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Bayesian Diagnostic Analysis at JET:

Interferometry and Polarimetry

Oliver Ford

Thanks to:

Dr J. Svensson PhD Supervisor (JET) Dr A. Boboc Inteferometry/Polarimetry

JET Contributors



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Talk Outline

- Project Outline
- Bayesian / forward modelling principles.
- A simple example: Interferometry inversion.
- Polarimetry:
 - Basics
 - Complications
 - A side result: High Temperature Effects



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Integrated Analysis





Model for the basic physical quantities.



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Integrated Analysis





Model for the basic physical quantities.

Models for the prediction of other required quantities.



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Integrated Analysis



Model for the basic physical quantities.

Models for the prediction of other required quantities.

Complete model for each diagnostic system, predicting the data it reports. The *"Forward Function"*.



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Bayesian Inference

From probability theory... Bayes Theorem:

 $P(A \mid B) = P(B \mid A) P(A)$ P(B)



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Bayesian Inference

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$P(\mu \mid D) \propto P(D \mid \mu) P(\mu)$

• μ – All free parameters in the system: Physical states n_e , T_e and **j** as well as calibration parameters e.g. Lines of sight.



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- P(µ|D) Encapsulates our knowledge of the system Including all statistical and systematic erors, calibration uncertainty, correlations etc.
- Combining diagnostics: $P(n_e, T_e, j | All Data) =$

• $P(D_{Polarimetry} | n_e, T_e, j) \times P(D_{Interferrometry} | n_e) \times P(D_{Magnetics} | j) \times P(n_e, T_e, j)$



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Interferometry Inversion I



As an example of Bayesian Methods

Physical Parameters: Density parameterised as 1D linear interpolation of 30 nodes over Ψ_{N} .



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Forward Function: Linear combination of the 30 nodes

$$f_i\left({{{{
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Weights determined by surfaces and lines of sight.



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• Likelihood: Each channel is Gaussian around $f(n_p)$ with $\sigma_d = 3 \times 10^{17} m^{-2}$.



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- Prior: Density must be positive: $P(\underline{\mathbf{n}_e}) = 0$ for $\mathbf{n}_e < 0$. For positive n_e , use a Gaussian centered at 0 with $\sigma_p = 2 \times 10^{20} \text{ m}^{-3}$ to suggest each node should be in JETs operating regeime.



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- Posterior: Rewrite likelihood and prior as multivariate Gaussians:

$$P\left(\underline{\mathbf{n}_{e}} \mid \underline{\mathbf{D}}\right) \propto \begin{cases} \mathcal{G}\left(\underline{\mathbf{D}}; \ \underline{\mathbf{W}} \ \underline{\mathbf{n}_{e}}, \ \underline{\boldsymbol{\sigma}_{\mathbf{D}}}\right) \mathcal{G}\left(\underline{\mathbf{n}_{e}}; \ \underline{\mathbf{0}}, \ \underline{\boldsymbol{\sigma}_{\mathbf{P}}}\right) & \text{for all } n_{e_{j}} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



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Interferometry Inversion II

• Multiply Gaussians and rewrite as $f(n_{p})$:



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$$P\left(\underline{\mathbf{n}}_{\underline{\mathbf{e}}} \middle| \underline{\mathbf{D}}\right) \propto \mathcal{G}\left(\underline{\mathbf{n}}_{\underline{\mathbf{e}}}; \ \underline{\mathbf{n}}_{\underline{\mathbf{e}}0}, \underline{\underline{\sigma}}_{\underline{\mathbf{n}}_{\underline{\mathbf{e}}}}\right) \text{ for all } n_{e_{j}} \ge 0 \qquad \text{Truncated} \\ \text{with} \quad \underline{\mathbf{n}}_{\underline{\mathbf{e}}0} = \underline{\underline{\sigma}}_{\underline{\mathbf{n}}_{\underline{\mathbf{e}}}} \underline{\underline{\mathbf{W}}}^{T} \underline{\underline{\sigma}}_{\underline{\mathbf{D}}}^{-1} \underline{\underline{\mathbf{D}}} \\ \underline{\underline{\sigma}}_{\underline{\mathbf{n}}_{\underline{\mathbf{e}}}} = \left[\underline{\underline{\mathbf{W}}}^{T} \underline{\underline{\sigma}}_{\underline{\mathbf{D}}}^{-1} \underline{\underline{\mathbf{W}}} + \underline{\underline{\sigma}}_{\underline{\mathbf{P}}}^{-1}\right]^{-1} \qquad \text{Gaussian.}$$

• Single (non-trivial) matrix inversion gives full posterior for density profile.



