# Bayesian Analysis Results on JET - Flux surface and equilibrium uncertainty

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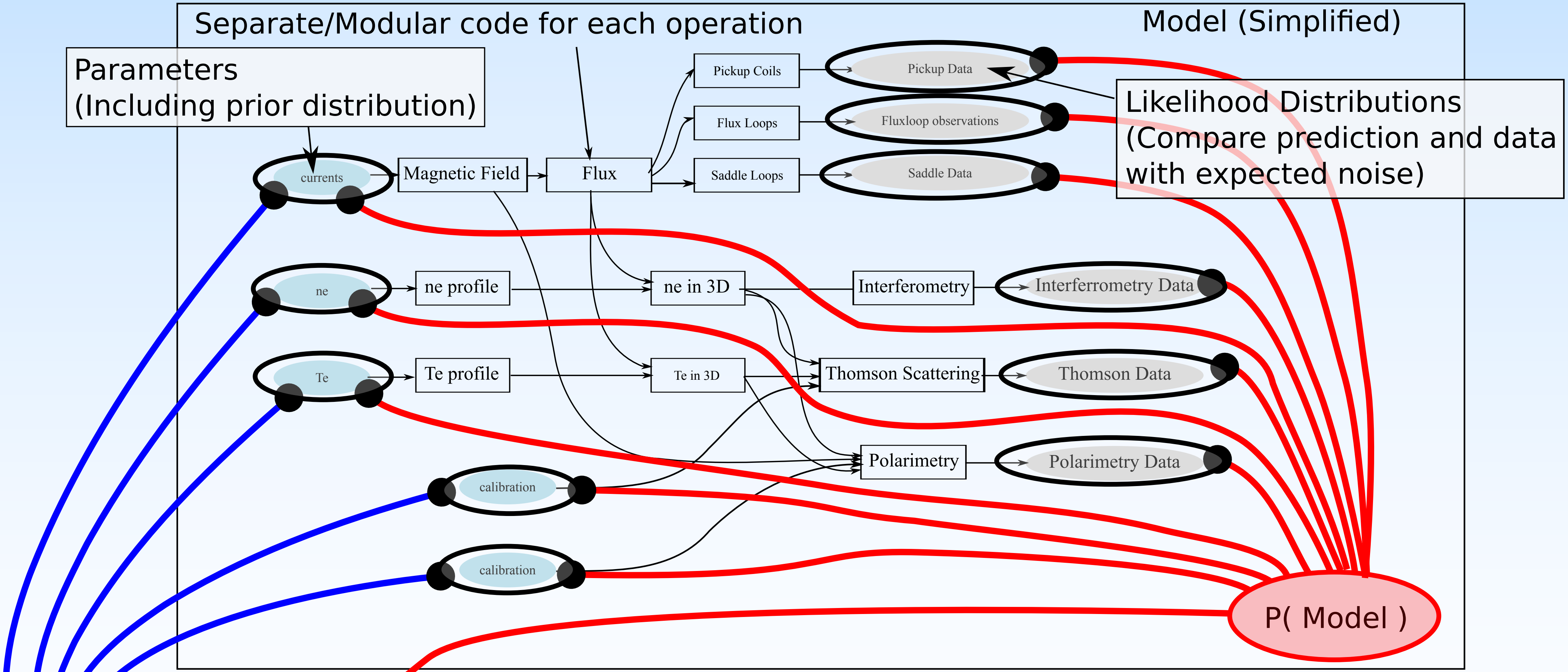
* See the Appendix of F. Romanelli et al., Fusion Energy Conference 2008 (Proc. 22nd Int. FEC Geneva) IAEA

# Forward Modelling and Bayesian Inference

The basic idea:




Bayes Theorem: P( Te, Ne, J | Data ) ~ P( D | Ne, Te, J ) P( Te, Ne, J)

Practically: Solve and explore using external algorithms:

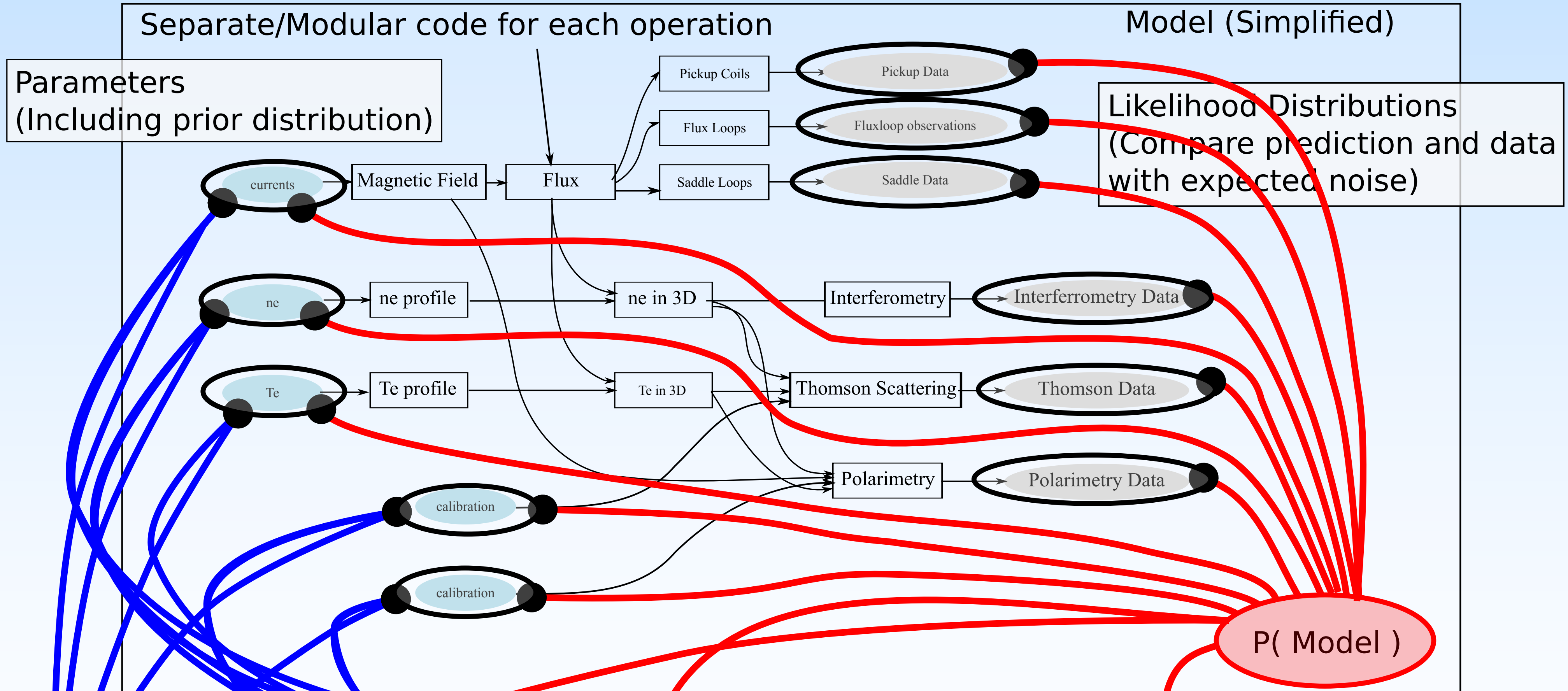
Linear Gaussian Solver
(Best fit and PDF
covariance)

Genetic Algorithms
(Non-linear best fit)

Metropolis Hastings
MCMC Non-linear Exploration:
--> Uncertainty

# The principals: Forward Modelling and Bayesian Inference

The basic idea:

