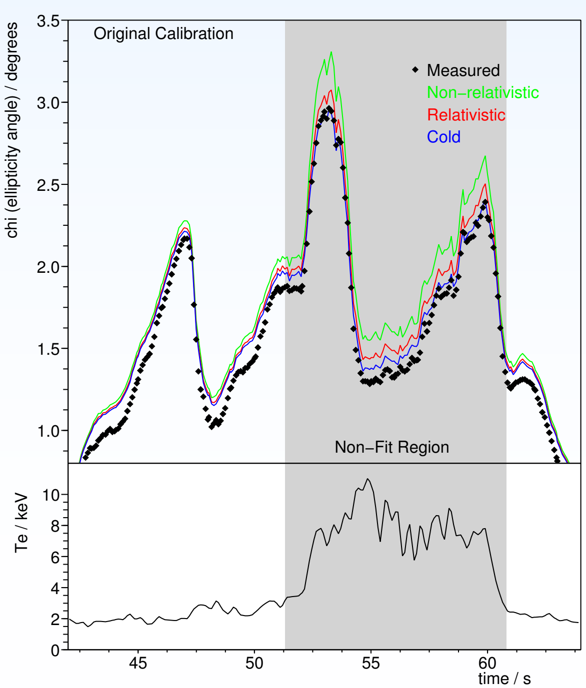
Solutions: Run session of pulses at very high T_e to get ~10 pulses with effect bigger than uncertainty?

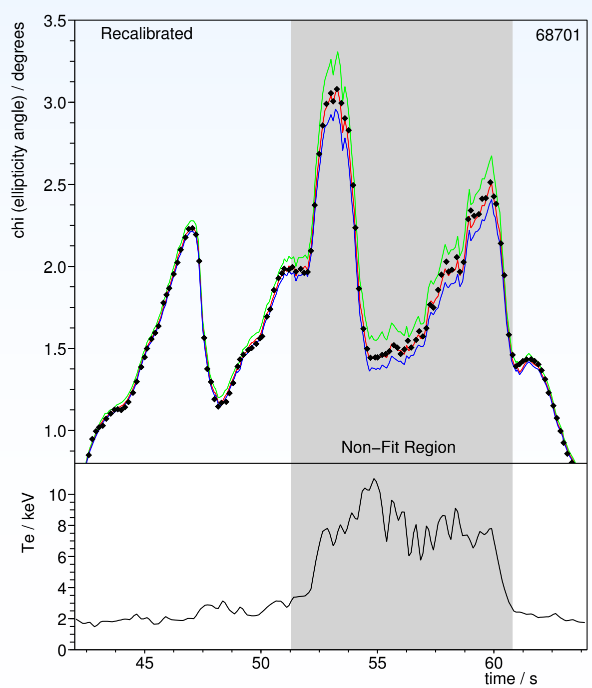
NO! Relativity **does not** '*switch on*' at 8keV.

Lot of stats --> accurate diagnostic: 10,000 points with +/-50% is better than 10 points with +/-10%.
At JET, we have **LOTS** of stats!

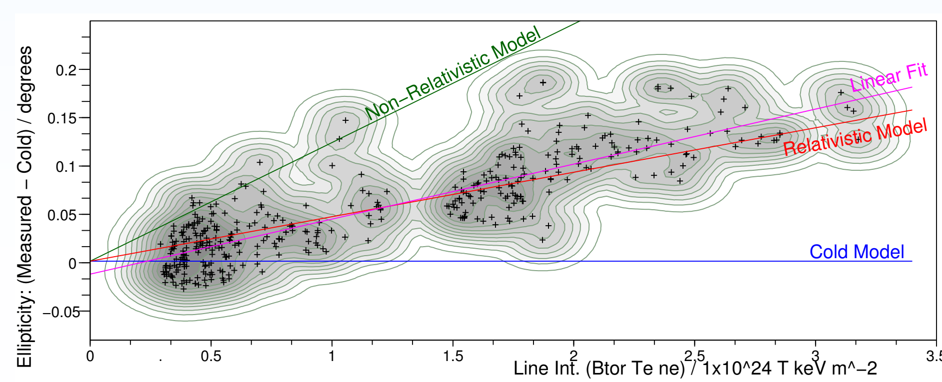


Assume only that calibration cannot depend on plasma core T_e.
(whatever it is, it is in the optics and electronics)



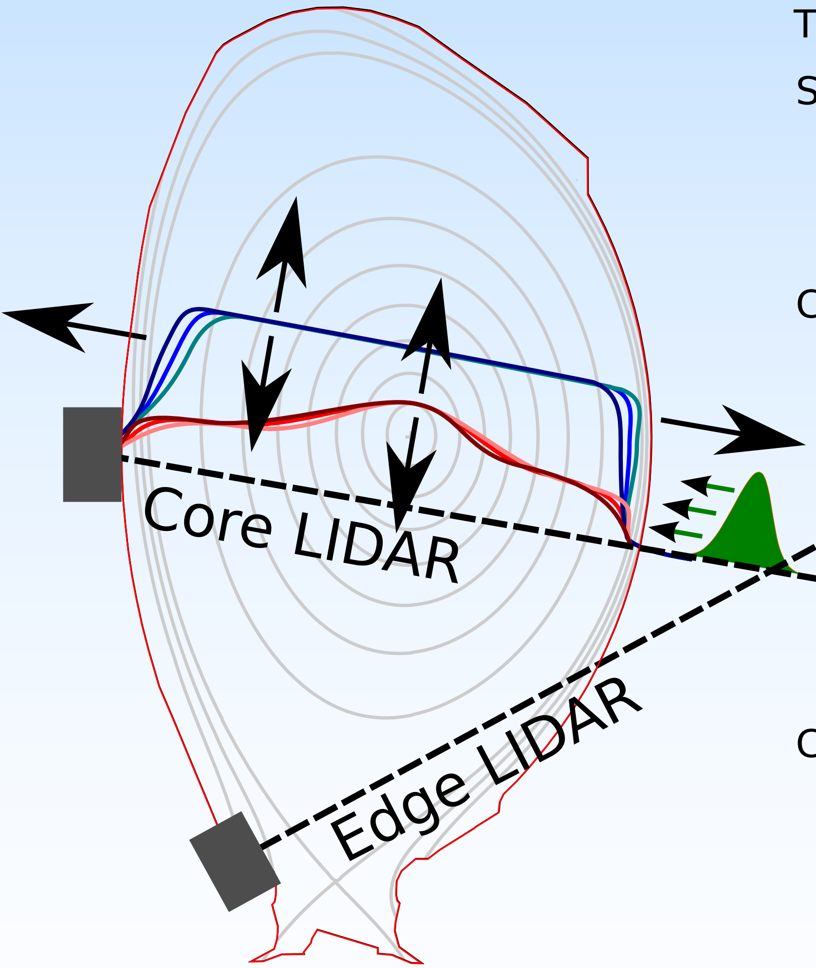


For some pulses:
1) Use cold plasma model on cold plasma, to fix calibration params of forward model for diagnostic optics.
2) Compare data in high T_e period.