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Interferometry Inversion II

• Multiply Gaussians and rewrite as $f(n_{p})$:

$$P\left(\underline{\mathbf{n}_{e}} \mid \underline{\mathbf{D}}\right) \propto \mathcal{G}\left(\underline{\mathbf{n}_{e}}; \ \underline{\mathbf{n}_{e0}}, \underline{\underline{\sigma}_{n_{e}}}\right) \text{ for all } n_{e_{j}} \geq 0$$

$$\text{with} \quad \underline{\mathbf{n}_{e0}} = \quad \underline{\underline{\sigma}_{n_{e}}} \underline{\underline{W}}^{T} \underline{\underline{\sigma}_{D}}^{-1} \underline{\underline{\mathbf{D}}}$$

$$\underline{\underline{\sigma}_{n_{e}}} = \quad \left[\underline{\underline{W}}^{T} \underline{\underline{\sigma}_{D}}^{-1} \underline{\underline{W}} + \underline{\underline{\sigma}_{P}}^{-1}\right]^{-1}$$

$$\text{Truncated multivariate Gaussian.}$$

- Single (non-trivial) matrix inversion gives full posterior for density profile.
- To examine the posterior, show maximum posterior





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with $\underline{\mathbf{n}_{e0}} = \underline{\underline{\sigma}_{n_{e}}} \underline{\underline{\mathbf{W}}}^{T} \underline{\underline{\sigma}_{D}}^{-1} \underline{\underline{\mathbf{D}}}$
 $\underline{\underline{\sigma}_{n_{e}}} = \left[\underline{\underline{\mathbf{W}}}^{T} \underline{\underline{\sigma}_{D}}^{-1} \underline{\underline{\mathbf{W}}} + \underline{\underline{\sigma}_{P}}^{-1}\right]^{-1}$

Truncated multivariate Gaussian.

- Single (non-trivial) matrix inversion gives full posterior for density profile.
- To examine the posterior, show maximum posterior and draw samples to represent uncertainty.
- Looks messy! but this is representing what the interferometry alone says about the n profile. Any oscillating profile that does not change the line integral

doesn't change the likelihood so is equally likely as a smooth one.





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Truncated multivariate Gaussian.

- Single (non-trivial) matrix inversion gives full posterior for density profile.
- To examine the posterior, show maximum posterior and draw samples to represent uncertainty.
- Looks messy! but this is representing what the interferometry alone says about the n_a

n_{e (j+1)}

profile. Any oscillating profile that does not change the line integral doesn't change the likelihood so is equally likely as a smooth one. Posterior is very narrow and aligned to these oscillations.





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Interferometry Inversion III

 If it is believed it should be smoother, this is *prior knowledge*, e.g Impose low probability for large difference between neighbouring nodes:

$$P(\underline{\mathrm{N}_{\mathrm{e}}}) \propto \prod_{j=1}^{N-1} \mathcal{G}\left(rac{\left(N_{e_{(j+1)}} - N_{e_{j}}
ight)}{\Delta \psi}; \ 0, \ \sigma_{\frac{dN_{e}}{d\psi}}
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 Drawing 300 samples and plotting sample density for each node gives marginal for each node:





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 Drawing 300 samples and plotting sample density for each node gives marginal for each node:



- Can now see density is better known where integration is more heavily weighted, i.e where the lines of sight are almost tangential to surfaces.
- This 'looks nicer', but the former posterior better represents what is actually <u>known</u>.
- Many non-physical profiles are given a high probability in the former but the 'real' profile will also be given a high probability.
- With the extra prior, sharp changes in the real profile will mean its given a low probability.



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Polarimetry: Basic Principles

• Polarisation state described by rotation ψ and ellipticity tan χ .





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- Effect of plasma usually split into two cases:
- **B** // **z**: Faraday rotation:

$$\Delta \psi \propto \int B_{\parallel} n_e \, dz$$

 $\chi = 0$



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Polarimetry: Basic Principles

• Polarisation state described by rotation ψ and ellipticity tan χ .



- Effect of plasma usually split into two cases:
- **B** // **z**: Faraday rotation:

B₁**z**: Cotton-Mouton effect:

 $\Delta \psi \propto \int B_{\parallel} n_e \, dz \qquad \qquad \psi = 45^{\circ} \text{ to } B$ $\chi = 0 \qquad \qquad \chi \propto \int B_{\perp}^2 n_e \, dz$

• For **B** at an arbitary angle θ to **z** and with large effects, these are not valid!