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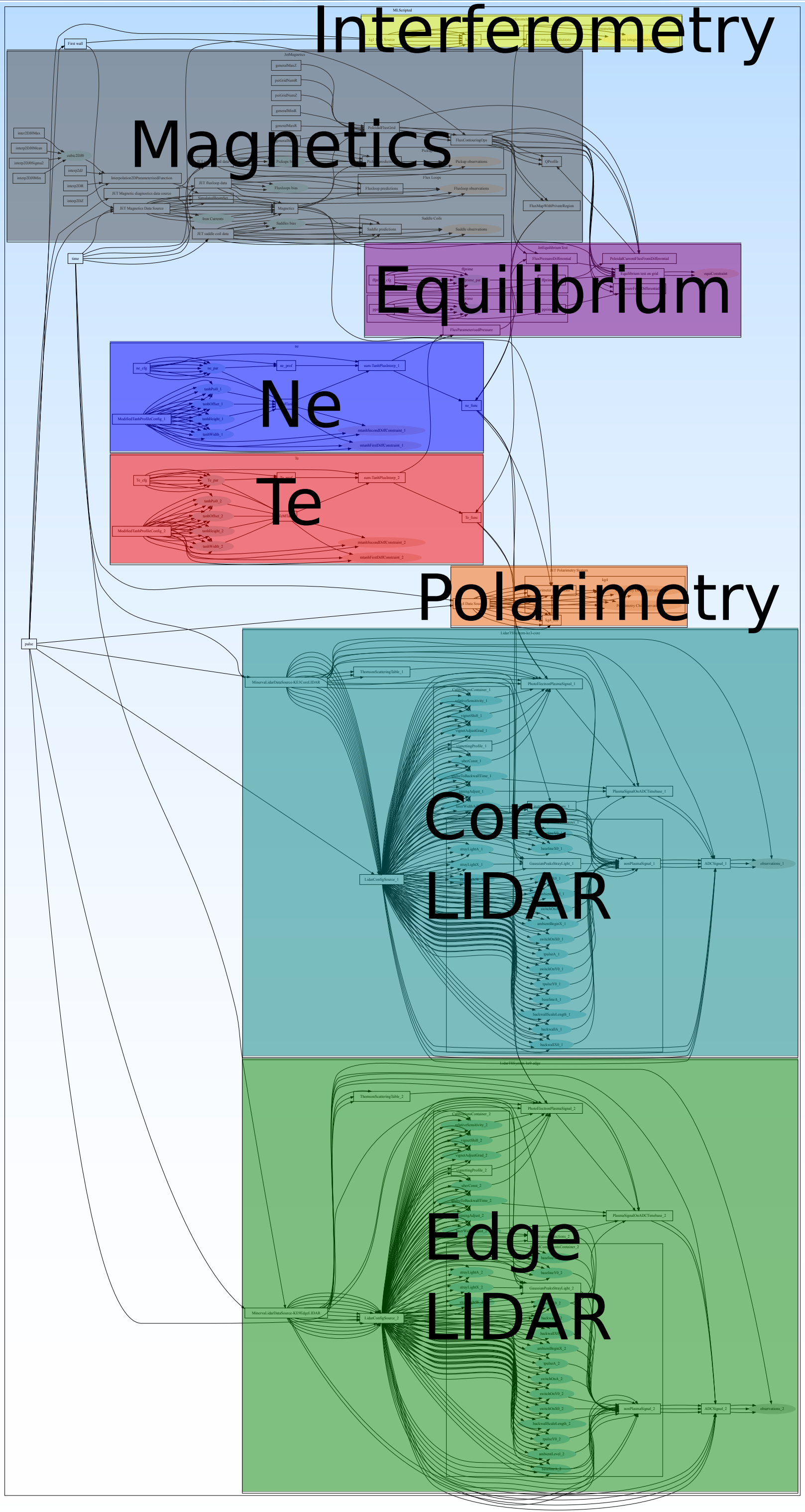



## Software and Models

Write nodes and wire them together.
Software framework handles the rest.
Even automatically generates the graphical representation.
We can re-wire the graph and redefine/modify the problem at will, even during a run.

Parts previously written:
- Magnetics (field/flux calculations and JET magnetics)
- Interferometry.

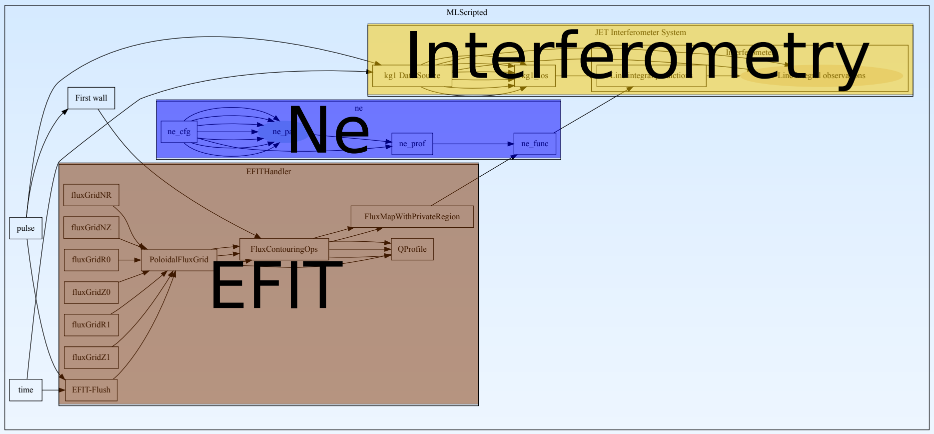
Parts I've written as part of my PhD:
- Polarimetry
- Core LIDAR
- Edge LIDAR
- Equilibrium (Grad-Shafranov Test)
- Various Ne/Te profile models.
- +(Parallelised and developed outer algorithms)

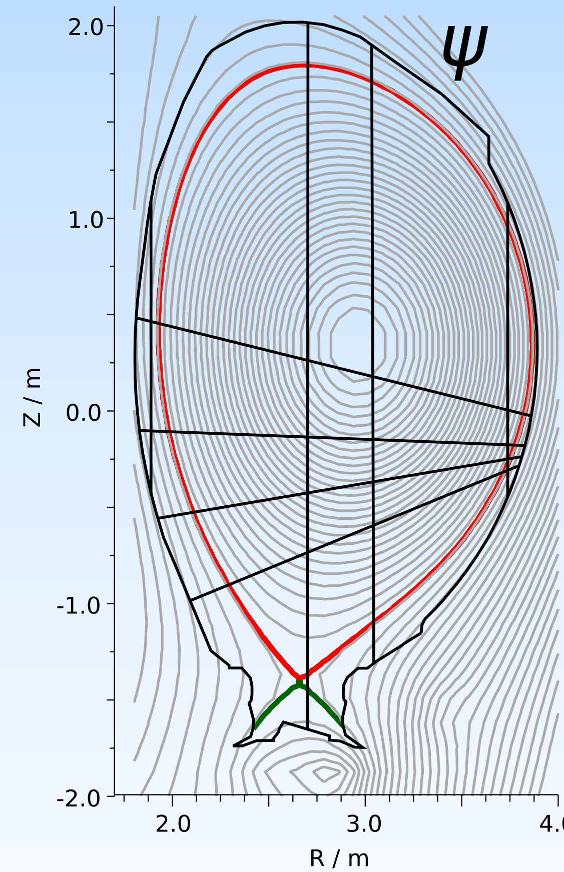
Other parts written during the past 3 years:
- JET MSE
- JET Reflectometry
- JET Infrared strikepoint camera
- MAST Magnetics
- MAST MSE
- MAST Thomson Scattering
- ... and a few others ...

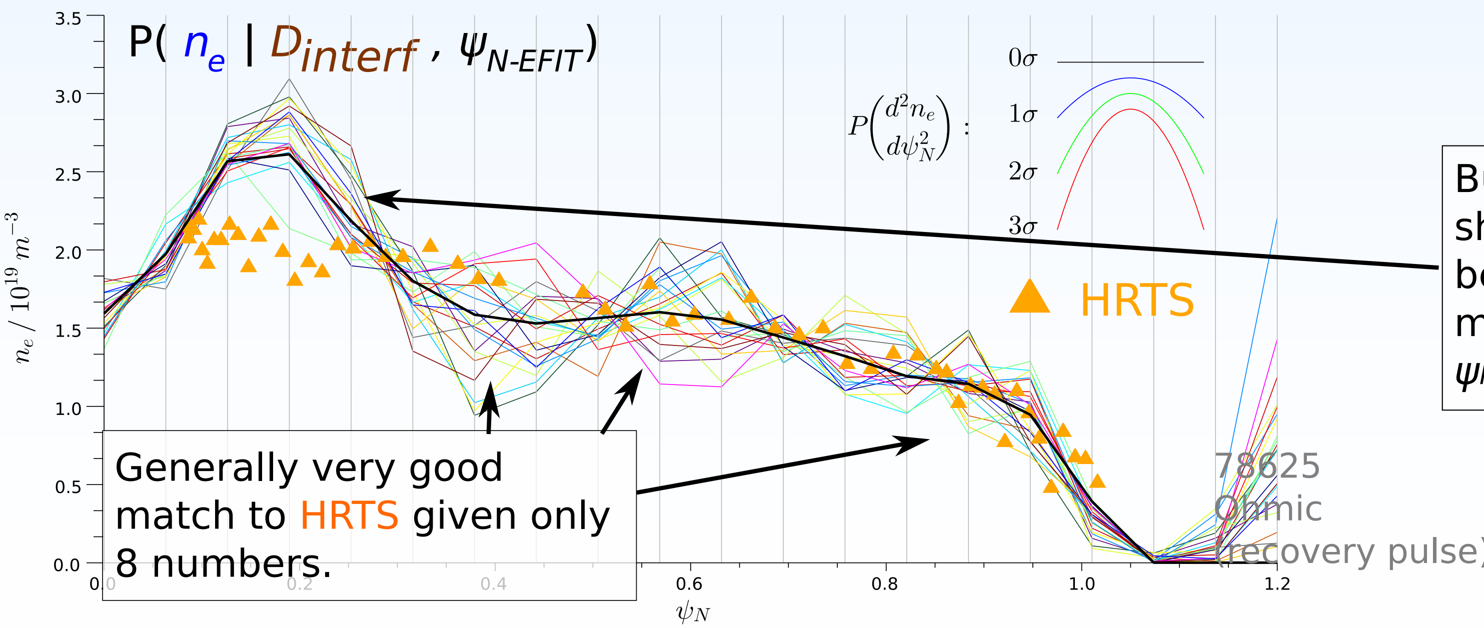
# Interferometry

A simple Bayesian + forward modelling practical demo:

We have 8 line integrated density measurements.
Assume $n_e(\psi_N)$ and invert to $n_e$ using weak smoothing prior based on magnetics only EFIT flux surfaces.