Core + Edge LIDAR I: The systems

Thomson Scattering diagnostics each using a single spectrometer set and time of flight for positioning.



TS physics well understood but hardware system very complex.

Spatial Resolution:

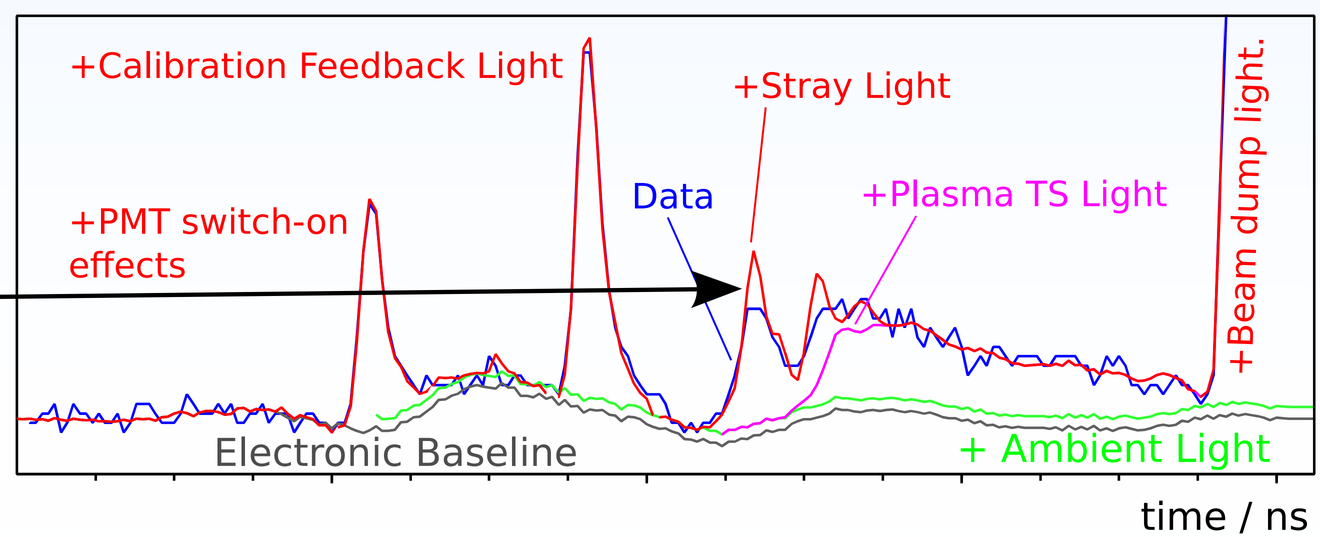
- Effective convolution of light signal.
- If ignored (chain1): Convolves n_e but complex effect on T_e.
- No problem for forward modelling: we just convolve the signal.

Calibrations:

- Beam dump position + timing --> Uncertain position.
- Optical transmission + laser energy --> n_e magnitude.
- Spectrometer Relative Sensitivities --> T_e magnitude.
- Relative Channel timing --> T_e + n_e shape!

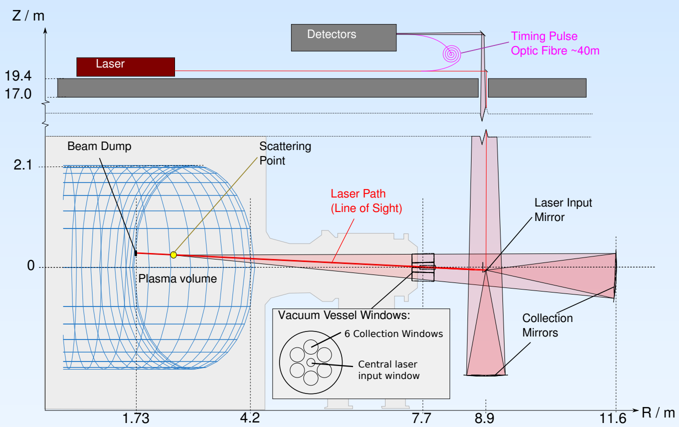
Created full detailed forward model including every part of the system:

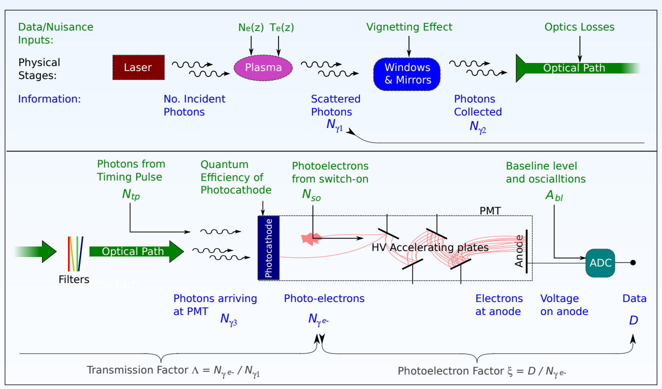
Stray light effects low signal (low n_e) data on both systems but is **vital** for proper edge LIDAR analysis.

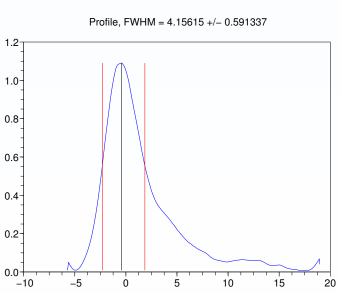


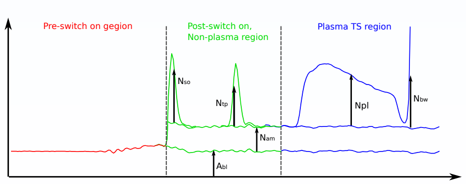
Core + Edge LIDAR I: The model

1) Really understand how each part of the system works:
Laser Pulse, TS physics, Optics, Filters, Photomultipliers, Counting Noise (PDFs), ADCs.