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Polarimetry: Full Model

[1]: S. E. Segre, Plasma Phys. Controlled Fusion 41, R57-R100 (1999)

Better model^[1] describes polarisation as stokes vector <u>s</u>:





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$$\frac{d\underline{s}}{dz} = \underline{\Omega} \times \underline{s}$$
 \leftarrow Integrate along line of sight.



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 \leftarrow Integrate along line of sight.

Cold plasma model (Te < 5 keV):

$$\underline{\Omega} = \frac{\omega_p^2}{2c\omega^3 \left(1 - \omega_c^2/\omega^2\right)} \begin{bmatrix} \left(e/m\right)^2 & \left(B_x^2 - B_y^2\right) \\ \left(e/m\right)^2 & 2B_x B_y \\ 2\omega \left(e/m\right) & B_z \end{bmatrix}$$



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- Typically one effect is minimised to avoid this, e.g. lateral channels.
- Forward modelling and Bayesian approaches allow maximum use of both effects.



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- Forward modelling and Bayesian approaches allow maximum use of both effects.
- To test model: Perform interferometry inversion and draw 300 sample n_e profiles. Predict ψ and χ for each sample using **B** from EFIT.
- This gives $P(\psi \mid D, B)$ and $P(\chi \mid D, B)$ what can be inferred about what ψ and χ 'should be', given only the interferometry data (and EFIT B).



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Polarimetry: Comparison I (Channel 5)

Compare these with measured values (at 500ms intervals for 1313 pulses)... Channel 5:





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Polarimetry: Comparison I (Channel 5)

Compare these with measured values (at 500ms intervals for 1313 pulses)... Channel 5: Good agreement for both ψ and χ .



Can only make predictions for this channel with the full model. χ is heavily dependent on ψ – provides extra information on Bpol.







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Polarimetry: Comparison II (Channel 3)

Channel 3 (core) shows a systematic disgreement on ψ during H-mode phase. Suspected due to inaccurate magnetic axis position from EFIT. σ_{16}







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Polarimetry: Comparison II (Channel 3)

Channel 3 (core) shows a systematic disgreement on ψ during H-mode phase. Suspected due to inaccurate magnetic axis position from EFIT.





Systematic disgreement at very high n_{e} . - Calibration Suspected!!







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Polarimetry: High Te Effects

- For Te > \sim 5keV, the cold plasma model (Ω_{0}) is not sufficient...
- High velocity electrons see Doppler shifted incident wave.
 - Calculate polarisation evolution Ω in terms of dielectric tensor ε .
 - Take $\boldsymbol{\varepsilon}$ from integration of Vlasov equation over Maxwellian.
 - Expand in T_a and take linear terms [2].
 - Leads to correction from the cold plasma of:

$$\underline{\Omega_1} \approx \underline{\Omega_0} + \frac{T_e}{mc^2} \begin{bmatrix} 12 & \Omega_{0_1} \\ 12 & \Omega_{0_2} \\ 3 & \Omega_{0_2} \end{bmatrix}$$

- But high velocity electrons also have relativistic mass increase.
 - Take $\boldsymbol{\varepsilon}$ from integration of relativistic Vlasov equation over relativistic Maxwellian [3].
 - Now have more terms linear in T_a but in the opposite direction:

$$\underline{\Omega_2} \approx \underline{\Omega_0} + \frac{T_e}{mc^2} \begin{bmatrix} 4\frac{1}{2} & \Omega_{0_1} \\ 4\frac{1}{2} & \Omega_{0_2} \\ -2 & \Omega_{0_2} \end{bmatrix}$$

[2]: S. E. Segre et al, Phys. Plasmas 9 2919 (2002)[3]: V. V. Mirnov, Phys. Plasmas 14 102105 (2007)



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Polarimetry: High Te Effects

- Channel 3 passes through very core of plasma where Te can be up to 10KeV.
- For all but very high n_{a} , χ of channel 3 works.
- Evaluate full model based on interferometry invesion for cold model and both corrections.
- Pulses with Te > 8keV and where calibration has worked well show clearly better agreement with Ω_2 :





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Polarimetry: High Te Effects



- In general, calibration and noise uncertainty are bigger than difference between the models. To prove it we must look at lots of data...
- 300 pulses with Te > 5keV.
- Very slight systematic underestimate by $\Omega_0^{}$ and overestimate by $\Omega_1^{}$. $\Omega_2^{}$ looks straighter in comparison.



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Work In Progress

- Understand/fix polarimetry calibration!
- Develop probabalistic model for the polarimetry detectors.
- Combine magnetics, interferometry and polarimetry to simultaneously find $P(n_{e}, j \mid All Data)$
- Include LIDAR, Edge LIDAR (already built) and HRTS models to include Te.



. . .

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