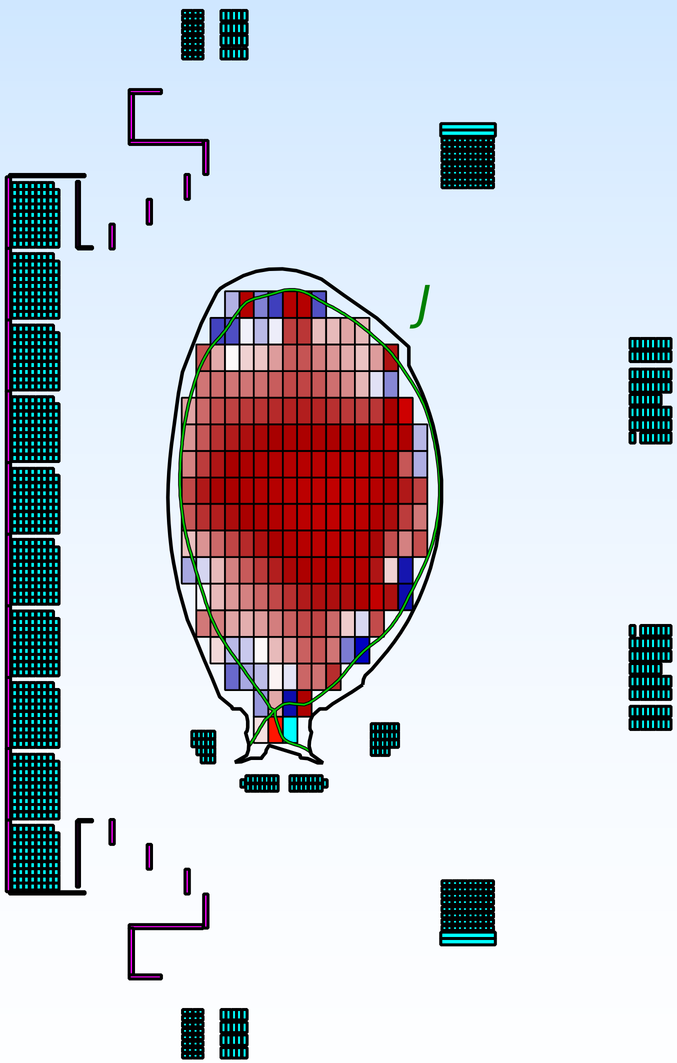



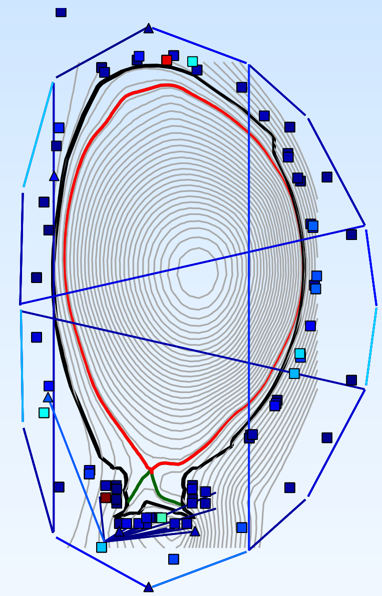

But, all possible profiles show structure we do not believe, so an assumption must be incorrect:
$\psi_N$ not perfect?

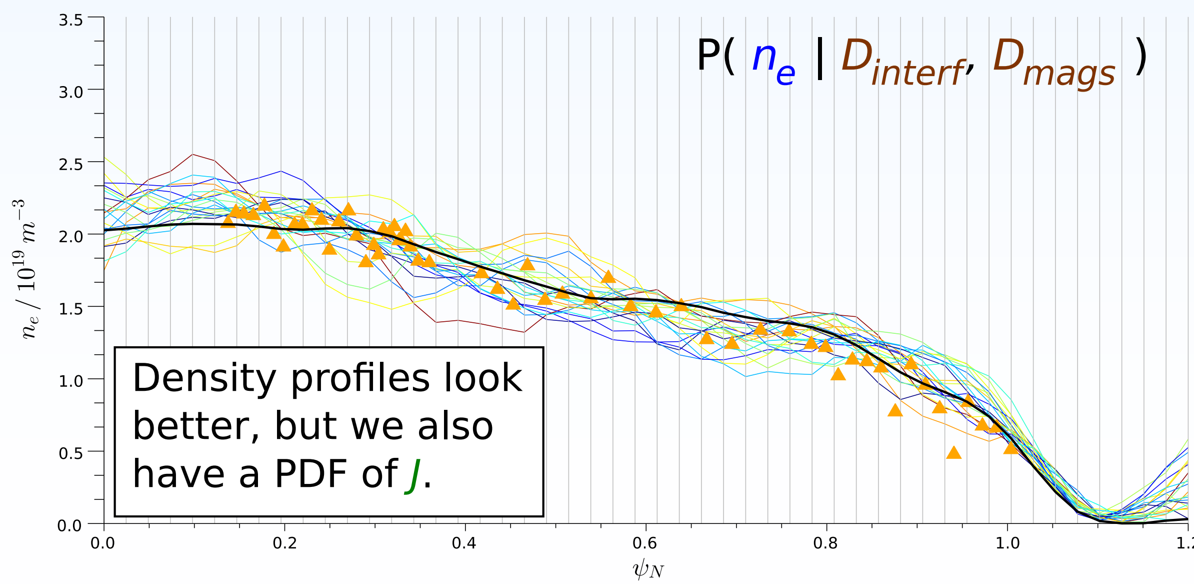
# Interferometry + Current Tomography I

Instead, calculate $\psi_N$ from toroidal currents $J$, include magnetics diagnostics and invert to full posterior:

i.e: Find combinations of $J$ and $n_e$ that are consistent with both interferometry and magnetics (and with $n_e$ and $J$ priors).



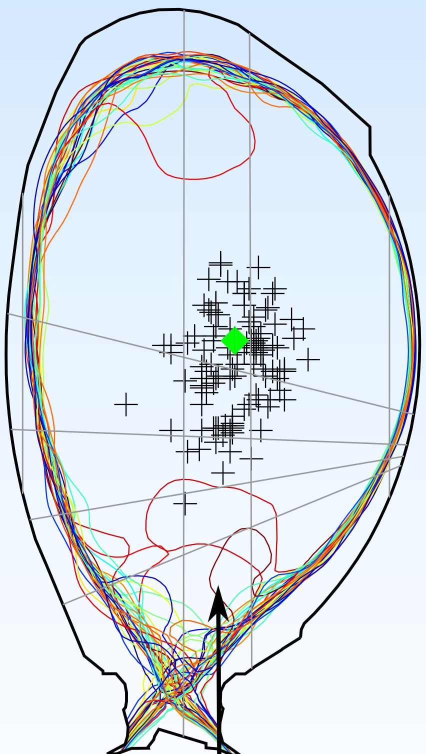





# Interferometry + Current Tomography II

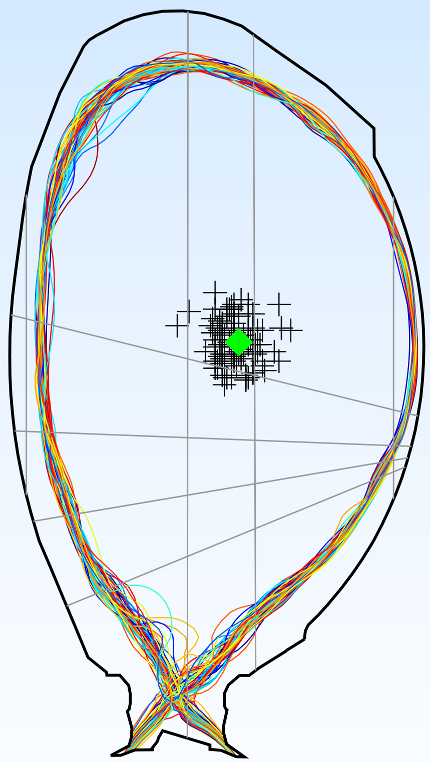
Each sample of the posterior contains a possible set of *J* given magnetics **and interferometry.**
Deliberatly using over-weak currents priors, that with only magnetics gives:

P( *J* | $D_{mags}$ )



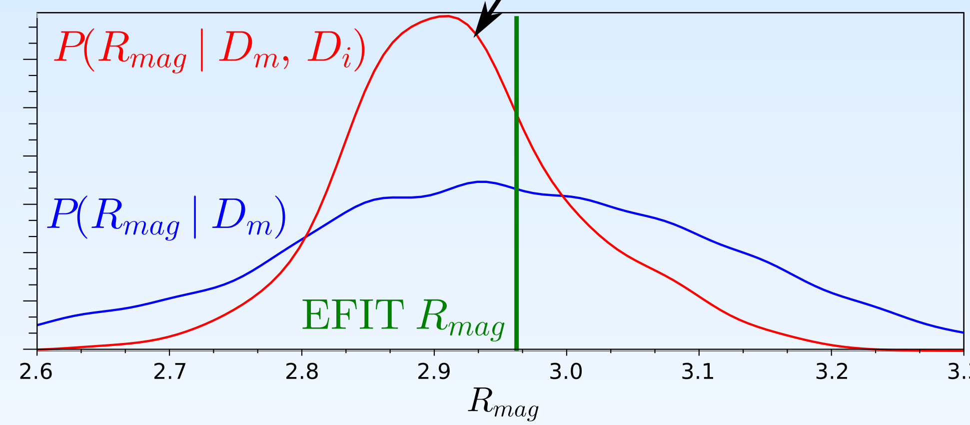
Magnetics
+Weak CAR prior

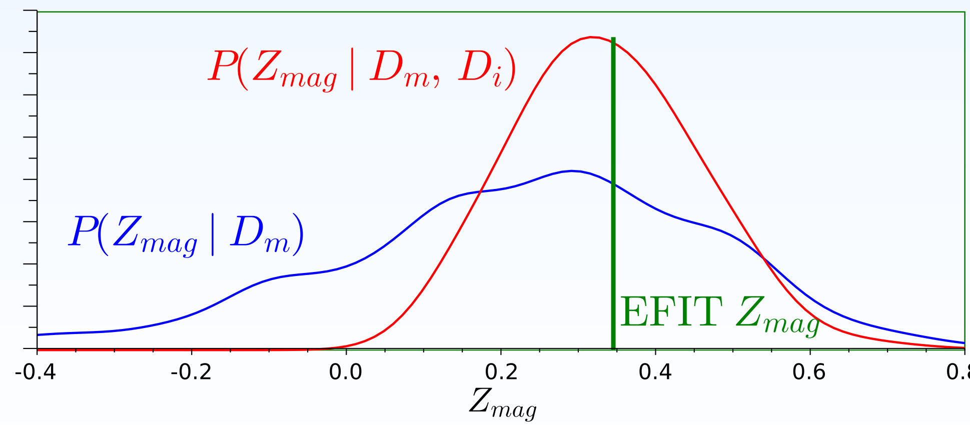
P( *J* | $D_{mags}$, $D_{interf}$ )



Magnetics + Weak CAR prior
+ Interferometry + Smooth *ne*

Less obviously, gives higher certainty magnetic axis

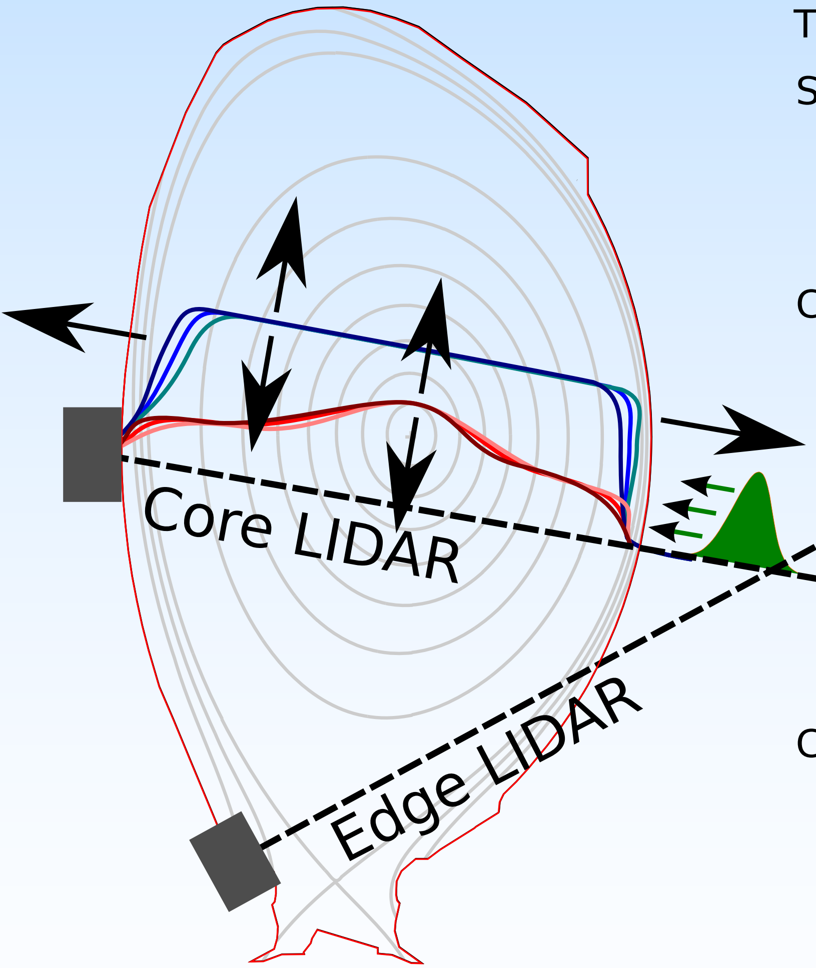



Interferometry combined with *ne* assumptions provides some information about plasma current:
i.e: Some currents give flux surfaces for which no *ne* profile can make interferometry data make sense.

# Core + Edge LIDAR I: The systems

Thomson Scattering diagnostics each using a single spectrometer set and time of flight for positioning.




TS physics well understood but hardware system very complex.

Spatial Resolution:

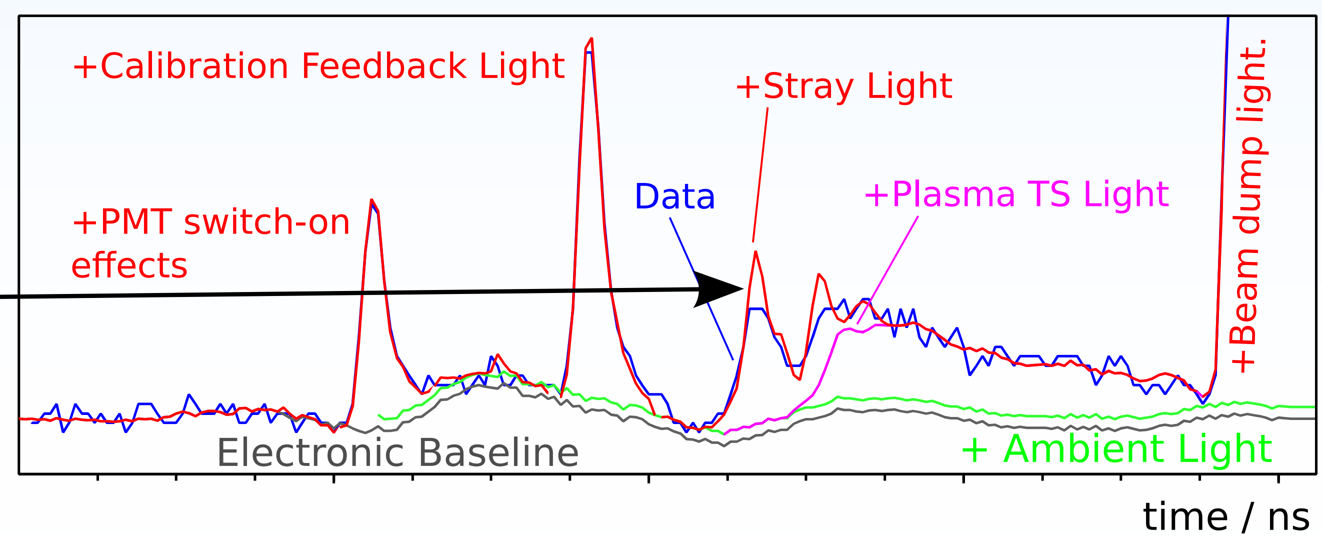
- Effective convolution of light signal.
- If ignored (chain1): Convolves $n_e$ but complex effect on $T_e$.
- No problem for forward modelling: we just convolve the signal.