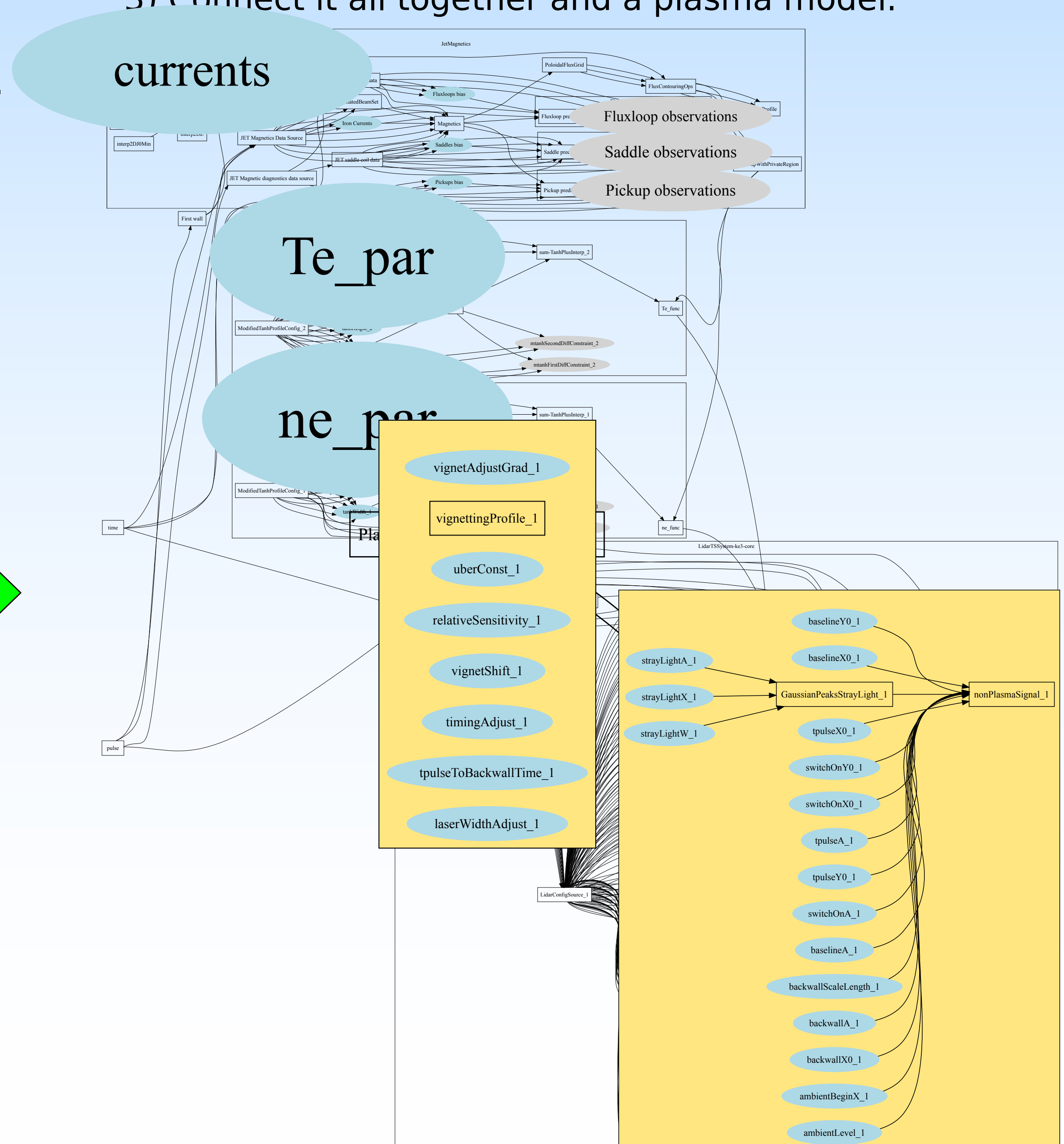








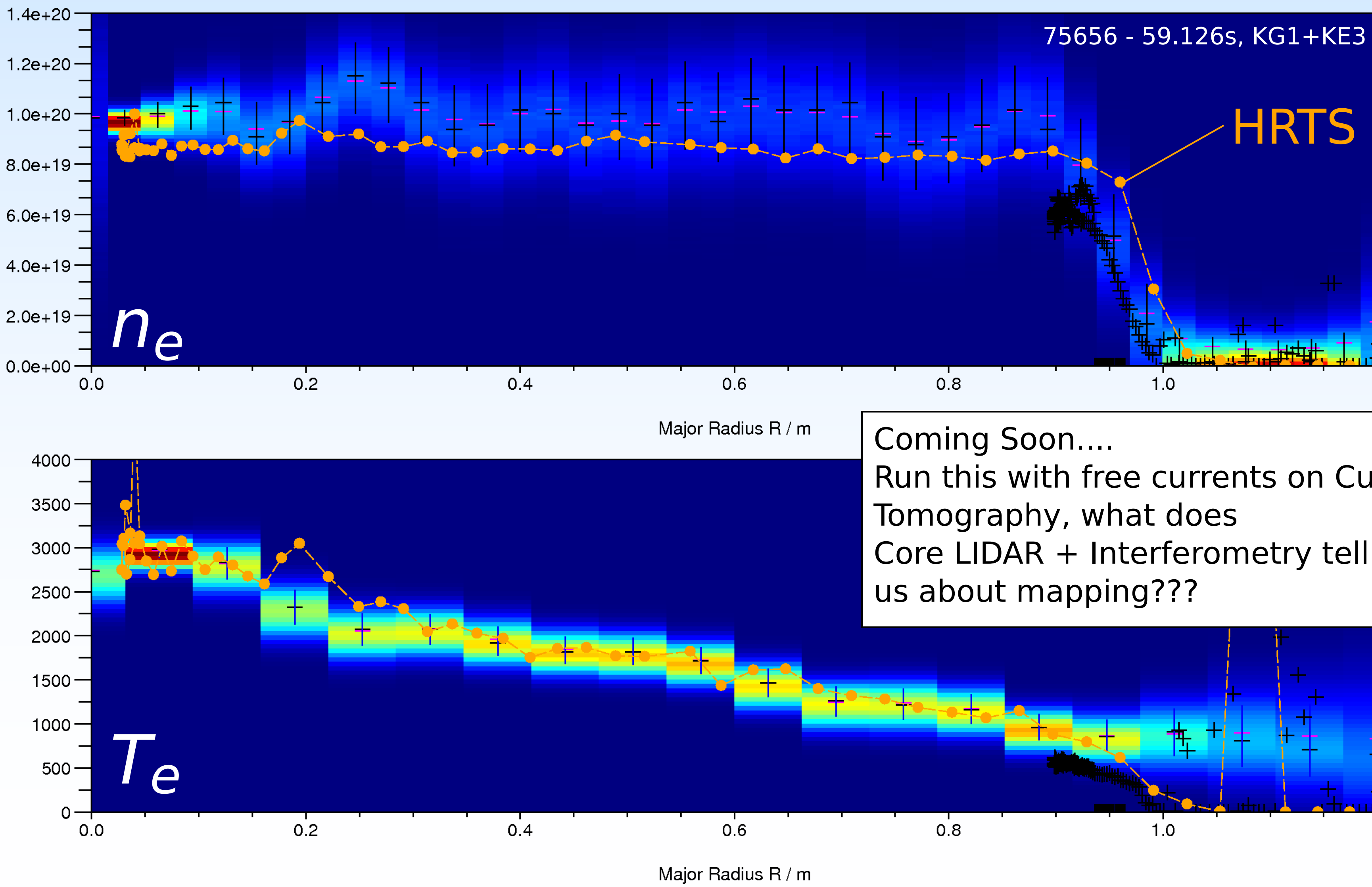
2) Develop MINERVA node for each part of the system.
3) Connect it all together and a plasma model.