Calibrations:

- Beam dump position + timing --> Uncertain position.
- Optical transmission + laser energy --> $n_e$ magnitude.
- Spectrometer Relative Sensitivities --> $T_e$ magnitude.
- Relative Channel timing --> $T_e$ + $n_e$ shape!

Created full detailed forward model including every part of the system:

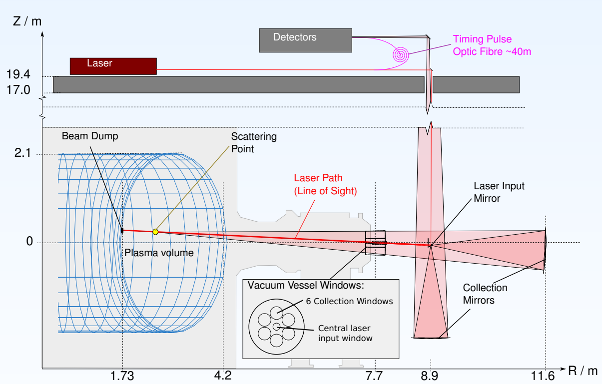
Stray light effects low signal (low $n_e$) data on both systems but is **vital** for proper edge LIDAR analysis.

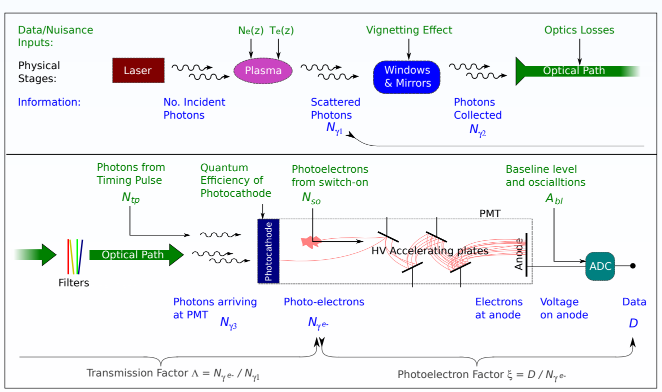


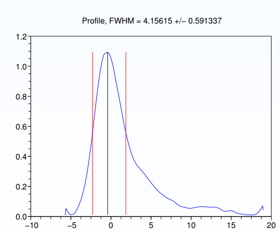
# Core + Edge LIDAR I: The model

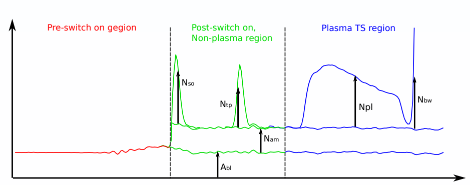
So how do we deal with disagreement with other diagnostics?
- Solution 1: Shift and scale output profiles to match...
  - Which diagnostic should we trust, can we remember which ones are reliable for which quantities.
  - What if calibrations effect profiles in complex ways?
- Solution 2: Build the model for each and wire up to minerva, tell it what we do know about the calibration parameters (the prior). and let it work out how to make everything consistent.

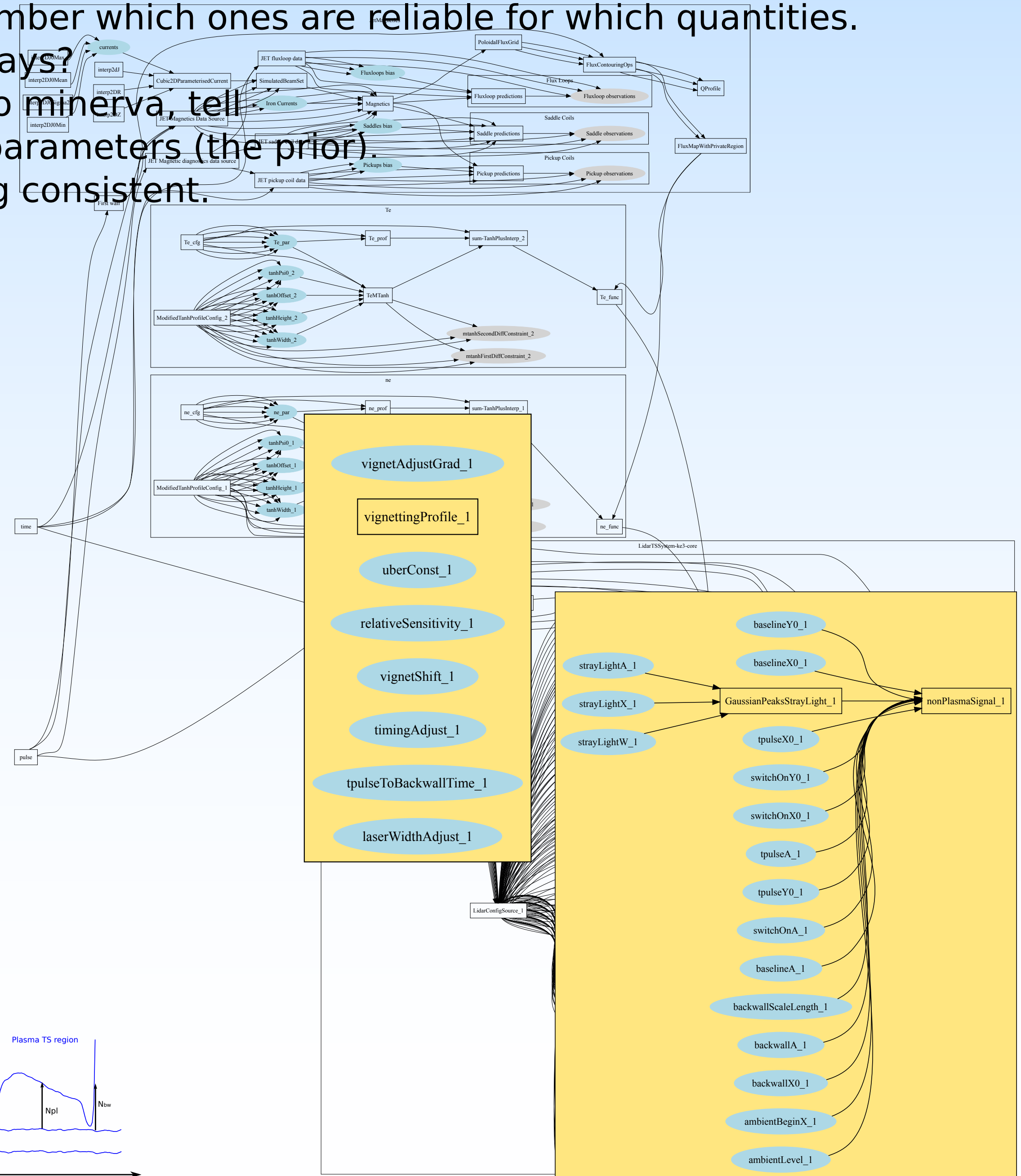
We must really understand how each part of the system works:
Laser Pulse, TS physics, Optics, Filters, Photomultipliers, Counting Noise (PDFs), ADCs.