Core + Edge LIDAR III: Early results (2008)

Early results:

- Core LIDAR + Interferometry on EFIT ψ_N.
- Weak priors on all calibration parameters except relative sensitivities (T_e magnitude calibration).
- Most calibrations are determined by consistency and data (either LIDAR or Interferometry).

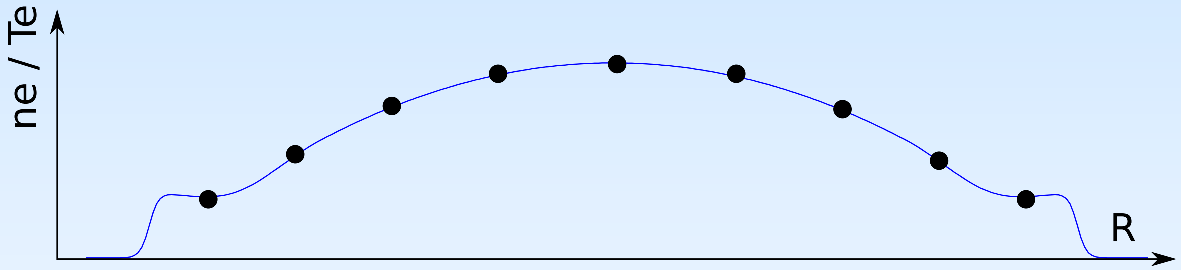


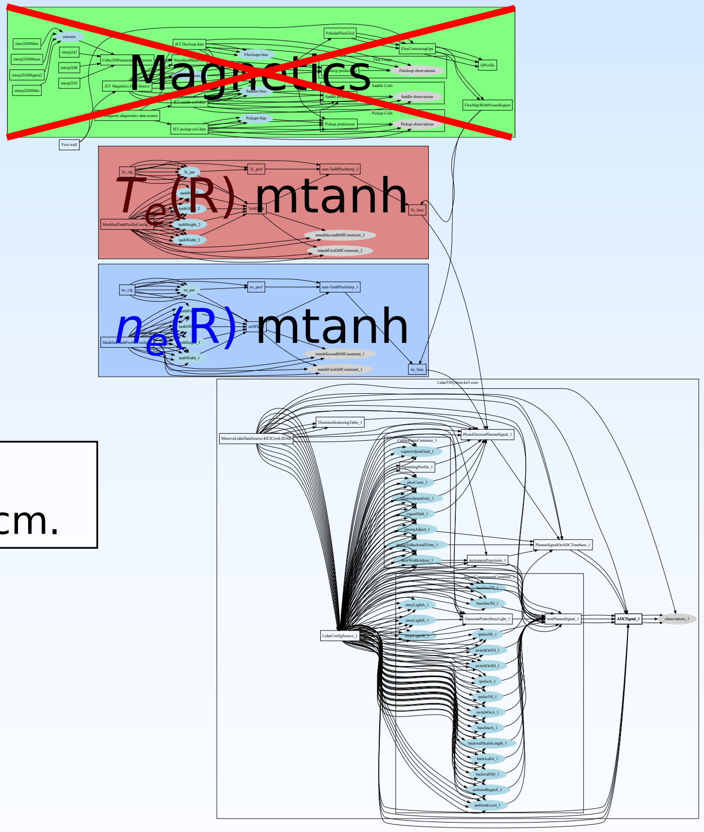
Coming Soon....
Run this with free currents on Current Tomography, what does Core LIDAR + Interferometry tell us about mapping???

Core + Edge LIDAR IV: Pedestal Information from core LIDAR.

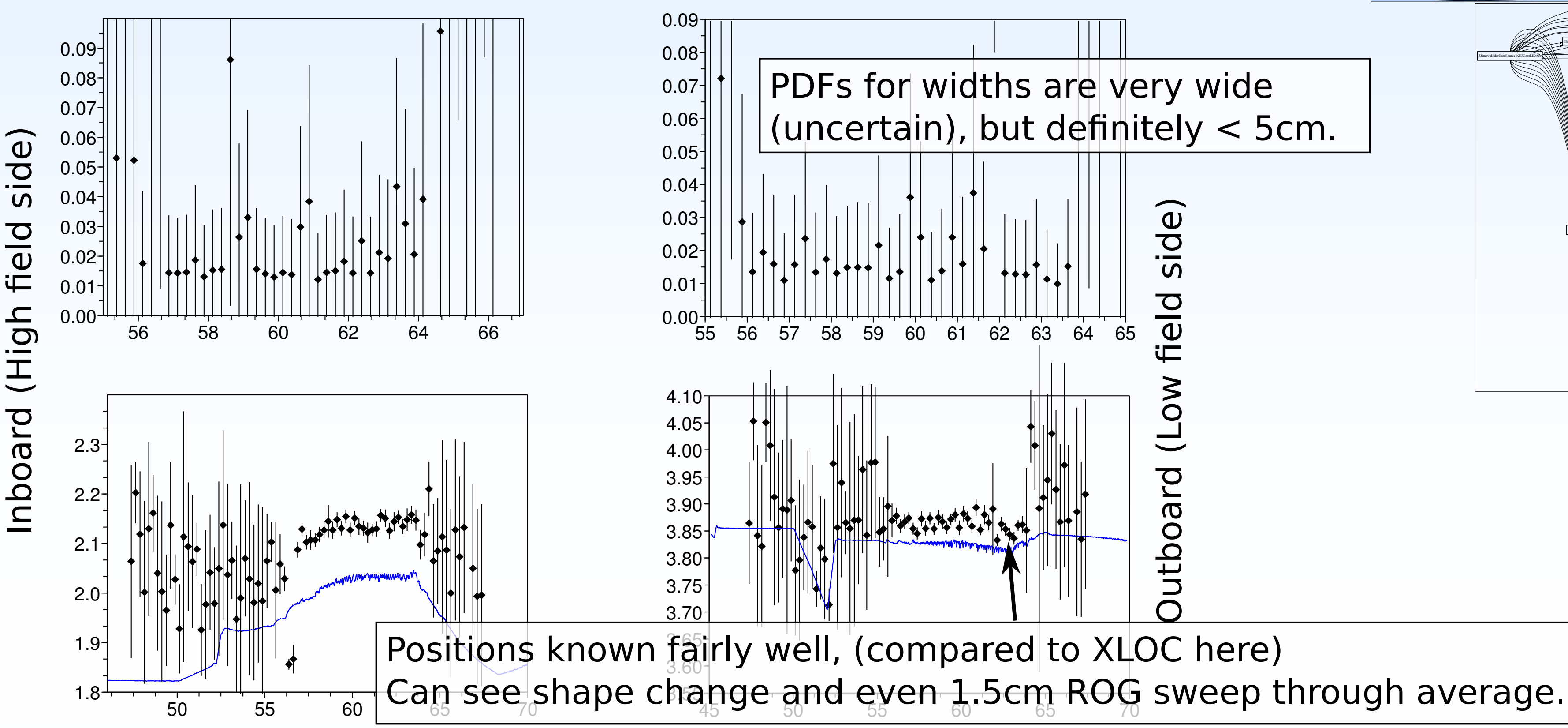
Core LIDAR has ~12cm convolution and data points for every 3cm - it will never completely 'resolve' the pedestal. But, can it tell us anything, if we help it out a bit?...

- Inversion directly to 1D *$n_e(R)$* (no flux mapping etc) and use **modified tanh** profile + knots for core.
- Hold all calibration and use Core LIDAR only.





Run for a whole shot:

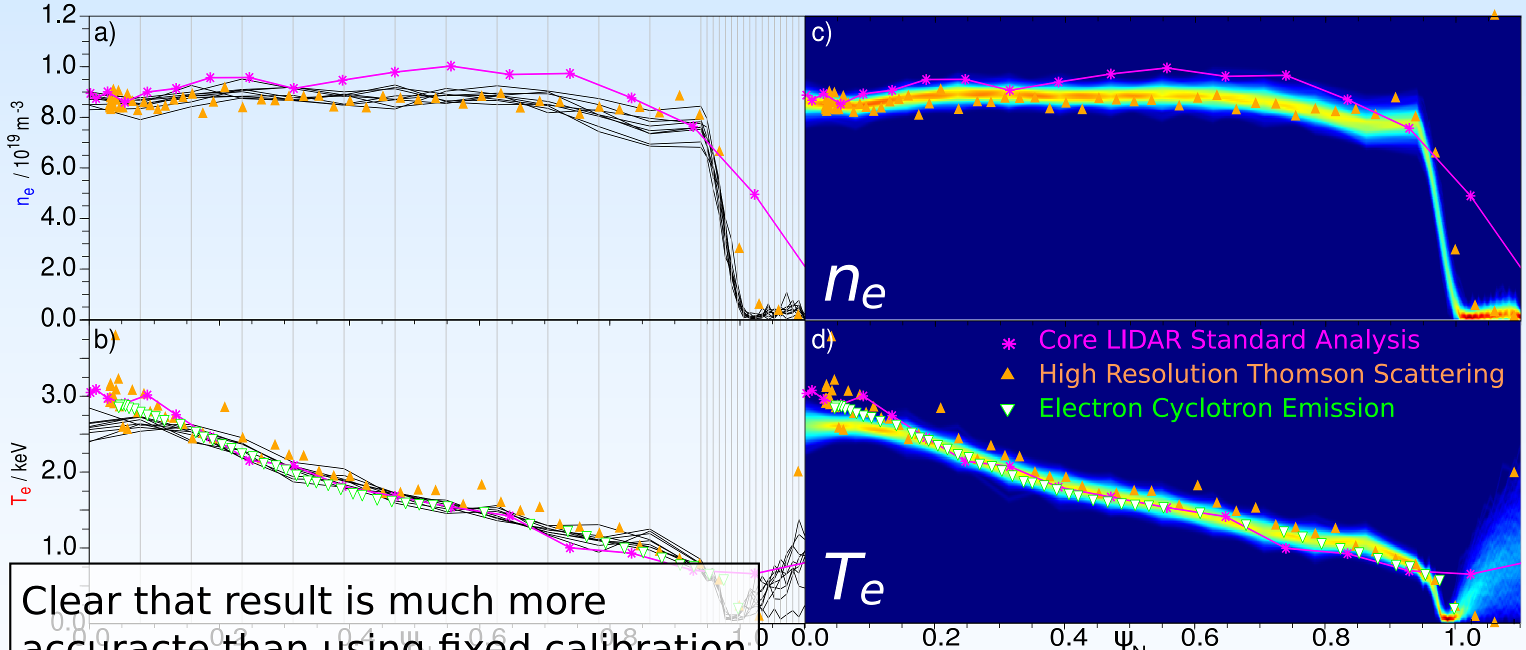


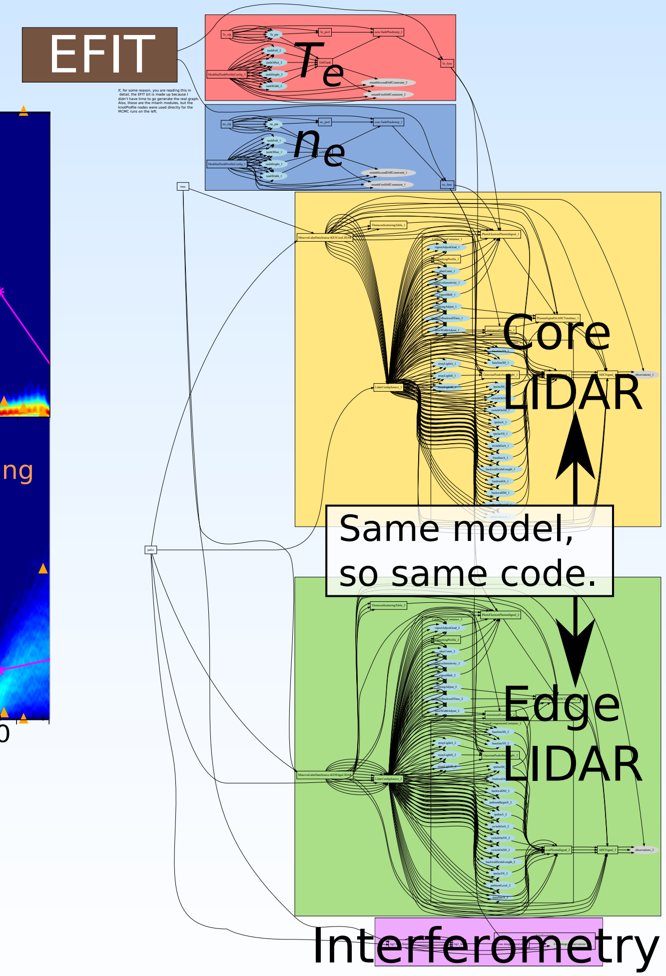
Core + Edge LIDAR V: Add edge LIDAR.

As with core LIDAR, calibrations (**position**, ***n_e* magnitude** etc) all have uncertainty (some large).