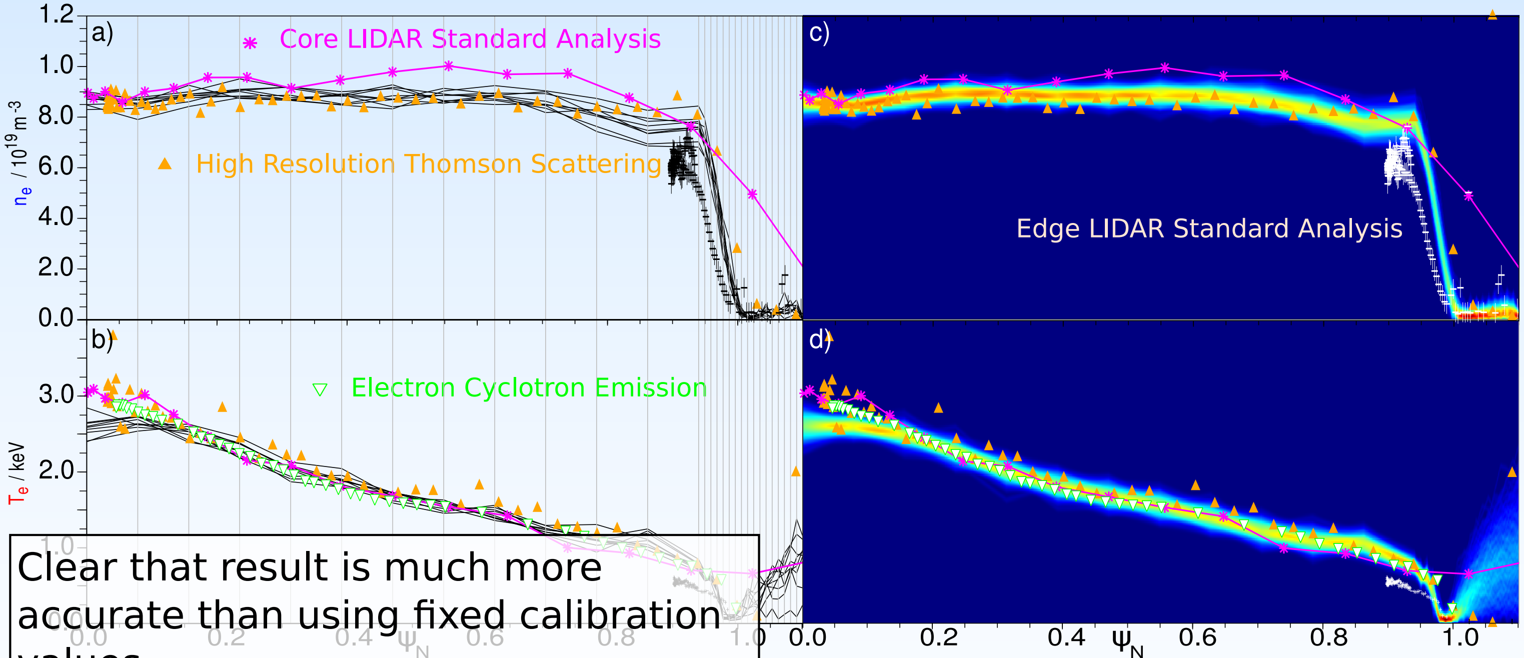



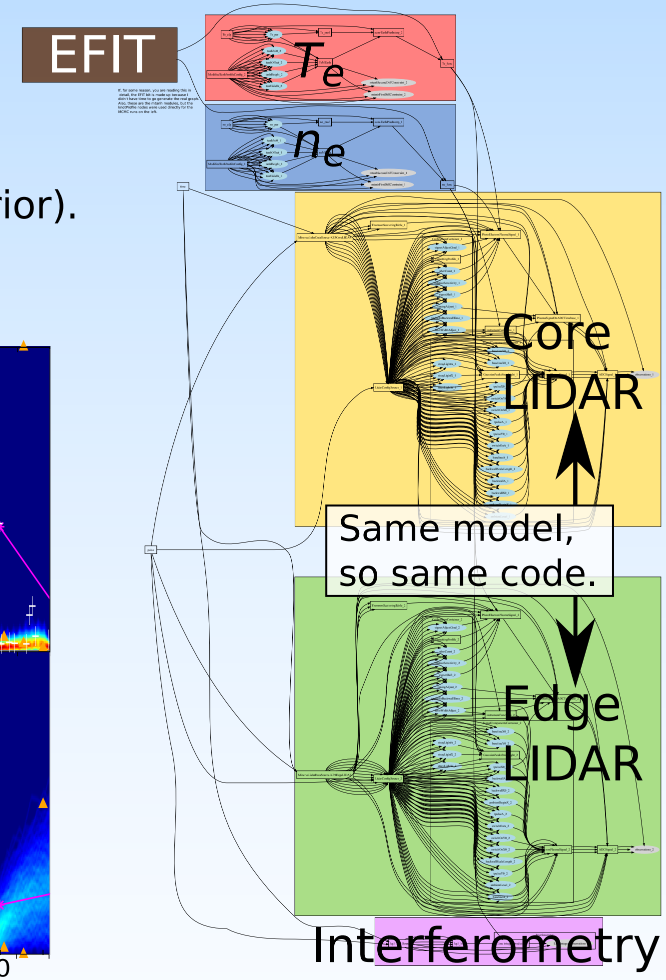



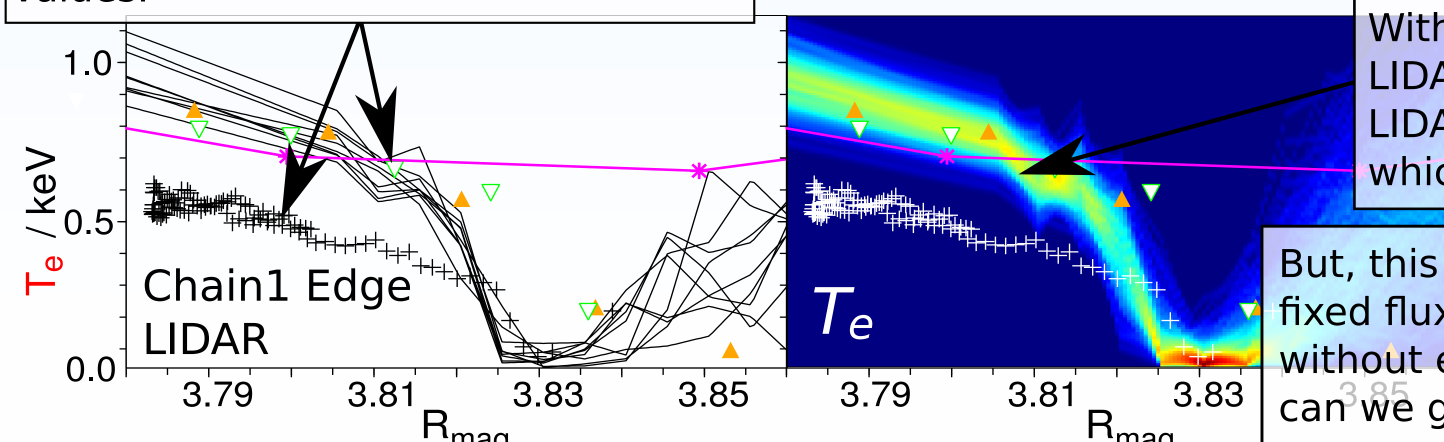
# Core + Edge LIDAR V: Add edge LIDAR.

A typical high density H-mode pulse:
- Connect up the model (Based on EFITs flux surfaces for the time being)
- Give all calibrations some uncertainty (what we believe).
- Give some less trusted calibrations almost complete freedom (uniform prior).
- Throw the complete problem at the GA for MAP (best fit) and then at the MCMC for distribution...




Clear that result is much more accurate than using fixed calibration values.




With completely free calibration, edge LIDAR provides shape with which Core LIDAR can give accurate Te ped height which feeds back to Edge LIDAR

But, this isn't complete - we are still using fixed flux surfaces. The Current tomography without equilibirum approach is useful but can we get more by assuming equilibrium...

# Equilbrium I

So mapping P( $\psi_N$ | ... ) is still the big problem.
Will try to explore using Current Tomography with CAR prior and all the diagnositcs (soon)

However, equilibrium condition may give enough constraint.

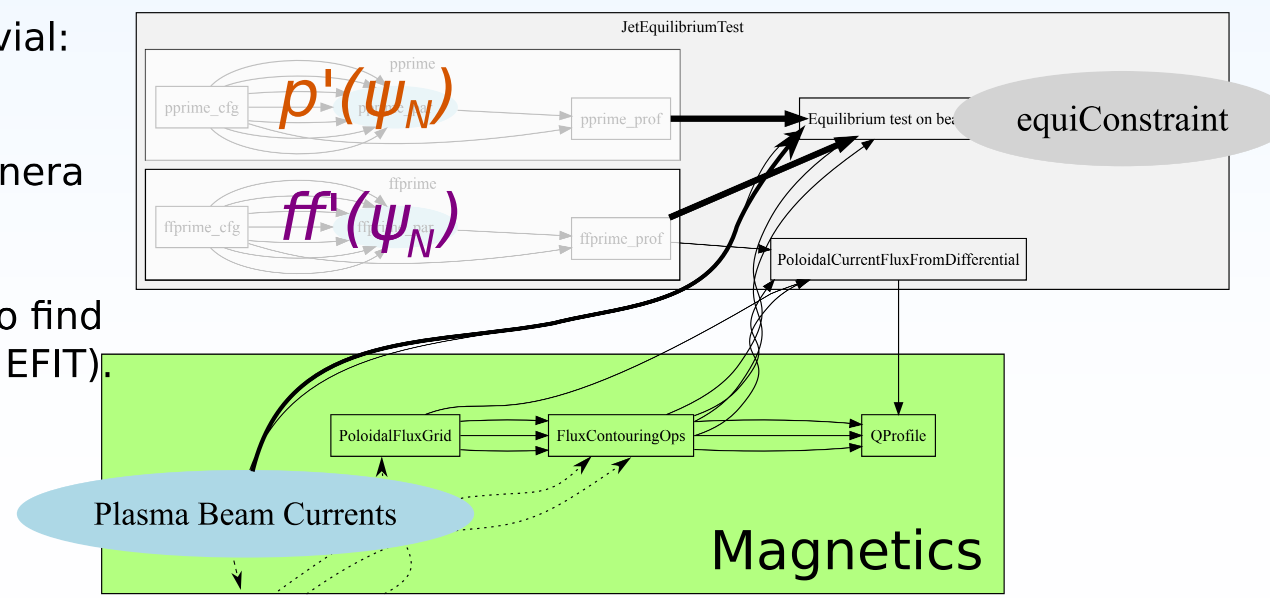
$$J_\phi = Rp' + \frac{\mu_0}{R} f f'$$

NB: It's not immediately clear how restrictive force balance (GS equation) actually is, since it is almost always used with strong prior constraints on *p*' (or *p* - the equilibirum pressure) and *ff*' (or *f* - the poloidal current flux). With weak (almost no) contraints on *p*' and *ff*', degeneracy of solutions is still huge.

Assume GS equality is, at least close to correct: assign a PDF on difference:
P( *J*, *p*', *ff*' ) = G( *J* - *Rp*' - *ff*'/*R*; 0, $\sigma_{GS}$) with relativly small $\sigma_{GS}$.

The posterior P( *J*, *p*', *ff*' | $D_{diags}$ + ~Equilibrium) should include all possible combinations of *J*, *p*' and *ff*' that are consistent with the diagnostics, the priors and describe a plasma very close to equilbrium.

Adding to model (and the code) is fairly trivial:




But, the problem is now very hard for the external algorithms to handle due to non-linera 1000D+ posterior.

1) Parallelise the linear solver and iterate to find MAP (much slower but more stable than EFIT).

2) Exporing the PDF only just possible (last week).

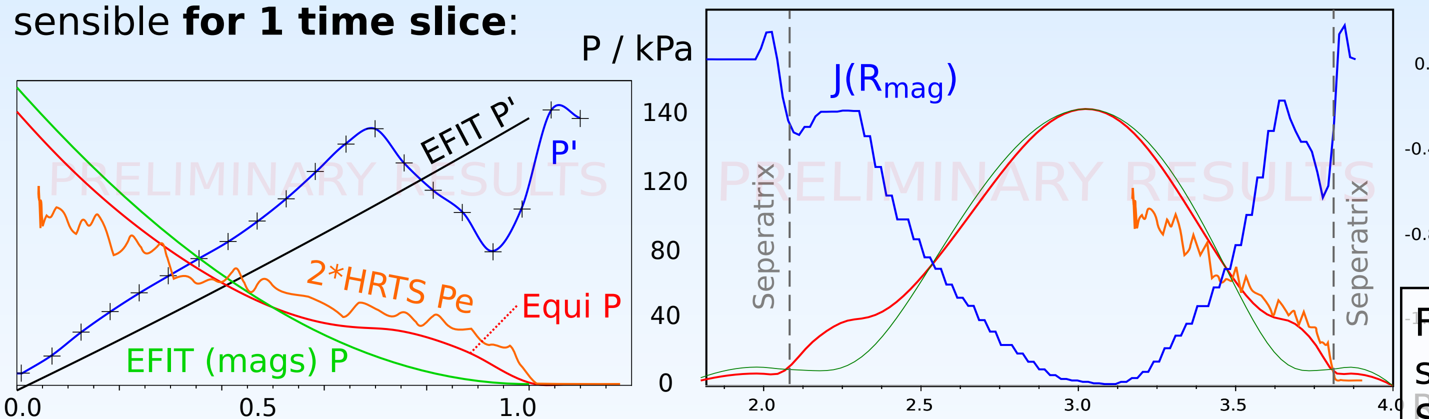
# Equilbrium II: Maximum Posterior (Magnetics Only)

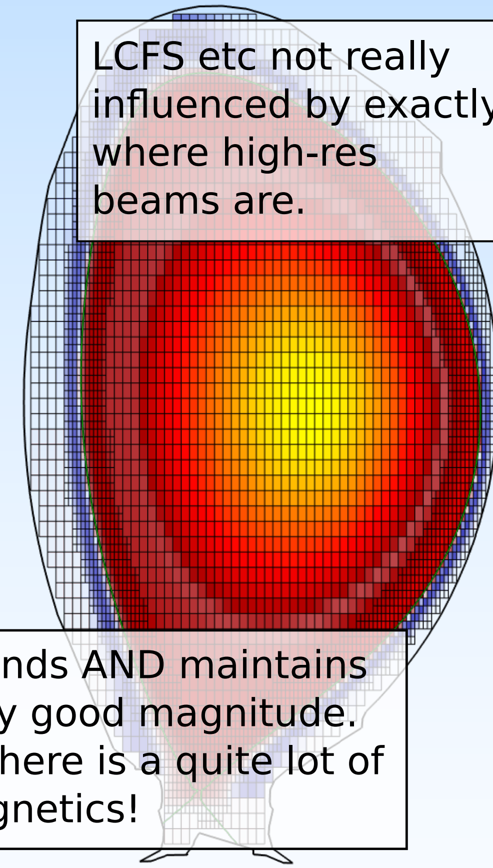
78601 High ne H-Mode (pellets)

Because of modularity, we can switch parametrisation and priors of *J*, *p*' and *ff*' at will and on-the-fly.
For H-Mode, fast changes at edge so:

- $J_{\phi}$: Current beams with higher resolution near edge (~**1cm**, ~5cm in core).
  No smoothing priors, just $J_{\phi}$ < 100MA m$^{-2}$.
- $p'(\psi_N)$, $ff'(\psi_N)$: 20 knots, weak smoothing priors.
  Fairly strong prior for small SOL p' and ff' (but not fixed)
  Has anyone measured $J_{SOL}$?

Clearly massively degenerate, so **adjust *p*' and *ff*' priors** to get something sensible **for 1 time slice**:

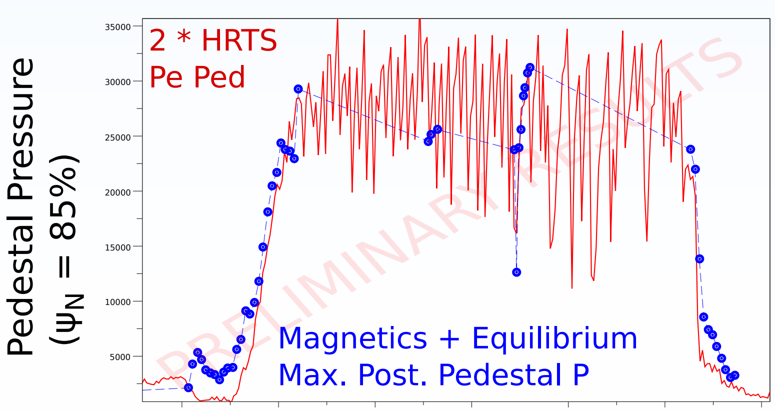


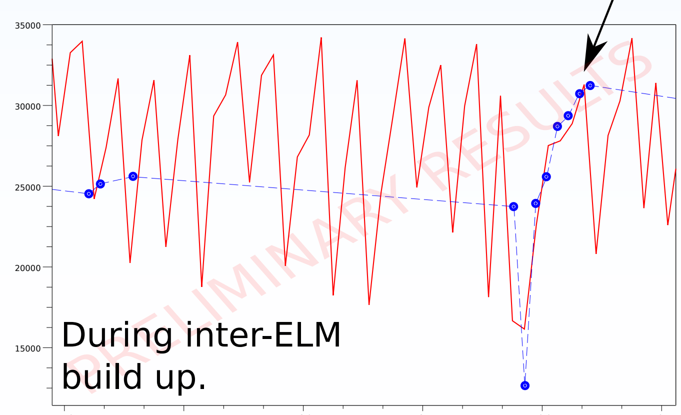



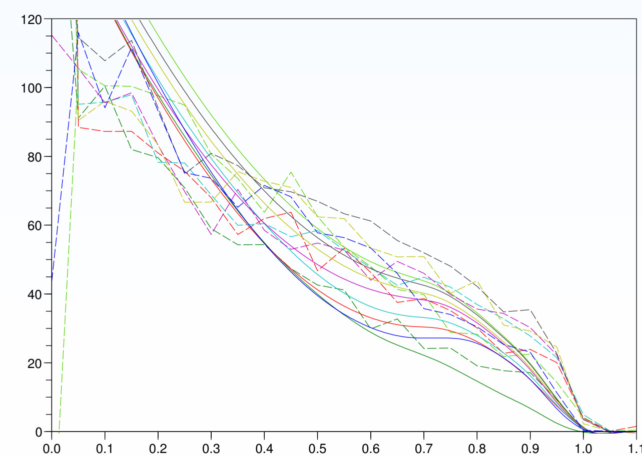
LCFS etc not really influenced by exactly where high-res beams are.