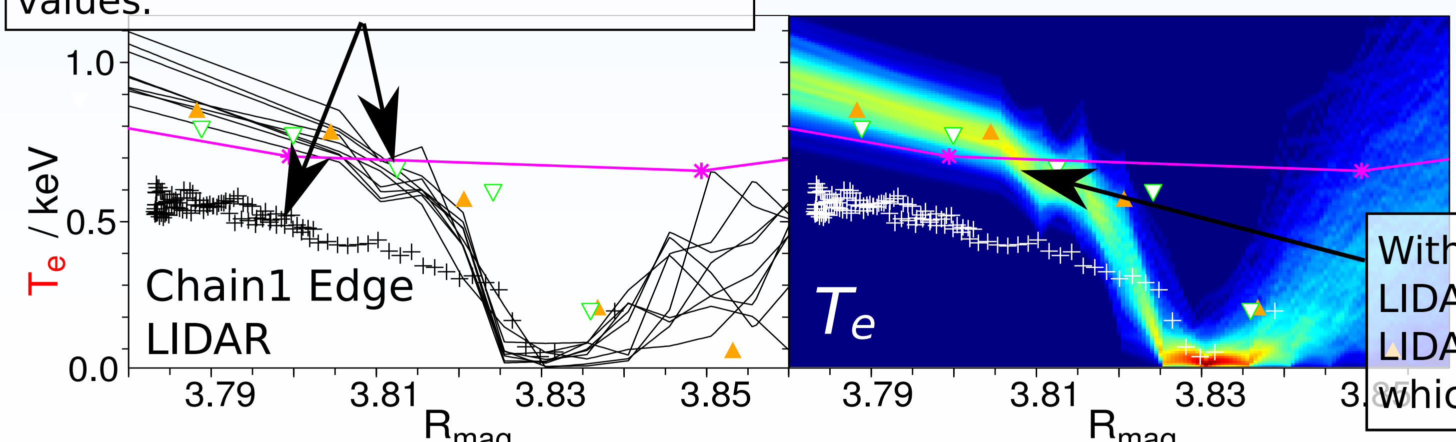
- Photoelectron calibration implies **noise level** much less than observed - recalculated from noise on data.
- Relative sensitivities (***T_e* calibration**) seem to disagree with other diagnostics, so give wide prior.
- **Stray light** a **BIG** problem for low *ne* shots - aquire from dry-runs.

Add Edge LIDAR to the mix...





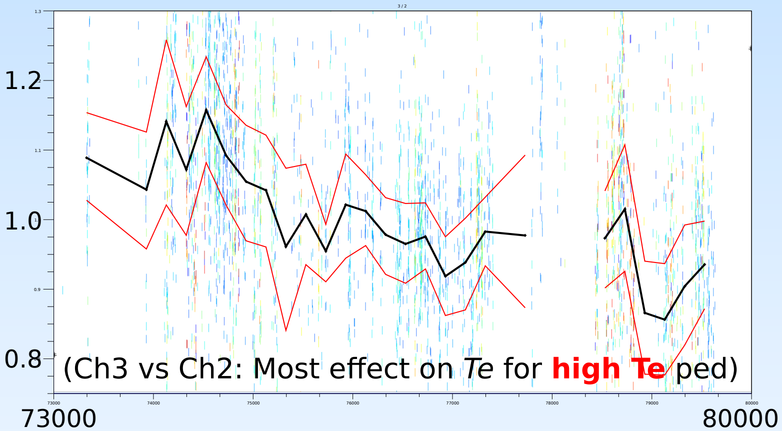
Clear that result is much more accuracte than using fixed calibration values.

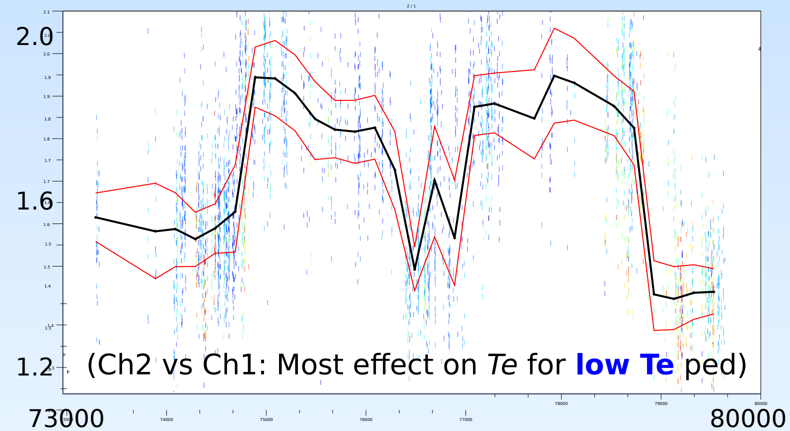


With completely free calibration, edge LIDAR provides shape with which Core LIDAR can give accurate T_e ped height which feeds back to Edge LIDAR

Core + Edge LIDAR VI: Calibrations Inference.

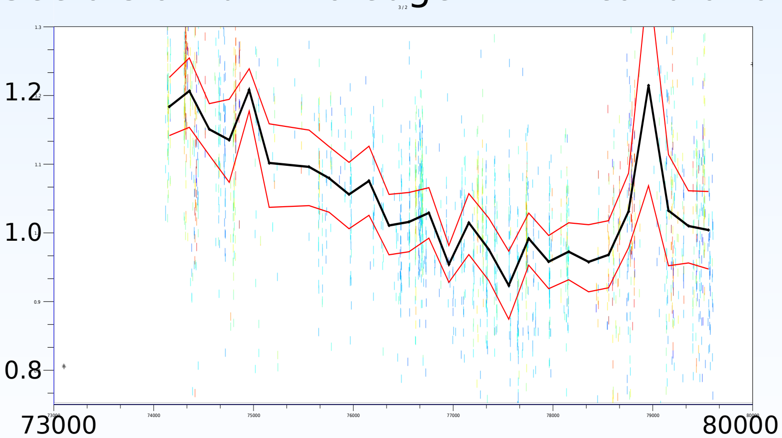
Find posterior maximum (best fit) for mtanh parametrised $n_e(\psi_N)$ and $T_e(\psi_N)$ with Core+Edge LIDAR with completely free edge LIDAR T_e calibrations. Look at inferred calibration for C25-present:

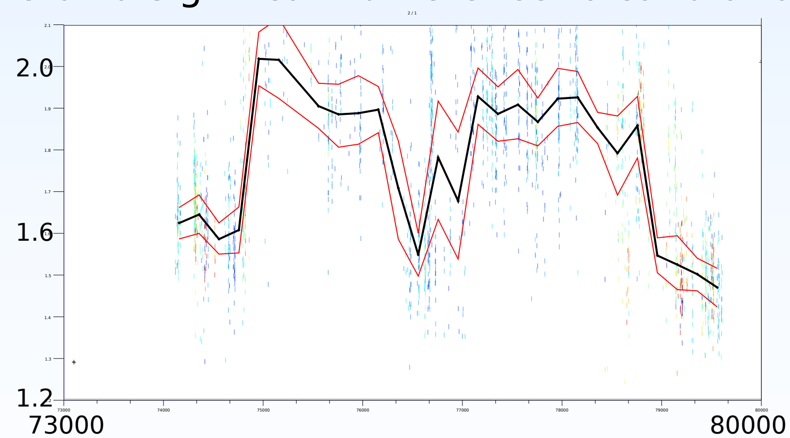




T_e ~ From Core Lidar

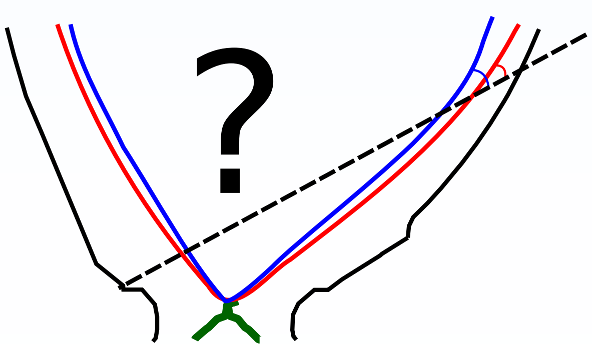
This is effectively a cross calibration with Core LIDAR (not easy normally).
See a slow drift to edge LIDAR calibration, but overall a significant difference to calibration usually used.