Follows trends AND maintains surprisingly good magnitude. Suggests there is a quite lot of info in magnetics!

Magnetics data seems to see edge current (and hence some *p*').
Exact magnitude you get does depend on priors.
But... **Hold priors** and run accross H-mode pulse. Is there any vague trend?



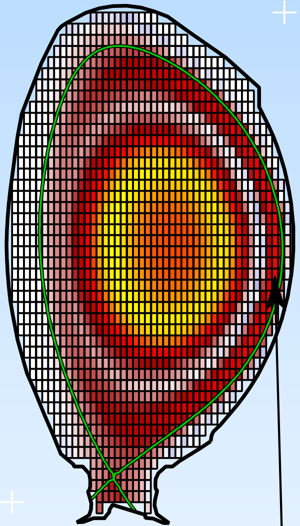



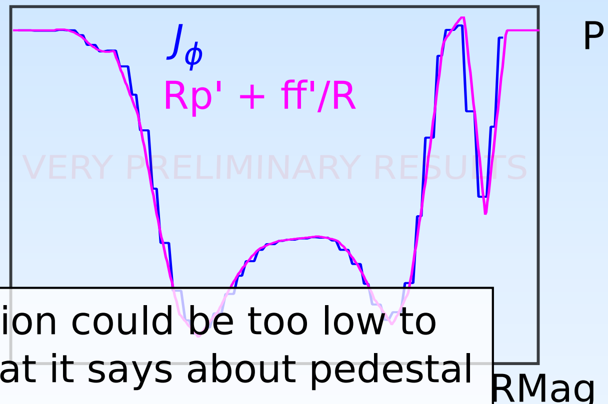



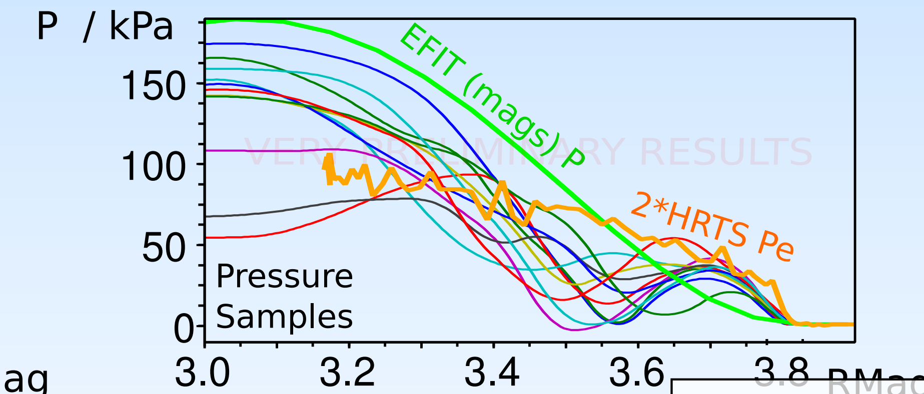
78601 High ne
H-Mode (pellets)

# Equilbrium III: Equilibria Exploration.

With the high-res beams, the posterior is 4732D. This is far too much for the MCMC algorithm (as it stands). So, for the moment, use a lower res (5cm beams).
Also, we need to allow a little more flexibility in GS difference (~1% of $J_\phi$) so it can explore.



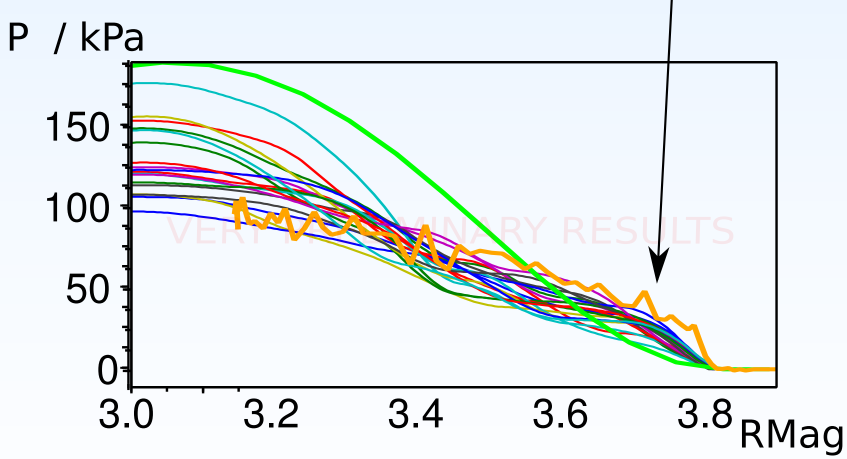


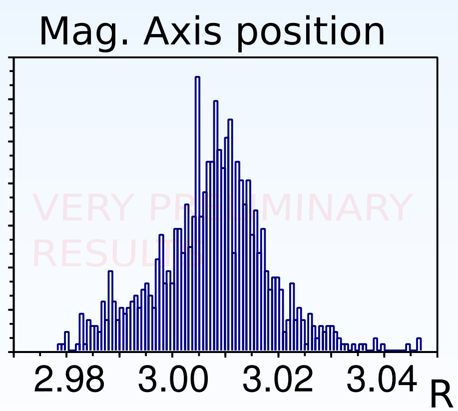



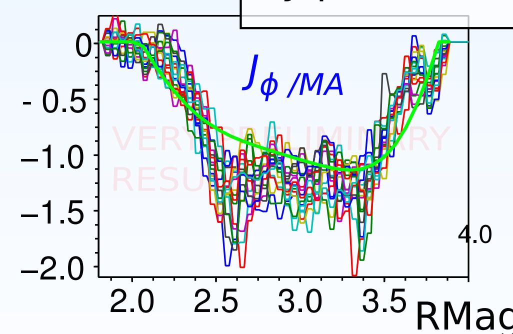

Beam resolution could be too low to really see what it says about pedestal

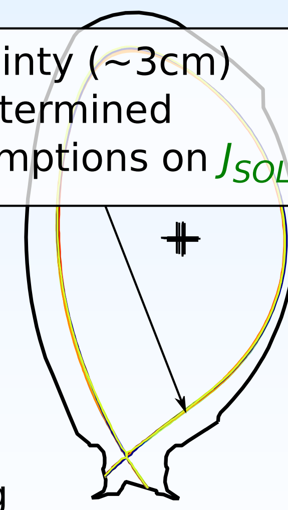
LCFS Uncertainty (~3cm)
- probably determined by prior assumptions on $J_{SOL}$

We can add a prior for monotonic P (-ve P'):



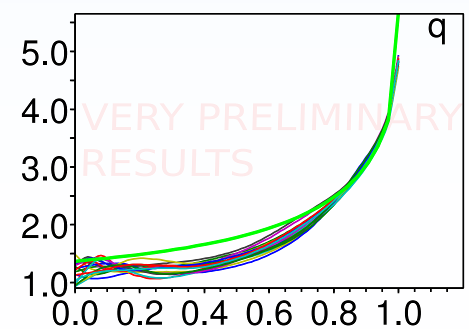



Of course, we can see how other diagnostics reduce this uncertainty, just by adding their forward model to the system and running it again.
This will be good for the obvious cases: MSE, Polarimetry etc, but maybe others too. e.g Interferometry and Edge LIDAR.

All of this still needs lots of investigating and validating...



# Conclusions so far and work to do...

✓ Developed full models for core and edge LIDAR and polarimetry, combined with existing magnetics and interferometry models.

✓ Used polarimetry model and lots of data to test theoretical models for relativistic polarimetry.
('O P Ford *et al* 2009 *Plasma Phys. Control. Fusion* **51** 065004' - In *IOP select* and PPCF highlights 2009).

✓ Have a framework for analysing diagnostics which not only can cope with mapping uncertainty, but also automatically feeds back information from diagnostic to make inference on the mapping (currents).

✓ Similarly, can deal with uncertain calibrations, no matter how complex the model, and then infer the calibration from the data or from consistency with other.

✓ Core+Edge LIDARs + Inteferometry give accurate $n_e$, $T_e$ profiles entirely independent of HRTS.

••• Need to look at what LIDARs + Interferometry can say about mapping/currents.

✓? *Appear* to be able to infer a surprising amount about the pedestal current/pressure from magnetics.

~✓ We can now (just) **explore** the PDF of possible equilibria - what can GS/force balance really tell us?

••• In the end (hopefully)....
P( $J$, $n_e$ , $T_e$ | Magnetics + Core LIDAR + Edge LIDAR + Interferometry + Polarimetry + Force Balance
+ MSE + Reflectometry + ECE + Strike Points)

••• Can we test pedestal scaling from edge LIDAR just with uncertain mapping (CT).
✓ [Have 7000 time points, type-I ELMy H-Mode, marked and clear of ELMS since Edge LIDAR upgrade C20-C27 ]

••• Do we get enough info to test current models at edge?