T_e ~ From HRTS

So now, have we constrained the picture enough to do some physics ?
Looked at pedestal widths using Core + Edge LIDAR + Interferometry:
- Works, but usually (pre-ELM) insists T_e pedestal is < 7mm.
- No JET diagnostic can outright disagree, does anyone know for certain?
- Only remaining reason might be mapping (since we're fixed to so far).



Equilbrium I

So mapping P(ψ_N | ...) is still the big problem.
Will try to explore using Current Tomography with CAR prior and all the diagnositcs (soon)

However, equilibrium condition may give enough constraint.

$$J_\phi = Rp' + \frac{\mu_0}{R} f f'$$

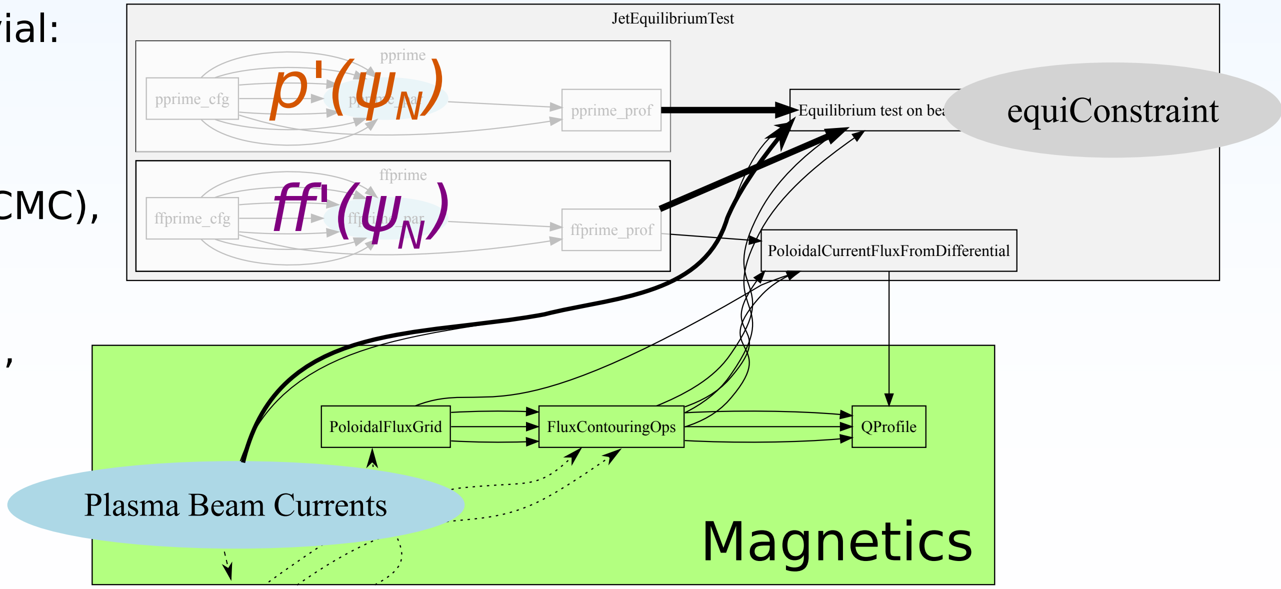
NB: It's not immediately clear how restrictive force balance (GS equation) actually is, since it is almost always used with strong prior constraints on *p*' (or *p* - the equilibirum pressure) and *ff*' (or *f* - the poloidal current flux). With weak (almost no) contraints on *p*' and *ff*', degeneracy of solutions is still huge.

Assume GS equality is, at least close to correct: assign a PDF on difference:

$P(J, p', ff') = G(J - Rp' - ff'/R;\ \ 0,\ \ \sigma_{GS})$ with relativly small σ_{GS}.

The posterior P(*J*, *p*', *ff*' | D_{diags} + ~Equilibrium) should include all possible combinations of *J*, *p*' and *ff*' that are consistent with the diagnostics, the priors and describe a plasma very close to equilbrium.

Adding to model (and the code) is fairly trivial:



Exploring the PDF is currently beyond the capability of our the present algorithms (MCMC), even for low resolution parametrisations.

Even finding the maximum (best fit) is hard, but can now be done...

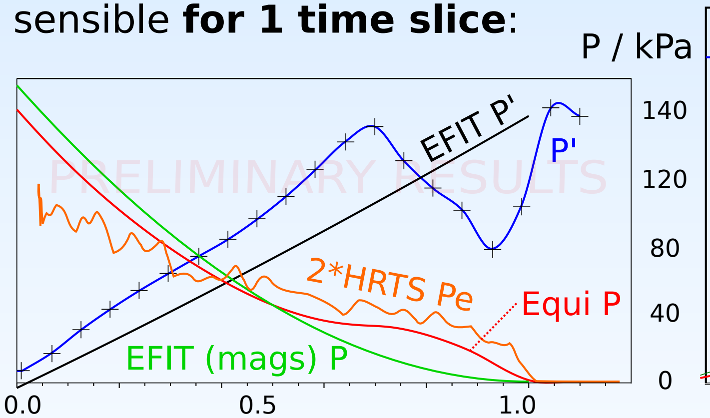
Equilbrium II: Maximum Posterior (Magnetics Only)

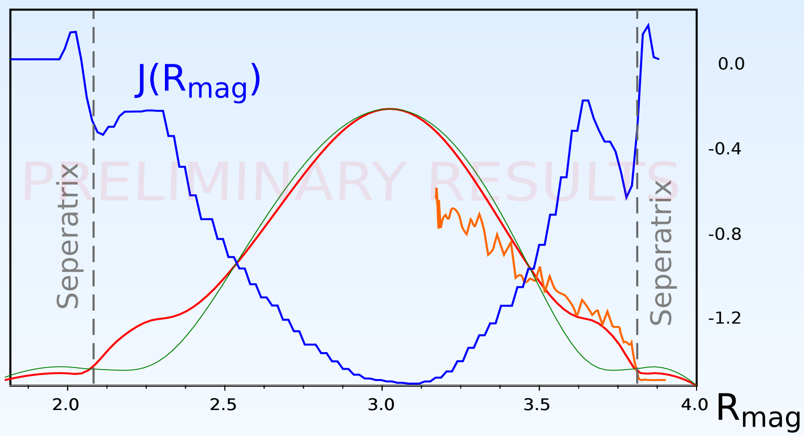
78601 High ne H-Mode (pellets)

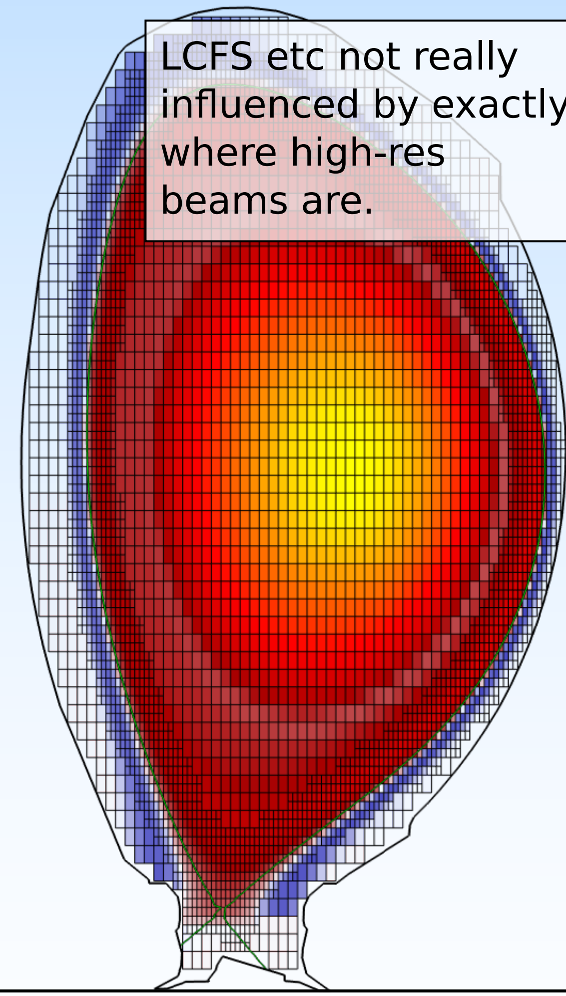
Because of modularity, we can switch parametrisation and priors of *J*, *p'* and *ff'* at will and on-the-fly.
For H-Mode, fast changes at edge so:

- J_ϕ: Current beams with higher resolution near edge (~**1cm**, ~5cm in core). No smoothing priors, just J_ϕ < 100MA m^{-2}.
- $p'(\psi_N)$, $ff'(\psi_N)$: 20 knots, weak smoothing priors. Fairly strong prior for small SOL p' and ff' (but not fixed). Has anyone measured J_{SOL}?

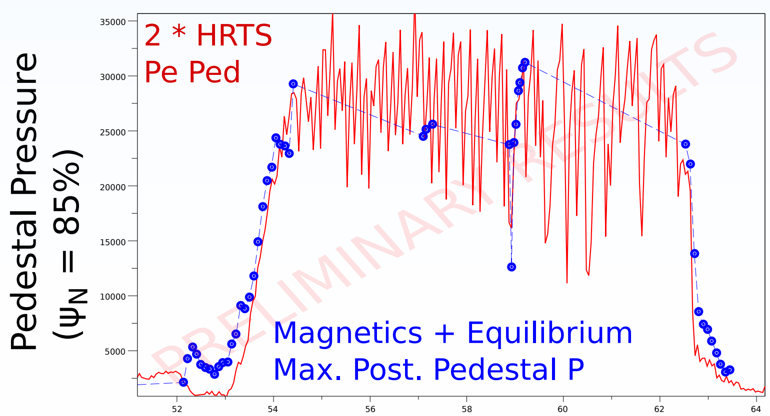
Clearly massively degenerate, so **adjust *p'* and *ff'* priors** to get something sensible **for 1 time slice**:

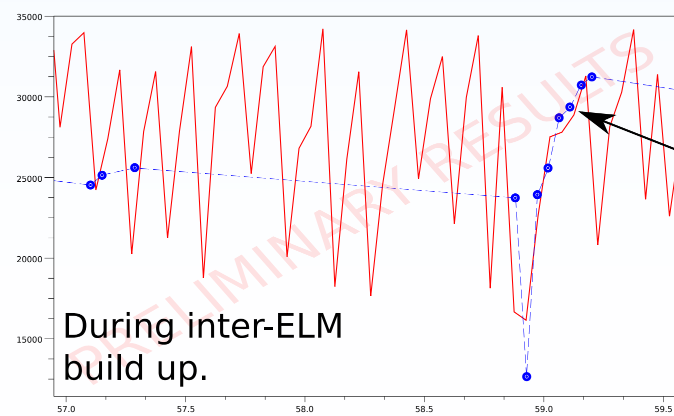






Magnetics data seems to see edge current (and hence some *p'*).
Exact magnitude you get does depend on priors.
But... **Hold priors** and run accross H-mode pulse. Is there any vague trend?





Follows trends AND maintains surprisingly good magnitude.
Suggests there is a quite lot of info in magnetics!
What is P(Jedge | Dmags)?
What if we constraint *P* against *Pe?*

Conclusions so far and work to do...

✓ Developed full models for core and edge LIDAR and polarimetry, combined with existing magnetics and interferometry models.

✓ Used polarimetry model and lots of data to test theoretical models for relativistic polarimetry.
('O P Ford *et al* 2009 *Plasma Phys. Control. Fusion* **51** 065004' - In *IOP select* and PPCF highlights 2009).

✓ Have a framework for analysing diagnostics which not only can cope with mapping uncertainty, but also automatically feeds back information from diagnostic to make inference on the mapping (currents).

✓ Similarly, can deal with uncertain calibrations, no matter how complex the model, and then infer the calibration from the data or from consistency with other.

✓ Having nailed down the calibrations, Core+Edge LIDARs + Inteferometry give accurate n_e, T_e profiles entirely independent of HRTS.

••• More work to do on effect of full combination on mapping/currents.

✓? *Appear* to be able to infer a surprising amount about the pedestal current/pressure from magnetics.
••• Need to **explore** the PDF - what can GS/force balance really tell us?

••• In the end (hopefully)....
P(J, n_e , T_e | Magnetics + Core LIDAR + Edge LIDAR + Interferometry + Force Balance)

••• Can we test pedestal scaling from edge LIDAR just with uncertain mapping (CT).
✓ [Have 7000 time points, type-I ELMy H-Mode, marked and clear of ELMS since Edge LIDAR upgrade C20-C27]

••• Do we get enough info to test current models at edge? - more use of the 'lots of stats'.

••• Can we see ∇P / $J_{//}$ evolution inter-ELM without assuming **anything** of where $J_{//}$ comes from?